BC Calculus Assignment 10.3

Parametric vectors and physics

1. Writing parametric functions in vector form is like writing a point, but with *x, y* _______________ in place of __________________.

2. To find the derivative of a parametric function in vector form, differentiate the $x(t)$ and $y(t)$ functions ___________________.

3. To find the speed of a particle, you use the ________________ ________________ with the two components of the velocity vector.

4. Arc length is found by evaluating the ___________________ of speed.

bold *numbers - calculator* ***permitted***

5. If f is a vector-valued function defined by $f(t) = (e^{-2t}, -\sin(3t))$, write the vector-valued function for $f''(t)$.

6. At time $t \geq 0$, a particle moving in the xy-plane has a velocity vector given by $v(t) = \langle \cos(4t), e^{4t} \rangle$. What is the acceleration vector of the particle?

7. A particle moves on a plane curve so that at any time $t > 0$ its x-coordinate is $-t^3 - t$ and its y-coordinate is $(2t - 3)^3$. The acceleration vector of the particle at $t = 2$ is

8. For time $t > 0$, the position of a particle moving in the xy-plane is given by the parametric equations $x = 2t + 3t^2$ and $y = \frac{1}{5t-6}$. What is the acceleration vector of the particle at time $t = 1$?

9. The position of a particle moving in the xy-plane is given by the vector $\langle 8t^3, y(4t)\rangle$, where y is a twice-differentiable function of t. At time $t = \frac{1}{2}$, what is the acceleration vector of the particle?

(A) $\langle 6, 4y''(2)\rangle$

(B) $\langle 12, 16y''(2)\rangle$

(C) $\langle 24, 4y''(2)\rangle$

(D) $\langle 24, 16y''(2)\rangle$

10. The velocity vector of a particle moving in the xy plane has components given by $\frac{dx}{dt} = \cos(t^2)$ and $\frac{dy}{dt} = e^{\sin t}$. At time $t = 4$, the position of the particle is (1, 3). What is the y-coordinate of the position vector at time $t = 2$?

11. A particle moves in the xy-plane so that its position at any time t is given by $x(t) = t^3$ and $y(t) = \cos(4t)$. What is the speed of the particle when $t = 2$?

12. The position of a particle moving in the xy-plane is given by the parametric equations $x(t) = \sin(3^t)$ and $y(t) = \cos(3^t)$ for time $t \geq 0$. What is the speed of the particle when $t = 1.4$?

13. The position of an object moving along a path in the xy plane is given by the parametric equations $x(t) = 5\cos(\pi t)$ and $y(t) = (2t + 3)^2$. The speed of the particle at $t = -1$ is

14. The position of a particle moving in the xy-plane is given by the parametric equations $x = t^3 - 6t^2$ and $y = 2t^3 - 6t^2 - 48t$. For what value(s) of t is the particle at rest?

15. The position of a particle moving in the xy-plane is given by the parametric equations $x(t) = t^3 + 3t^2$ and $y(t) = 12t + 3t^2$. At what point (x, y) is the particle at rest?

16. The length of the path described by the parametric equations $x = \cos^4 t$ and $y = \sin^3 t$, for $0 \le t \le \frac{\pi}{2}$, is given by

(A) $\int_0^{\frac{\pi}{2}} \sqrt{\cos^8 t + \sin^6 t}\, dt$

(B) $\int_0^{\frac{\pi}{2}} \sqrt{-4\cos^3 t \sin^2 t + 3\sin^2 t \cos t}\, dt$

(C) $\int_0^{\frac{\pi}{2}} \sqrt{16\cos^6 t \sin^2 t + 9\sin^4 t \cos^2 t}\, dt$

(D) $\int_0^{\frac{\pi}{2}} \sqrt{16\cos^6 t + 9\sin^4 t}\, dt$

(E) $\int_0^{\frac{\pi}{2}} \sqrt{4\cos^3 t + 3\sin^2 t}\, dt$

17. Write an integral that gives the length of the path described by the parametric equations $x = \frac{1}{5}t^5$ and $y = t^2$, where $0 \le t \le 1$.

18. Write an integral that gives the length of the path described by the parametric equations $x(t) = 17 - 2t$ and $y(t) = 2t^2 + 11$ from $t = 2$ to $t = 3$.

Free Response - calculator ***permitted***

An object moving along a curve in the *xy*-plane is at position $(x(t), y(t))$ at time t, where

$$\frac{dx}{dt} = \sin^{-1}(1 - 2e^{-t}) \text{ and } \frac{dy}{dt} = \frac{4t}{1 + t^3}$$

for $t \geq 0$. At time $t = 2$, the object is at the point $(6, -3)$. (Note: $\sin^{-1}x = \arcsin x$)

(a) Find the acceleration vector and the speed of the object at time $t = 2$.
1: acceleration
1: speed

(b) The curve has a vertical tangent line at one point. At what time t is the object at this point?
1: condition for vertical tangent
1: answer

(c) Let $m(t)$ denote the slope of the line tangent to the curve at the point $(x(t), y(t))$. Write an expression for $m(t)$ in terms of t and use it to evaluate $\lim_{t \to \infty} m(t)$.

1: m(t)
1: value

(d) The graph of the curve has a horizontal asymptote $y = c$. Write, but do not evaluate, an expression involving an improper integral that represents this value c.
1: integrand
1: bounds
1: initial value consistent with lower bound

2. An object moving along a curve in the xy-plane has position $(x(t), y(t))$ at time $t \geq 0$ with $\frac{dx}{dt} = 3 + \cos(t^2)$. The derivative $\frac{dy}{dt}$ is not explicitly given. At time $t = 2$, the object is at position (1, 8).

(a) Find the x-coordinate of the position of the object at time $t = 4$.
1: integral
1: handles initial conditions
1: answer

(b) At time $t = 2$, the value of $\frac{dy}{dt}$ is -7. Write an equation for the line tangent to the curve at the point $(x(2), y(2))$.
1: finds $\frac{dy}{dx}$ at $x = 2$
1: tangent line equation

(c) Find the speed of the object at time $t = 2$.

1: answer

(d) For $t \geq 3$, the line tangent to the curve at $(x(t), y(t))$ has a slope of $2t + 1$. Find the acceleration vector of the object at time $t = 4$.

1: x''(4)

1: $\frac{dy}{dt}$

1: answer