## **BC Calculus Assignment 10.3**

Parametric vectors and physics

1. Writing parametric functions in vector form is like writing a po	int, but with x, y in
place of	
2. To find the derivative of a parametric function in vector form, o	differentiate the $x(t)$ and $y(t)$ functions
3. To find the speed of a particle, you use the	with the two
components of the velocity vector.	
4. Arc length is found by evaluating the	of speed.
<b>bold</b> numbers - calculator <b>peri</b>	mitted
<b>5.</b> If f is a vector-valued function defined by $f(t) = (e^{-2t}, -\sin(3t))$ , vector-valued function defined by $f(t) = (e^{-2t}, -\sin(3t))$ , vector-valued function defined by $f(t) = (e^{-2t}, -\sin(3t))$ , vector-valued function defined by $f(t) = (e^{-2t}, -\sin(3t))$ , vector-valued function defined by $f(t) = (e^{-2t}, -\sin(3t))$ , vector-valued function defined by $f(t) = (e^{-2t}, -\sin(3t))$ .	write the vector-valued function for $f''(t)$ .
6. At time $t \ge 0$ , a particle moving in the <i>xy</i> -plane has a velocity v	vector given by $v(t) = \langle \cos(4t), e^{4t} \rangle$ . What
is the acceleration vector of the particle?	

7. A particle moves on a plane curve so that at any time t > 0 its x-coordinate is  $-t^3 - t$  and its y-coordinate is  $(2t - 3)^3$ . The acceleration vector of the particle at t = 2 is

8. For time t > 0, the position of a particle moving in the *xy*-plane is given by the parametric equations  $x = 2t + 3t^2$  and  $y = \frac{1}{5t - 6}$ . What is the acceleration vector of the particle at time t = 1?

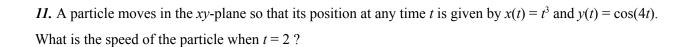
9. The position of a particle moving in the *xy*-plane is given by the vector  $\langle 8t^3, y(4t) \rangle$ , where *y* is a twice-differentiable function of *t*. At time  $t = \frac{1}{2}$ , what is the acceleration vector of the particle?

(A) 
$$\langle 6, 4y''(2) \rangle$$

(B) 
$$\langle 12, 16y"(2) \rangle$$

(C) 
$$\langle 24, 4y"(2) \rangle$$

10. The velocity vector of a particle moving in the xy plane has components given by  $\frac{dx}{dt} = \cos(t^2)$  and  $\frac{dy}{dt} = e^{\sin t}$ . At time t = 4, the position of the particle is (1, 3). What is the y-coordinate of the position vector at time t = 2?



12. The position of a particle moving in the *xy*-plane is given by the parametric equations  $x(t) = \sin(3^t)$  and  $y(t) = \cos(3^t)$  for time  $t \ge 0$ . What is the speed of the particle when t = 1.4?

13. The position of an object moving along a path in the xy plane is given by the parametric equations  $x(t) = 5\cos(\pi t)$  and  $y(t) = (2t + 3)^2$ . The speed of the particle at t = -1 is

14. The position of a particle moving in the *xy*-plane is given by the parametric equations  $x = t^3 - 6t^2$  and  $y = 2t^3 - 6t^2 - 48t$ . For what value(s) of *t* is the particle at rest?

15. The position of a particle moving in the xy-plane is given by the parametric equations  $x(t) = t^3 + 3t^2$ and  $y(t) = 12t + 3t^2$ . At what point (x, y) is the particle at rest?

16. The length of the path described by the parametric equations  $x = \cos^4 t$  and  $y = \sin^3 t$ , for  $0 \le t \le \frac{\pi}{2}$ , is given by

$$(A) \int_{0}^{\frac{\pi}{2}} \sqrt{\cos^8 t + \sin^6 t} dt$$

(B) 
$$\int_{0}^{\frac{\pi}{2}} \sqrt{-4\cos^{3}t \sin^{2}t + 3\sin^{2}t \cos t} dt$$
(C) 
$$\int_{0}^{\frac{\pi}{2}} \sqrt{16\cos^{6}t \sin^{2}t + 9\sin^{4}t \cos^{2}t} dt$$

(C) 
$$\int_{0}^{\frac{\pi}{2}} \sqrt{16\cos^{6}t \sin^{2}t + 9\sin^{4}t \cos^{2}t} dt$$

(D) 
$$\int_{0}^{\frac{\pi}{2}} \sqrt{16\cos^{6}t + 9\sin^{4}t} dt$$
(E) 
$$\int_{0}^{\frac{\pi}{2}} \sqrt{4\cos^{3}t + 3\sin^{2}t} dt$$

(E) 
$$\int_{0}^{\frac{\pi}{2}} \sqrt{4\cos^3 t + 3\sin^2 t} \, dt$$

17. Write an integral that gives the length of the path described by the parametric equations  $x = \frac{1}{5}t^5$  and  $y = t^2$ , where  $0 \le t \le 1$ .

18. Write an integral that gives the length of the path described by the parametric equations x(t) = 17 - 2tand  $y(t) = 2t^2 + 11$  from t = 2 to t = 3.

## Free Response - calculator permitted

An object moving along a curve in the xy-plane is at position (x(t), y(t)) at time t, where

$$\frac{dx}{dt} = \sin^{-1}(1 - 2e^{-t})$$
 and  $\frac{dy}{dt} = \frac{4t}{1 + t^3}$ 

for  $t \ge 0$ . At time t = 2, the object is at the point (6, -3). (Note:  $\sin^{-1}x = \arcsin x$ )

(a) Find the acceleration vector and the speed of the object at time t = 2.

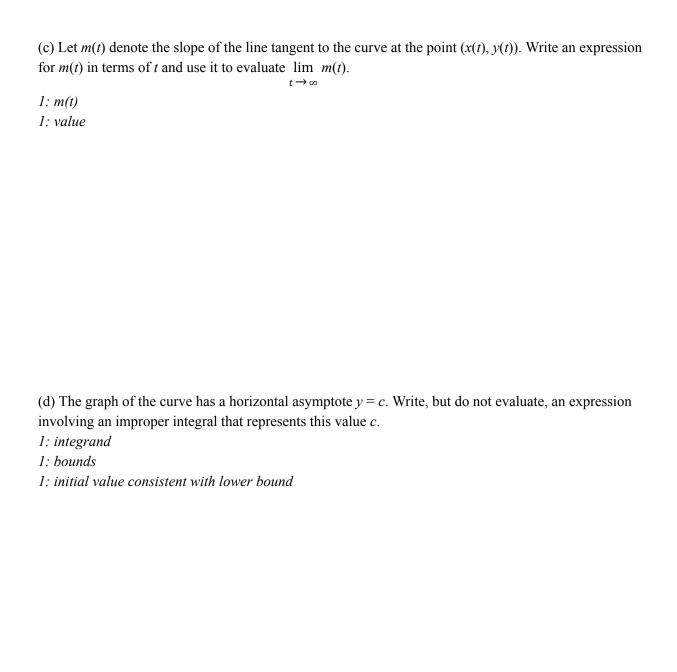
1: acceleration

1: speed

(b) The curve has a vertical tangent line at one point. At what time *t* is the object at this point?

1: condition for vertical tangent

1: answer



2. An object moving along a curve in the *xy*-plane has position (x(t), y(t)) at time  $t \ge 0$  with  $\frac{dx}{dt} = 3 + \cos(t^2)$ . The derivative  $\frac{dy}{dt}$  is not explicitly given. At time t = 2, the object is at position (1, 8).

- (a) Find the *x*-coordinate of the position of the object at time t = 4.
- 1: integral
- 1: handles initial conditions
- 1: answer

(b) At time t = 2, the value of  $\frac{dy}{dt}$  is -7. Write an equation for the line tangent to the curve at the point (x(2), y(2)).

1: finds 
$$\frac{dy}{dx}$$
 at  $x = 2$ 

1: tangent line equation

