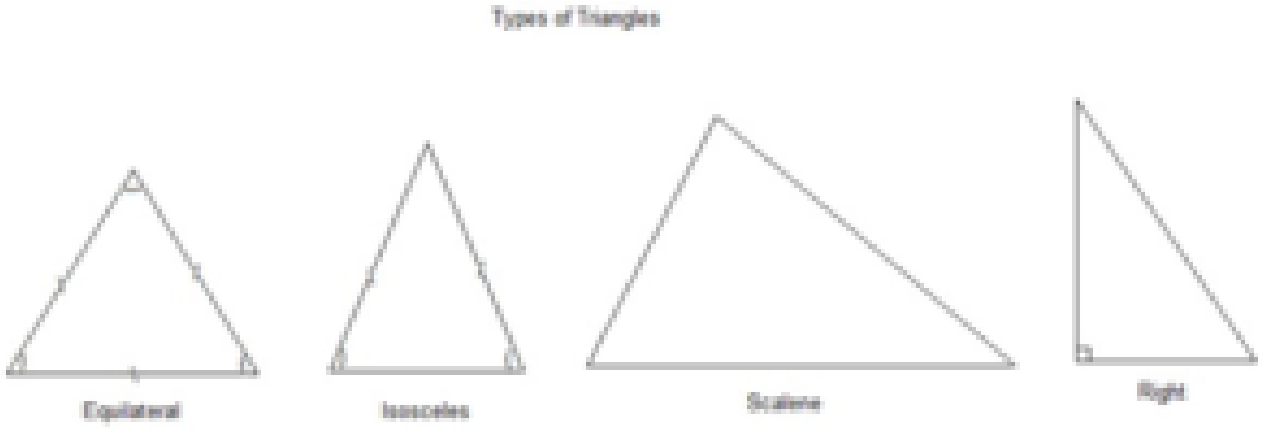
See this page in the course material.

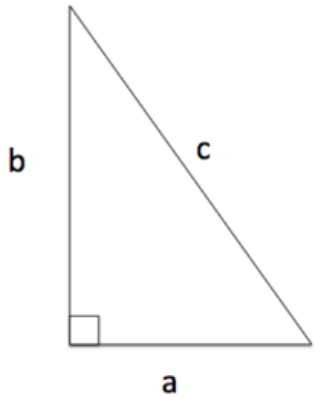
Learning Outcomes

- Solve application problems involving quadratic equations
- Recognize a right triangle from other types of triangles
- Use the Pythagorean theorem to find the lengths of a right triangle



Triangles

The **Pythagorean theorem,** or **Pythagoras's theorem,** is a statement about the sides of a right triangle. One of the angles of a right triangle is always equal to [latex]90[/latex] degrees. This angle is the right angle. The two sides next to the right angle are called the legs and the other side is called the hypotenuse. The hypotenuse is the side opposite to the right angle, and it is always the longest side. The image above shows four common kinds of triangle, including a right triangle.



Right Triangle with Sides Labeled

The Pythagorean theorem is often used to find unknown lengths of the sides of right triangles. If the longest leg of a right triangle is labeled c, and the other two a, and b as in the image on the left, The Pythagorean Theorem states that

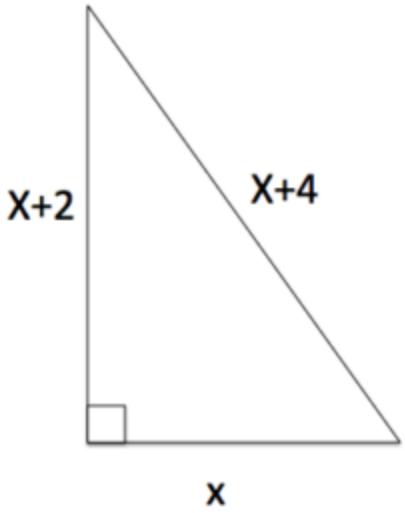
[latex]a^2+b^2=c^2[/latex]

Given enough information, we can solve for an unknown length. This relationship has been used for many, many years for things such as celestial navigation and early civil engineering projects. We now have digital GPS and survey equipment that have been programmed to do the calculations for us.

In the next example we will combine the power of the Pythagorean theorem and what we know about solving quadratic equations to find unknown lengths of right triangles.

Example

A right triangle has one leg with length x, another whose length is greater by two, and the length of the hypotenuse is greater by four. Find the lengths of the sides of the triangle. Use the image below.



Show Solution

Read and understand: We know the lengths of all the sides of a triangle in terms of one side. We also know that the Pythagorean theorem will give us a relationship between the side lengths of a right triangle.

Translate:

[latex]\begin{array}{l}a^2+b^2=c^2\\x^2+\left(x+2\right)^2=\left(x+4\right)^2\end{array}[/latex]

Solve: To solve this equation, we need to start by simplifying the equation and moving all the terms to one side. If we can factor it, then we can use the zero product principle to solve.

First, multiply the binomials and simplify so we can see what we are working with.

[latex]\begin{array}{l}x^2+\left(x+2\right)^2=\left(x+4\right)^2\\x^2+x^2+4x+4=x^2+8x+16\\2x^2+4x+4=x^2+8x+16\end{array}[/latex]

Now move all the terms to one side and see if we can factor.

[latex]\begin{array}{l}2x^2+4x+4=x^2+8x+16\\\underline{-x^2}\,\,\,\underline{-8x}\,\,\,\underline{-1

6}\,\,\,\,\,\underline{-x^2}\,\,\,\underline{-8x}\,\,\,\underline{-16}\\x^2-4x-12=0\end{array}[/latex]

This went from a messy looking problem to something promising. We can factor using the shortcut:

[latex]-6\cdot{2}=-12,\text{ and }-6+2=-4[/latex]

So we can build our binomial factors with [latex]-6[/latex] and [latex]2[/latex]:

[latex]\left(x-6\right)\left(x+2\right)=0[/latex]

Set each factor equal to zero:

[latex]x-6=0, x=6[/latex]

[latex]x+2=0, x=-2[/latex]

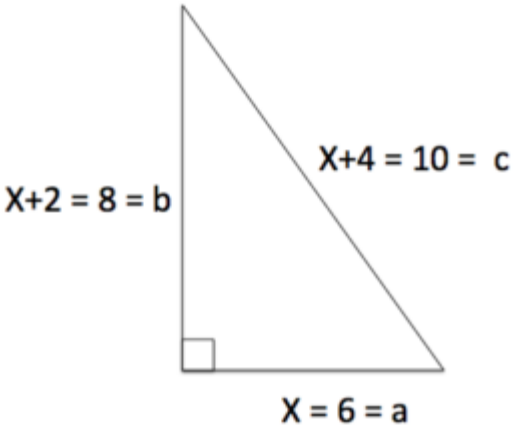
Interpret: Ok, it doesn't make sense to have a length equal to [latex]-2[/latex], so we can safely throw that solution out. The lengths of the sides are as follows:

[latex]x=6[/latex]

[latex]x+2=6+2=8[/latex]

[latex]x+4=6+4=10[/latex]

Check: Since we know the relationship between the sides of a right triangle we can check that we are correct. Sometimes it helps to draw a picture



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We know that [latex]a^2+b^2=c^2[/latex], so we can substitute the values we found:

[latex]\begin{array}{l}6^2+8^2=10^2\\36+64=100\\100=100\end{array}[/latex]

Our solution checks out.

Answer

The lengths of the sides of the right triangle are [latex]6, 8[/latex], and [latex]10[/latex]

This video example shows another way a quadratic equation can be used to find and unknown length of a right triangle.

Video Link

Try It

See this interactive in the course material.

If you are interested in celestial navigation and the mathematics behind it, watch this video for fun.

Video Link

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