ADVANCED MATHEMATICS 1

Time: 3 Hours   **MARCH 2023**

Instructions

1. This paper consists of a total of **ten (10)** questions.

2. Answer **ALL** questions.

3. Show clearly all the steps involved in arriving at a correct answer.

4. Cellular phones and any unauthorized materials are not allowed in the examination room.

5. The use of non-programmable calculator is allowed.

6. Write your **Examination Number (Name)** on every page of your answer booklet(s).

1. (a) Using a non – programmable calculator evaluates each of the followings;

The quantity x and y are connected by the relation $\sin x = \log(a+by)$

Where "a"and "b" are constants

x	10°	20°	30°	60°
y	3.2290			7.8639

i. By using calculator estimate the values of "a" and "b" into one decimal place

(b) Given that $A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & -1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$, use a calculator to find;

(i) $|A|$ (ii) A^{-1} (iii) A^{T} (iv) A^{3} **(4 Marks)**

(3 Marks)

ii. Fill the table with the values of y approximately into 4 decimal places. **(3 marks)**

2. (a) Show that $\sinh(m+1)x + \sinh(m-1)x = 2\sinh mx \cosh x$ **(3 Marks)**

(b) Give the condition for the equation $p\cosh x + \sinh x + q = 0$ to have real roots, where p and q are constants. **(3 Marks)**

(c) Show that $\coth^{-1} = \frac{1}{2}\ln\left(\frac{x+1}{x-1}\right)$ **(4 Marks)**

3. (a) Without drawing, explain how you can determine the solution set of the simultaneous inequalities $y + x \geq 3$ and $x + 2y \leq 9$ **(2 Marks)**

(b) An oil company has two deposits A and B with capacity of 7 000 litres and 4 000 litres respectively. The company has to supply oil to three petrol pumps P, Q and R whose requirements are 4500, 3000 and 3500 litres respectively. The distance between the deposits and petrol pumps are given in the table below.

From / To	A	B
P	7	3
Q	6	4
R	3	2

How should the delivery be scheduled in order to minimize the transportation cost? **(8 Marks)**

4. (a) The mean and variance of 7 observations are 8 and 16 respectively. If five of the observations are 2, 4, 10, 12 and 14. Find the other two observations. **(4 Marks)**

(b) The scores of 38 students in a basic mathematics test were grouped and tabulated in a table as shown below;

Class interval	21 − 25	26 − 30	31 − 35	36 − 40	41 − 45	46 − 50
Frequency	5	4	12	6	3	8

Calculate;

    (i)   Mean by coding method.   **(4 Marks)**

    (ii)   Mode.   **(1 Mark)**

    (iii)   Median.   **(1 Mark)**

5. (a) Use the laws of algebra to show that $A \cap (A \cup B) \equiv A$.   **(3 Marks)**

    (b) In a recent pool of 500 men and 500 women; it was observed that a total of 650 were married. Of those married, 275 were men and 500 claimed to be happy. Out of 750 claimed to be happy, 400 were men of which 200 were married.

    (i)   Represent the information above in a venn diagram.   **(4 Marks)**

    (ii)   How many married people are happy?   **(1.5 Marks)**

    (iii)   How many unmarried people are not happy?   **(1.5 Marks)**

6. (a) (i)   Given that $f(x) = \sqrt{19 - x^2}$ and $g(x) = x^2 - 3$, solve for x if fog(x) = 0.   **(2 Marks)**

    (i)   If goh(x) = $2x^2 + 4x - 1$ and h(x) = x + 1, find g(x).   **(3 Marks)**

    (b) Sketch the graph of the function $y = \dfrac{x^3 - 2x}{2(x^2 - 5)}$ hence determine its domain and range.   **(5 Marks)**

7. (a) (i)   Derive the Newton's Raphson Method for numerical approximations. **(2 Marks)**

    (ii)   Verify that the equation $x^2 - 2x - 1 = 0$ has a root lying between x = 2 and x = 3. With 4 iterations, apply the method in (i) above to obtain an approximation of the roots.   **(3 Marks)**

    (b)   Approximate $\int_{0}^{1} xe^{x^2}\, dx$ with 6 ordinates. Use both Simpson's and Trapezoidal rules, hence evaluate and calculate the percentage error committed for each case. Give comments.   **(5 Marks)**

8.   (a) Determine the ratio in which the line 3x + y − 9 = 0 divides the line segment joining points (1, 3) and (2, 7)   (3

Marks)

(b) The sum of the distance of a moving point from the points (c, 0) and (−c, 0) is always

2a units. Find the equation to the locus of the moving point. **(3 Marks)**

(c) Find the equation of the circle which passes through point A (4, −3) and touches the line

x + 2y − 7 = 0 at the point B(3, 2). **(4 Marks)**

9. (a) Show that $\int \frac{f'(x)}{f(x)} dx = \ln \ln [kf(x)]$, where k is a constant. **(3 Marks)**

(b) $\int e^{\frac{x}{2}} \cos 2x dx$. **(3 Marks)**

(c)(i) Show that the area formed by a certain curve, f(x) under the x − axis from certain point

x_1 to x_2 along the x-axis is given by; Area $= \int_{X_2}^{X_1} f(x)dx$ **(2 Marks)**

(ii).   Use the formula in (i) above to find the area bounded by the curve $f(x) = 2x - x^2$ and
the   x − axis.

(2 Marks)

10. (a) Use the first principle to show $\frac{d(x^n)}{dx} = nx^{n-1}$. **(3 Marks)**

(b)   Obtain the Taylor's expansion of the function $x^2 \ln x$ in ascending powers of (x − 1) as far
as the term in $(x - 1)^4$. **(3 Marks)**

(c)   A piece of wire which forms the circumference of a circle of radius of 0.12 m is cut and

bent so as to form two new circles. Find the radius of each circle in order that the sum of

their areas of the two circles shall be as small as possible. **(4 Marks)**