Assignment 1

PM1/04 (Group B)

(Symbols have their usual meanings)

1. (a) What do you mean by reparametrization of a curve? Prove that a parametrized curve has a unit-speed reparametrization if and only if it is regular.

(b) Find the unit speed reparametrization of a logarithmic spiral $\gamma(t) = (e^{kt}cost, e^{kt}sint)$, where k is a non-zero constant.

2. Let $\gamma(t)$ be any regular curve in R^3 with nowhere vanishing curvature. Then denoting $\frac{d}{dt}$ by a dot, prove that its torsion is $\frac{(\dot{\gamma}\times\ddot{\gamma}).\dddot{\gamma}}{\|\dot{\gamma}\times\ddot{\gamma}\|^2}$.

3. Find the torsion of the circular helix $\gamma(\theta) = (acos\theta, asin\theta, b\theta)$, $\theta \in R$, where a and b are constants.

4. Define regular curve. Which of the following curves are regular?

(i) $\gamma(t) = (cos^2 t, sin^2 t)$ for $-\infty < t < \infty$

(ii) $\gamma(t) = (t. cosht)$ for $-\infty < t < \infty$.

Find the unit speed reparametrization of regular curve (s).

5. Define the signed curvature κ_s. Let $\gamma(s)$ be a unit speed curve and $\varphi(s)$ be the turning angle of γ. Prove that $\kappa_s = \frac{d\varphi}{ds}$. Find the signed curvature of the catenary $\gamma(t) = (t, cosht)$

6. Let $\gamma(t)$ be a regular curve and λ be constant. The parallel curve γ^λ of γ is defined by $\gamma^\lambda(t) = \gamma(t) + \lambda n_s(t)$. Show that if $\lambda\kappa_s(t)\neq 1$, prove that γ^λ is a regular curve and its signed curvature is $\frac{\kappa_s}{|1-\lambda\kappa_s|}$.

7. State and prove Frenet–Serret equations.

8. Let γ be a unit-speed curve in R^3 with constant curvature and zero torsion. Prove that γ is a parametrization of (part of) a circle.

9. Compute κ, τ, **t, n** and **b** of the following curve and verify that Frenet–Serret equations are satisfied:

(i) $\gamma(t)=\left(\frac{1}{3}(1+t)^{3/2},(1-t)^{3/2},\frac{t}{\sqrt{2}}\right)$, (ii) $\gamma(t)=\left(\frac{4}{5}\cos t,1-\sin t,-\frac{3}{5}\cos t\right)$

Show that the curve in (ii) is a circle, and find its centre, radius and the plane in which it lies.