# Quantitative Logistics

## Extensions of Vehicle Routing in Last-Mile Logistics

### Background

The vehicle routing problem (VRP) is one of the most important combinatorial optimization problems; it determines the optimal set of routes performed by a fleet of vehicles to serve a predetermined set of customers (Toth & Vigo, 2002). The various VRP models and algorithms can be used in a wide range of applications to solve problems concerning the delivery or collection of goods and many other problems originating in the transportation system.

Over the years, VRP models have been extended in various ways to cope with real-life characteristics and aspects (Braekers et al., 2016). This has given rise to many sub-classes of the classical VRP, such as the Time-dependent VRP, Multi-depot, Multi-echelon VRP, Multi-period VRP, and Dynamic VRP, among others.

One interesting version of VRP that is not studied as frequently as other versions, is when a delivery must be made but where the delivery location can be chosen by the transportation company. In many traditional problems, the customer may very well decide on alternative addresses, e.g., your own, a specific pick-up store, or a specific box-delivery location, but often the transportation company has no flexibility of where to do the delivery.

The project is part of the so-called SPARA-project, where David Gundlegård at ITN is the project leader. A project studying time flexibility has been conducted as a Master's thesis in 2026 (Harriesson & Mejman, 2026)

### Purpose and aim

The aim of this project is to investigate how flexible delivery locations can be incorporated into existing VRP models within the context of last-mile delivery. Furthermore, the project will examine the ramifications of this flexibility on overall system efficiency, including cost reduction and how these changes influence routing complexity, solution efficiency, and potential trade-offs between flexibility and logistical constraints.

### Organization

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## Goals and requirements

The goal of this project is to is to modify existing VRP models to account for flexibility in space and to analyze the overall system efficiency. This process includes several steps:

- A literature study covering related VRP variants (i.e., VRP and their extensions), especially related to last-mile delivery
- Modification of an existing VRP model to incorporate delivery flexibility between locations.
- Implementation (possibly including with the help of heuristics), solution and validation of the model.
- Analysis of overall system efficiency and logistical ramifications.

This project will include mathematical modeling and programming related to optimization and data analytics. The group should therefore be knowledgeable in programming, preferably Python, R, and/or AMPL. Datasets with customer locations, individual demands, and alternative delivery addresses will be made available for the group.

## Project group

The project group should be conducted by a group of at least three and at most five students.

## Project grading

Except for the common requirements related to course grading specified in the course information, project specific grading is given according to the scale Fail; 3,4, or 5.

For grade 3, at least the following steps must be carried out:

- Write a detailed project specification and time plan in correct and readable English.
- Write a final report in English and make an oral presentation of the project results.
- Give an overview on the problem background, perform a literature review and describe related work.
- Modify an existing VRP model to incorporate flexibility in delivery location, where alternative delivery locations do not have a limited receiving capacity.

- Validating the model, by implementing the model with a "toy" dataset[1].
- Analyze the results compared to the basic version of the model (for e.g. solution speed, objective function, required vehicles), identify improvements (or deteriorations), and discuss the potential reasons for these.

For grade 4, the grade 3 steps must be carried out with very good results and the following additional steps must be carried out:

- Design and implementation of a solution algorithm to solve a realistic instance (based on a heuristic algorithm or a fast commercial solver).
- Comparison of the results with and without flexibility.
- Solve the problem, incorporating flexibility in delivery location(s), where alternative delivery locations may have a limited receiving capacity.
- Analysis of the resulting vehicle routes and comparison with actual routes recorded by the transport company.

For grade 5, the grade 3 and 4 steps must be carried out with very good results and the following additional steps must be carried out:

- Solve the problem, incorporating flexibility in delivery location(s), where alternative delivery locations may have a limited receiving capacity, and where capacity increases of existing locations, and/or included in the model, suggestions of new alternative locations.

## References

Braekers, K., Ramaekers, K., & Nieuwenhuyse, I. V. (2016). The vehicle routing problem: State of the art classification and review. Computers & Industrial Engineering, 99, 300–313.

Harriesson, J., Mejman, M. (2026). Patience cuts the distance. Master's thesis LiU-ITN-TEK-A–26/047–SE, Linköping University.

Toth, P. & Vigo, D. (2002). *The vehicle routing problem*. SIAM.

[1] However, a larger data set than the one used in TNK124, vt2026. Thus there will be a need for more work and/or other solution approaches than was used in the 15-customer problem that was considered in TNK124.