

Calculus 2 — Answer Key

Q1 Week 6 — September 8–11

Name:

1. FTC Part I — Accumulation

$F(x)$ represents the

net accumulation of f from a to x .

$F'(x)$ represents how fast the

accumulation is changing.

$F'(x) =$

$f(x)$

The expression on the right represents the average value of f on $[x, x+h]$.

As $h \rightarrow 0$, $av(f) \rightarrow$

$f(x)$

Therefore $F'(x) =$

$f(x)$

2. FTC with Changing Bounds

$F'(x) =$

$f(g(x))g'(x)$

Upper bound: multiply by

the derivative of the upper bound.

Lower bound: multiply by

the derivative of the lower bound.

Change the

sign.

Both bounds:

$f(h(x))h'(x) - f(g(x))g'(x)$

3. Practice

1. $x^3 - 4x + 1$

2. $2x\sqrt{1+(x^2+1)^4}$

3. $(1+\sin^2 x)^3 \cos x$

4. $2x(1-x^4) - \cos^3 x$

5. $3x^2\sqrt{1+(x^3+1)^2} - 2x\sqrt{1+x^4}$

Answer Summary: $x^3 - 4x + 1$; $2x\sqrt{1+(x^2+1)^4}$; $(1+\sin^2 x)^3 \cos x$; $2x(1-x^4) - \cos^3 x$; $3x^2\sqrt{1+(x^3+1)^2} - 2x\sqrt{1+x^4}$

4–5. FTC Part II

1. 10

2. 121/12

3. 37/6

4. 5

6–9. Concept Notes

Final Amount = Initial Amount +

Net Change

The integral of a rate gives

net change.

$\int_0^5 v(t) dt = 12$ represents

net change in position.

If $s(0)=7$ and net change=12, $s(5)=$

19

$F(b)=$

$F(a) + \int_a^b F'(x) dx$

The sign of $v(t)$

matters for net change.

Before total distance, $v(t)=$

0

If velocity changes sign, split the integral.

For total area, make each area positive.

10. Practice

1. $\int_0^3 (3t^2 - 6t + 2) dt = [t^3 - 3t^2 + 2t]_0^3 = 6.$

2. $\int_0^4 (2t + 5) dt = [t^2 + 5t]_0^4 = 36$ gallons.

3. $F(3) = 7 + \int_1^3 (3x^2 - 4x) dx = 7 + 10 = 17.$

4. $P(5) = 10 + \int_2^5 (4t - 3) dt = 10 + 33 = 43.$

5. Zero at $x=2$. Total area = $2 + 9/2 = 13/2$.

6. Zero at $x=1$. Total area = $2 + 2 = 4$.

7. Net change = $\int_0^5 (t-2) dt = 5/2$. Total distance = $2 + 9/2 = 13/2$. Negative motion before $t=2$ reduces net change but still contributes to distance.

Answer Summary: 6, 36, 17, 43, $13/2$, 4, net change $5/2$ and total distance $13/2$

11. Mixed Practice — 5.2–5.4

1. 615

2. 20

3. -22

4. $-20 \leq \int_1^5 [2f(x) - 1] dx \leq 20$

5. 2

6. $2x\sqrt{1+(x^2+1)^4}$

7. $\sin x(1+\cos^2 x)^4$

8. $2x(1-x^4)-\cos^3 x$

Answer Summary: 615, 20, -22, $[-20, 20]$, 2, $2x\sqrt{1+(x^2+1)^4}$, $\sin x(1+\cos^2 x)^4$, $2x(1-x^4)-\cos^3 x$

12. Mixed Practice

1. 14

2. -9

3. $-42 \leq \int_{-1}^5 [3f(x)+2]dx \leq 84$

4. 10/3

5. $-2x/[1+(x^2+1)^4]$

6. $3x^2\sqrt{1+x^6} + \sin x\sqrt{1+\cos^2x}$

7. Net change = 5; total distance = 13.

8. $2x[(x^2+1)^2+2] - \cos x(\sin^2x+2)$

Answer Summary: 14, -9, $[-42,84]$, 10/3, $-2x/[1+(x^2+1)^4]$, $3x^2\sqrt{1+x^6}+\sin x\sqrt{1+\cos^2x}$, 5 and 13, $2x[(x^2+1)^2+2]-\cos x(\sin^2x+2)$