Working Paper

Assessing the Uncertainty in Integrated Climate Economy Models.

Abstract

This paper and the associated CAGE (Climate And economic Growth Explorer) Excel workbook provide a tool to explore some climate change uncertainties and better assess our ability to effectively mitigate climate change. It is an integrated assessment model based the DICE (Dynamic Integrated model of Climate and the Economy) developed by William Nordhaus. However, the economic growth model has been significantly modified and it has two additional features. First, there several factors that can be varied and evaluated easily. There are various economic, climate and optimization options. Economic variations include four projected total factor productivity curves and two options for projecting the capital stock. Climate variations include two damage functions (plus a no damage option), five options for applying damage to capital and output, two options for abatement costs, and three climate sensitivities. Finally, there are options on whether to include carbon price optimizations and maximum temperature limits (three temperatures). Once all of the choices have been made, the projected carbon dioxide emission price must be optimized. This optimization is computationally demanding. However, the second additional feature of CAGE is a library of 1440 optimized carbon dioxide emission price curves for all the input combinations though some goals are not be achievable. This allows several outputs to be projected. These include GDP, capital stock, atmospheric temperature, energy use and cost, abatement fraction and cost, renewable energy production cost, and several others. Using this information several general conclusions are drawn including that underreacting to climate change appears more destabilizing to the economy than over reacting.

Introduction

Slowly the world is starting to mitigate climate change. Renewable energy costs have plummeted and are already the lowest cost option for new capacity. In many cases, renewables are lower cost than running costs for existing coal power plants. Yet, we have one hundred years of infrastructure built around fossil fuels. Powerful fossil fuel interests will fight against this existential threat to their businesses. Renewable energy sources are intermittent and will require enormous energy storage capacity. An entire new energy infrastructure will be required to power our transportation system. All of this will require enormous investment.

Most economists agree that putting a price on carbon emissions is the most efficient method to achieve this transition. This allows the power of the market to sort out winning solutions. So far, implementation has been spotty. Progress has not been steady. The political will is often lacking. However, some progress is being made. Carbon emission markets are starting, stopping, being restructured, and slowly we are learning how to make them work. At the same time, governments are incentivizing some solutions like renewable energy, electric cars, building energy efficiency improvements and ending fossil fuel subsidies. This requires governments to pick winning solutions. Unfortunately, governments do not have a good track record of picking winning solutions. Yet, progress is being made.

To help understand the scale and speed needed for climate change mitigation, we need tools to that help guide us. One approach is an integrated assessment model. These models include economic and climate models and interactions between them. They necessarily have long time scales; generally, 100 to 300 years. One such model is the DICE (Dynamic Integrated model of Climate and the Economy) developed by William Nordhaus. All of these models make many assumptions about how the future will unfold, and DICE is no exception. Nordhaus made the best assumptions he could and his efforts won him a Nobel prize, but there is considerable uncertainty in several areas. CAGE allows some different assumptions for comparison. For example, DICE uses a climate sensitivity of a 3.2°C increase in equilibrium temperature for a doubling of atmospheric CO_2 content. This is still the good estimate, but the current estimate of the 95% confidence limits is between 2.3°C and 4.7°C. These can make a significant difference. At a climate sensitivity of 3.2°C and above it appears that we cannot limit the temperature increase to 1.5°C, but at a climate sensitivity of 2.6°C we can. At climate sensitivity of 4.7°C, the model indicates it is impossible to limit the temperature increase to 2.0°C.

Since climate change is driven by CO_2 emissions from burning fossil fuels for energy and energy use is highly correlated with GDP, we need a good long-term estimate of GDP. So, there are economic uncertainties as well. An example is the projection of the total factor productivity. DICE uses a modified exponential curve that is concave upward. CAGE has three additional total factor productivity curves that represent futures that are at least as likely. One is a simple linear extrapolation of historical data. The second is a logistic extrapolation of historical data that is concave downward in the future. The third is an endogenous option that depends on the previous year's GDP. These options significantly affect GDP projection. Since the economic and climate parts of the model are integrated, the effects ripple through the rest of the model.

The rest of this paper is divided into the following sections. In the *Modeling Approach* section, the overall structure of the model is discussed including a quick review of the Excel workbook. The *Detailed Equations* section describes the equations used in the model. Significant parts of the CAGE model follow the method used in DICE. Therefore, this section focuses on the differences between DICE and CAGE. The *Instructions for CAGE* section provides instructions on how to use CAGE and some guidance on how the various options can be used to explore the effects of different assumptions. The *Examples* section shows a few examples of the kinds of things that can be explored with CAGE. There are certainly many others. Some of these are suggested in the *Future Work* section. The paper ends with the *Conclusions* which include an assessment of the usefulness of CAGE and some general conclusions based on the examples discussed.

Modeling Approach

The overall approach of the model is straightforward. Like DICE the model is global and there is an economic model that uses a Solow growth equation to project GDP. However, it is significantly different than the one used in DICE. It adds human capital to model and uses the number of employed people instead of population. All the projected inputs into the model are based on 60 years of historical data. The economic model uses 1-year time steps. That is used to determine energy use and CO_2 emissions. The CO_2 emissions feed the climate model to project an atmospheric temperature increase. The climate model uses 5-year times (as DICE does). This in turn causes damage to economic output and/or the capital stock which reduces economic growth.

To combat climate change, a price on is applied to CO_2 emissions. This drives abatement of CO_2 emissions by converting to renewable energy sources. There are two cases for the price on CO_2 emissions; the base case which represents a business-as-usual scenario, and a case where the price on CO_2 emissions is optimized. In the base case the price on CO_2 emissions increases very slowly over time. This represents the slow adoption of a CO_2 emissions price and other incentives that are used to convert to renewable energy sources and energy efficiency. In the optimized case, the price on CO_2 emissions is optimized by maximizing the discounted economic utility over the life of the model (2020 to 2300) subject to some constraints. The price on CO_2 emissions must be greater than or equal to the base price. It must also be less than or equal to a backstop price which decreases over time. The backstop price is the price of technologies that can replace all fossil fuels. When the price on CO_2 emissions equals the backstop price, all fossil fuels have been replaced. In the base case, this occurs in the year 2245. After that all prices on CO_2 emissions equal the backstop price. At this point there are no CO_2 emissions, but the CO_2 emissions price prevents backsliding. The optimization process is computationally intensive. CAGE optimizes two parameters which are used to calculate the prices on CO_2 emissions. This scheme reduces that optimization time by about 2 orders of magnitude compared to optimizing CO_2 emission prices directly and generally gives slightly better optima, and it is automated. It is not used in real time but is included to facilitate future modifications. In real time, CAGE restricts input to discrete choices for several parameters and has a library of optimized carbon for all possible inputs.

Detailed Equations

The CAGE model has several parts that are well integrated. There is no particular preferred order in which to describe it. Here the order roughly follows the order in Excel workbook. For those who would like to dig deeply into the way the workbook works, the row number of each variable in the Calc tab will be shown in parentheses where it is first defined. To extent possible, the descriptions will be grouped into sections as is the workbook. When there are options in an equation or in the row number of a variable, the options will be shown separated with an “or”. The choice of which one to use depends on the input choices in Main tab. The subscript “t” is the time period and similarly “t-1” is the previous time period. As each equation is presented only new variables are defined.

Utility. This section calculates the discounted sum of the population-weighted utility of per capita consumption. This value is maximized when the price on CO_2 emissions is optimized and is calculated by summing the discounted utilities for each year over the range of 2020 to 2300.

$W = \sum U_t M_W + A_W$, where Eq. 1

W = Discounted sum of the population-weighted utility of per capita consumption (4)

U_t = Utility function of consumption at time period (5)

t = Year of each time period (2)

M_W = Multiplicative scaling coefficient in utility function, 0.0164 (6)

A_W = Additive scaling coefficient in utility function, -3823.87 (7)

$U_t = L_t (c_t^{1-\alpha}/(1-\alpha)) R_t$, where Eq. 2

L_t = World population at time period, billions (69)

c_t = Annual per capita consumption at time period, 2011 $ (12)

α = Elasticity of the marginal utility of consumption, 1.45 (10)

R_t = Discount factor (social time preference factor) at time period (8)

$L_t = L/(1+e^{-k(t-x)})$, where Eq. 3

L = Maximum world population, billions, 12.97 (70)

k = Slope at the sigmoid's midpoint, 0.0204 (71)

x = Time at the sigmoid's midpoint, 2004.5 (72)

$c_t = C_t/L_t$, where Eq. 4

C_t = Annual consumption at time period, trillions 2011 $ (11)

$C_t = Y_n - I_d$, where

Y_n = Global output net of damages and abatement at time period, trillion$/year (19)

I_d = Gross investment at time period, trillion 2011$ per year (55)

$R_t = (1+\rho)^{-(t-2010)}$, where Eq. 5

ρ = Pure rate of social time preference, 0.015 (9)

This section is directly from DICE. Only the population projection has been updated.

Gross Domestic Product (GDP). The calculation of global output (GDP) has some significant changes from DICE. The number of employed individuals is used instead of population. Human capital is added to the equation along with its additional exponent. The method of projecting capital has been replaced by a choice of two new methods; one is based on investment calculated from the previous year's GDP, the second is a projection of historical data. In addition to an adjusted DICE total factor productivity, three new choices are added; one is a linear extrapolation of historical data, the second is a logistic extrapolation of historical data, the third is endogenous where the total factor productivity depends on the previous year's GDP. Output is calculated in two steps. First, gross global output is calculated including damage to capital if applicable, and then the cost of abatement and damage to output, if applicable, are subtracted to get net global output. The gross output actually exists, but some of it consumed by damage to output and abatement costs. Only net output is available for investment and consumption.

$Y_{g\,t} = K_t^{\gamma}\, H_t^{\beta}\, (L_{e\,t}\, A_t)^{(1-\gamma-\beta)}$, where Eq. 6

$Y_{g\,t}$ = Global gross output at time period, trillion 2011\$/year (13)

K_t = Global capital stock at time period, trillion 2011\$ (54 or 65)

γ = the output elasticity of the capital stock, 0.36 (14)

H_t = the global index of human capital per person at time period (78)

β = the output elasticity of the human capital stock (15)

$L_{e\,t}$ = the global number of persons employed at time period, billions (73)

A_t = the total factor productivity at time period (40, 41 or 44)

To calculate the net output, the cost of abatement and cost of damage to output must be calculated.

$Y_{n\,t} = Y_{g\,t} - C_{a\,t} - C_{d\,t}$, where Eq. 7

$Y_{n\,t}$ = Global net output at time period, trillion 2011\$/year (19)

$C_{a\,t}$ = Cost of abatement at time period, trillion 2011\$/year (17)

$C_{d\,t}$ = Cost of damage at time period, trillion 2011\$/year (18)

$C_{a\,t} = \Lambda_t\, Y_{g\,t}$ + (0 or $C_{PV\,t}$), where Eq. 8

Λ_t = Cost for CO2 abatement at time period, fraction of gross output (21)

$C_{PV\,t}$ = Abatement cost of Photovoltaics at time period, trillion 2011\$/year (31)

$C_{d\,t} = Y_{g\,t}\, D_t^{Y}$, where Eq. 9

D_t^{Y} = Climate damage born by output degradation at time period, fraction of output (16)

One other related calculation is simply calculating the growth rate of net output.

$\Delta Y_{n\,t} = Y_{n\,t} / Y_{n\,t-1} - 1$, where Eq. 10

$\Delta Y_{n\,t}$= The net output growth rate at time period, % (20)

Abatement Cost. The cost of abatement is the cost of reducing CO_2 emissions. It depends the emission control rate fraction of total emissions, the CO_2 emission price and carbon efficiency of the economy if energy were supplied by fossil fuels.

$\Lambda_t = \theta_1 \mu_t \theta_2$, where Eq. 11

θ_1 = Abatement cost control function coefficient (22)

μ_t = Emissions control rate at time period, fraction of total (24)

θ_2 = Abatement cost control function exponent, 2.8 (23)

$\mu_t = (P_{C\,t} / P_{B\,t})^{1/(\theta_2 - 1)}$, where Eq. 12

$P_{C\,t}$ = Carbon emission price at time period, 2011$ per ton CO_2 (27)

$P_{B\,t}$ = Backstop price at time period, 2011$ per ton CO_2 (25)

$\theta_{1\,t} = P_{B\,t} \sigma_t \theta_2$, where Eq. 13

σ_t = Carbon efficiency of the economy at time period, $MTCO_2$/1000 2011$ of GDP (85)

$P_{B\,t} = 9051600\ e^{-0.0050636\,t}$ Eq. 14

This section follows DICE. The backstop price is a curve fit through the DICE backstop prices to calculate for each year. CAGE uses an improved optimization scheme described below.

Photovoltaics. This section projects the production of photovoltaics to meet the renewable energy demand and replace depreciation. It also projects the price of utility scale photovoltaic based on production. While electricity generated by wind turbines is expected to contribute significantly, photovoltaics is expected to dominate and their projected costs are similar. Therefore, modeling renewable energy costs with only photovoltaics is reasonable.

$E_{p\,t} = E_{p\,t\text{-}1} (Y_{g\,t} / Y_{g\,t\text{-}1}) (\sigma_t / \sigma_{t\text{-}1})$, where Eq. 15

$E_{p\,t}$ = Primary Energy Usage at time period, tWh (28)

$E_{r\,t} = \mu_t E_{p\,t}$, where Eq. 16

$E_{r\,t}$ = Energy associated with abated emissions at time period, tWh (29)

$C_{UPV\,t} = 1000\ (7.952\ S_{PV\,t}^{-0.263})$, where Eq. 17

$C_{UPV\,t}$ = Utility scale unit installed cost of PV at time period, 2011$/KW (30)

$S_{PV\,t}$ = Cumulative PV capital stock installed at time period, GW (35)

$C_{PV\,t} = \Delta S_{PV\,t}\ C_{UPV\,t} / 1E6$, where Eq. 18

$\Delta S_{PV\,t}$ = PV added to stock at time period, GW (32)

$\Delta S_{PV\,t} = \text{Max}\left((E_{r\,t} - E_{r\,t-1})\,PV_c + PV_d,\ 0\right)$, where Eq. 19

PV_c = Conversion factor, GW/tWh, 0.887 (38)

PV_d = PV Depreciation, 0.0089 (39)

This section is new and is not included in DICE. The PV unit cost estimate uses an adjusted production to reflect the current PV share of renewable energy. It underestimates the cost reduction since the PV share is expected to increase. The PV conversion factor converts from tWh to GW based on cumulative global PV installed and electricity generated over the years 2014 to 2019. The depreciation is based on a 20% reduction in output over 25 years. Current and future PV is expected to depreciate slower.

Total Factor Productivity. The projection of the total factor productivity (TFP) has a significant effect on the projection of output or GDP. The projection used in DICE is a slightly modified exponential form. This is reasonably consistent with historical data and is based on a survey of mainstream economists, but it results in a GDP over 50 times larger than today by the year 2300. This seems highly unlike given the constraints on energy and materials. An option of using an adjusted DICE total factor productivity is provided in CAGE for comparison. It adjusted to work in the CAGE growth equation (Eq. 6). CAGE has three additional total factor productivity options. Linear and logistic based on curve fits of historical data. These match historical data about as well as DICE, but results more reasonable GDP in 2300. The linear projection is 7 times todays GDP and the logistic projection is about 4 times todays GDP. Even these may run into insurmountable constraints. The last one is endogenous based on the previous year's GDP.

$TFP = (TFP_{Linear\,t},\ TFP_{Logistic\,t},\ TFP_{DICE\,t}$ or $TFP_{Endo})$, where Eq. 20

$TFP_{Linear\,t} = -1{,}571 + 0.8043\,t$ (40) Eq. 21

$TFP_{Logistic\,t} = 114.9 / (1 + e^{-0.04812\,(t - 2016)})$ (44) Eq. 22

$TFP_{DICE\,t} = TFP_{DICE\,t-1} / (1 - g_t)$, where (41) Eq. 23

g_t = Growth rate for technology per year at time period, $g_0 = 0.0412$ (42)

$g_t = g_{t-1} / (1 - \delta)$, where Eq. 24

δ = Decline rate of technology change per year, 0.00509 (43)

$TFP_{Endo} = =0.7457\,Y_{g\,t-1} + 0.4454$ (46) Eq.25

Capital Stock. In CAGE, there are two options for projecting the capital stock. One uses a logistic curve fit from the last 60 years of global data. This projection is fixed. The other one is endogenous. This one responds to other choices made. The equations used are shown below. The constants δ, κ, and γ are determined to fit historic global data.

$K_t = (K_{te} \text{ or } K_{tl})$, where Eq.26

K_t= Capital stock at time period, trillion 2011$ (53)

K_{te} = Endogenous capital stock at time period, trillion 2011$ (54)

K_{tl} = Logistic capital stock at time period, trillion 2011$ (65)

Endogenous Capital Stock. The endogenous capital stock incorporations two climate damage functions. One, designated as high, developed by Weitzman and adopted by Rezai. A less severe climate damage function, designated as low, was developed by Nordhaus. DICE applied the damage only to output. CAGE applies the damage to output, capital or proportioned between the two.

$K_{te} = K_{te-1} + (I_d - DA)$, where Eq. 27

DA = Depreciation allowance at time period, trillion 2011$ (62)

$DA = (\delta + D^K)\, K_{te-1}$, where Eq. 28

δ= Constant rate of depreciation, 0.0337 (56)

D^K = Rate of capital degradation due to climate damage (57)

$D^K = f_K\, (D_H, D_L, 0)$, where Eq. 29

f_K = the fraction of damage born by capital (0, 0.333, 0.5, 0.667 or 1) (58)

D_H = High damage function (Weitzman, Rezai) due to climate change (82)

D_L = Low damage function (Nordhaus) due to climate change (83)

0, Only for reference in an imaged world with no climate change damage

$D_H = 1 - 1/(1+(T/20.2)^2+(T/6.08)^{6.76})$, where Eq. 30

T=temperature increase from 1900, °C (114)

$D_L=0.00266375\, T^2$, where Eq. 31

$K^T=\kappa\, Y_{n\,t-1}$, where

K^T= the target capital stock for the at time period (63)

κ = target fixed capital to output ratio, 11.34 (61)

$I_d= \gamma\, (K^T- K_{t-1}) + \delta\, K_{t-1}$, where Eq. 32

γ= partial adjustment for fixed capital, 0.0237 (60)

From this system of equations, the some of the structure of CAGE starts to become apparent. This year's capital stock depends on last year's capital stock, and this year's investment and depreciation including damage due to climate change. Investment depends indirectly on last year's net GDP and this year's depreciation.

Logistic Capital Stock.

$K_{tl} = 1414 / (1 + e^{-0.04736\,(t - 2037)})$ Eq. 33

Persons Employed. An important part of the projection of output is the global number of persons employed. In CAGE that is determined by the product of the global population projection and the percent employed projection.

$L_{e\,t} = L_t\,E_f$, where Eq. 34

L_t = World population at time period, billions (69)

E_f = Fraction of population employed at time period (74)

$L_t = 12.97 / (1 + e^{-0.0204\,(t - 2005)})$ Eq. 35

$E_f = 0.520 / (1 + e^{-0.020\,(t - 1919)})$ Eq. 36

Human Capital. Human capital represents the education, skill and experience of the workforce.

$H_t = 3.71 / (1 + e^{-0.019\,(t - 1980)})$ Eq. 37

Intensity of the economy. DICE projects the carbon efficiency of the economy (CO_2 emissions per dollar of GDP) using a modified exponential curve assuming is supplied by fossil fuels. This decreases over time as the economy becomes more efficient. CAGE also requires a projection of the energy efficiency of the economy. This also decreases following an exponential curve. It turns out that, as might be expected, the rate of decrease of is nearly identical. CAGE uses the DICE projection rates of decrease for both purposes. The initial growth rate, σ_0, is -0.01.

$\sigma_t = \sigma_{t-1} e^{g_{\sigma t}}$, where Eq. 38

$g_{\sigma t}$ = Growth rate of σ at time period (86)

$g_{\sigma t} = g_{\sigma t\text{-}1}\,(1 + \Delta g_{\sigma t})$, where Eq. 39

$\Delta g_{\sigma t} = -0.001$ (87)

Climate Model. The climate model is from DICE using its 5-year time step with a few changes. In CAGE the most substantive change is to the coefficient of heat gain by deep oceans. It was changed from 0.0250 to 0.0175 to be consistent with the latest data. From 2009 to 2019 the deep temperature increased from 0.02 to 0.04 °C. Using the coefficient of heat gain by deep oceans of 0.175 results in a deep ocean temperature increase of 0.03°C over the ten-year time period. The climate sensitivity (equilibrium atmospheric temperature increase associated with a doubling of atmospheric CO_2 content) used in DICE (3.1°C) is still a reasonable estimate, but CAGE uses a more recent mean of 3.5 and adds the options of using the 95% upper or lower confidence limits (2.3°C or 4.7°C). Other differences are CAGE interpolates to the 1-year time steps for the total CO_2 emissions used for plotting and the atmospheric temperature increase for use in the climate change damage functions.

$T_{at\ t} = T_{at\ t-5}\ \xi_1 (F_t - \xi_2 T_{at\ t-5} - \xi_3 (T_{at\ t-5} - T_{Lo\ t-5})$, where Eq. 40

$T_{at\ t}$ = Atmospheric temperature (°C above 1900), 0.830 at t=0 (114)

ξ_1 = Speed of adjustment parameter for atmospheric temperature, 0.104 (108)

F_t = Total increase in forcing since 1750 at time period, W/m^2 (113)

$\xi_2 = \eta / T_{at2x}$=forcing/temperature for CO2 doubling (109)

ξ_3 = Coefficient of heat loss from atmosphere to oceans, 0.088 (110)

$T_{Lo\ t}$ = Temperature of deep oceans at time period (°C above 1900), 0.0068 at t=0 (115)

$F_t = \eta \text{ Log } (M_{at\ t} / M_{at\ eq}) / \text{Log } (2) + F_{ex}$, where Eq. 41

η = Forcing at CO2 doubling, 3.8, W/m^2 (107)

$M_{at\ t}$ = CO_2 in atmosphere, GTC (100)

$M_{at\ eq}$ = Equilibrium CO_2 in atmosphere. At 1750, 588, GTC (112)

F_{ex} = Exogenous non-CO2 forcing, W/m^2 (103)

$M_{at\ t} = 5\ E_{total\ t} / 3.666 + \varphi_{11} M_{at\ t-5}\ 0.99873^5 / 100 + \varphi_{21} / 100\ M_{up}$, where Eq. 42

$E_{total\ t}$ = Total emissions at time period, GT CO_2 per year (92)

φ_{11} = Flow parameter, atmosphere to atmosphere, 91.200 (93)

φ_{21} = Flow parameters biosphere/shallow oceans to atmosphere, 3.833 (94)

$M_{up\ t}$ = CO_2 in biosphere/shallow oceans at time period, GTC (101)

Note: 0.99873 accounts for the natural annual reduction in atmospheric CO_2

$E_{total\ t} = E_{land\ t} + E_{ind\ t}$, where Eq. 43

$E_{land\ t}$ = Land use changes CO_2 emissions at time period, GT CO_2 per year (89)

$E_{ind\ t}$ = Industrial use changes CO_2 emissions at time period, GT CO_2 per year (90)

$E_{land\ t} = E_{land\ t-5} (1 - \Delta E_{land})$, where Eq. 44

ΔE_{land} = Decline rate of land emissions per 5-year period, 0.2 (91)

$E_{ind\ t} = \sigma_t (1 - \mu_t) Y_{g\ t}$ Eq. 45

$M_{up\ t} = \varphi_{12} / 100\ M_{at\ t-5} + \varphi_{22} / 100\ M_{up\ t-5} + \varphi_{32} / 100\ M_{Lo\ t-5}$, where Eq. 46

φ_{12} = Flow parameter, atmosphere to biosphere/shallow oceans, 8.800 (95)

φ_{22} = Flow parameter, biosphere/shallow oceans to biosphere/shallow oceans, 95.917 (96)

φ_{32} = Flow parameters deep oceans to biosphere/shallow oceans, 0.034 (97)

$M_{Lo\ t}$, = CO_2 in deep oceans, 10,010 at t=0, GTC (102)

$M_{Lo\ t} = \varphi_{23} / 100\ M_{up\ t-5} + \varphi_{33} / 100\ M_{Lo\ t-5}$, where Eq. 47

φ_{23} = Flow parameter, biosphere/shallow oceans to deep oceans, 0.250 (98)

φ_{33} = Flow parameter, deep oceans to deep oceans, 99.966 (99)

$F_{ex} = \mathrm{Min}\ (F_{ex}\ 2100, F_{ex}\ 2000 + 0.05\ (F_{ex}\ 2100 - F_{ex}\ 2000)\ (2 + t - 1))$, where Eq. 48

F_{ex} 2100 = Non-CO_2 GHG and influences in 2100, 0.62 (105)

F_{ex} 2000 = Non-CO_2 GHG and influences in 2000, -0.06 (104)

$\xi_2 = \eta / T_{at2x}$, where Eq. 49

T_{at2x} = Equilibrium temperature increase for CO_2 doubling (climate sensitivity), °C, 2.3, 3.5 or 4.7 (106)

$T_{Lo\ t} = T_{Lo\ t-5} + \xi_4 (T_{at\ t-5} - T_{Lo\ t-5})$, where Eq. 50

ξ_4 = Coefficient of heat gain by deep oceans, 0.0175 (111)

Carbon Price Optimization. The carbon emission price at each time period is determined by an optimization scheme which maximizes the discounted utility of consumption over the whole-time range of the model. This is subject to a constraint that the carbon emission price must be less than or equal to the backstop price. Raising the carbon emission price above the backstop price has no positive effect. The Excel version on DICE provides little guidance for the optimization scheme. It only suggests that the Excel Solver can be used. The most straightforward way is to optimize the carbon price for each year. However, this approach is impossible in Excel for 1-year time periods since it exceeds the limit on the number of decision variables. It is possible with Excel to optimize with 5-year time periods, but it has two problems. First, it is too slow to be practical for a large number of variations. The other problem was that the results had all kinds of waves and abrupt discontinuities that made no sense based on how the model works.

A better approach was needed. CAGE optimizes three parameters which are used to calculate the prices on CO_2 emissions. This scheme reduces that optimization time by about 2 orders of magnitude compared to optimizing CO_2 emission prices directly, generally gives slightly better optima, and avoids spurious results. It uses a logistic curve from the year 2020 to the year in which the price on CO_2 emissions first equals the backstop price, after that it follows the backstop price. The three parameters. The first is tfb, the year in which the price on CO_2 emissions first equals the backstop price, the second is x, the average of that year and 2020, and the third is k, the slope of the logistic curve at time x.

The Solver is set up to maximize W, the discounted utility of consumption, by varying the values of tfb and k, subject to constraints. To ensure that the logistic curve passes through the carbon prices at the year 2020 and tfb, a constraint is required. This is done by calculating L, the maximum value of the logistic curve, at 2020 and tfb. These must be equal as shown below.

$$P_{C\ 2020}\ (1+e^{-k(2020-x)}) = P_{C\ tfb}\ (1+e^{-k(tfb-x)}) \qquad \text{Eq. 51}$$

The next two constraints limit the range of tfb. The carbon price must be between the base carbon price (business as usual) and the backstop price. The base carbon price increases slowly until the year 2045 and then follow the backstop price down. Therefore, tfb must be less than or equal to 2245. At the other end, tfb must be greater than or equal to 2020. As a practical matter, this allows the impossible case where we instantly stop emitting CO_2. To avoid this, a minimum of 2025 was chosen. Analysis shows that even that is virtually impossible because of the investments required.

The next constraint, k > =0.01, ensures the slope of the logistic curve at the midpoint is positive.

Finally, one additional constraint is only used for the optimizations where the maximum atmospheric temperature increase is limited. It is simply, MAX ($T_{at\ t}$) <= (2.5, 2.0 or 1.5).

This optimization scheme is fast enough to be used in real time. However, it can occasionally crash. This is easily fixed by changing the starting parameters. CAGE avoids this by using a library of optimized parameters.

Instructions for CAGE

Overview. There are a few ways to use CAGE. The most basic way is to manually change each parameter on the Main tab and view the 12 graphs of results which can be visible at once. It may take a little adjustment of the zoom to about 75%. There are 5 more graphs which can be viewed by scrolling to the right. To compare different input conditions, click on the Copy Lines button and adjust the input for the next line. Marcos have to been enabled to use any of the buttons. This can be done for a total of 10 lines. To start over, click on the Clear Graphs button which clear every line but the one for the current input.

There is another set of graphs on the Results tab. These show the results for a particular parameter, such as maximum temperature increase, for all the possible input cases. If a point is of particular interest, it is possible to load this case into the Main tab.

These are described in more detail below.

Manual. CAGE can be run entirely from the Main tab. The sheets are protected and only the required cells are unlocked. This is to prevent accidentally changing something. However, there are no passwords for the sheet protection so they can be unprotected and changed. I just urge caution. Also, CAGE uses macros which can be confused by seemingly simple changes. Of course, the macros are also accessible.

CAGE provides a method to easily explore the effects of various options for analyzing the interaction of climate and the economy. The choices below can be changed and evaluated individually or in any combination. The results are shown in graphs which will update after each change. This takes a few seconds. To compare a set of inputs to another set, click on the “Copy Lines” button in cell F25. Then both the changed inputs and copied lines will be visible. Up to 10 lines can be shown (1 live and 9 copied). If this is exceeded, the oldest copied lines will be written over. In practice, the graphs get pretty messy after about 5 lines. If the graphs get too messy, use the “Clear Graphs” button in cell F34 to start fresh. This will clear all the copies lines.

The parameters and options are explained below in the order they appear, but they may be changed in any order. To enter the input options manually, set cell F21 to “Manual” by using the drop-down menu. This allows the graph to update with each change in input. Since this takes a few seconds, it may be inconvenient to make several changes at once. To avoid this, set F21 to “Auto” while making changes and then back to “Manual” to update the graphs.

1. Pick a Total Factor Productivity projection in cell F4. The three options are Linear, Logistic and DICE. Click on cell F4, then click on the down arrow, and click on the choice.
   Total Factor Productivity.
   a. Linear; a linear extrapolation of historical total factor productivities.
   b. Logistic; a logistic extrapolation of historical total factor productivities.
   c. DICE; the total factor productivity used in DICE which is adjusted to be compatible with the modified Solow equation used in CAGE.
   d. Endogenous; the total factor productivity is determined from the gross output of the previous year.
2. Pick a damage function in cell F6. The three options are High (Weitzman), Low (Nordhaus), No climate effects. The High and Low options only show a large difference when using the base carbon prices because when using the optimized carbon prices the temperature increases are below about 3°. In this range there is little difference between the two. The No Climate Effects option decouples the climate model from the economic model as if climate had no effect.
   Climate Damage.

   a. High; a damage curve from Weitzman. This curve approaches a damage fraction of 1.0 at an atmospheric temperature increase of 12°C.
   b. Low; the damage curve from DICE. This curve approaches a damage fraction of 0.4 at an atmospheric temperature increase of 12°C.
   c. No Climate Effects; this is only used for comparison of how the economy might perform if we didn't have to account for climate change.

3. Pick a capital damage fraction in cell F8. Capital Damage Fraction is the fraction of damage that is born by capital, the remainder is born by output. There is limited research that suggests 0.3333 may be appropriate, but it is worth checking the whole range. This option cannot be used with Logistic capital stock projection.

   Capital Damage Fraction

   a. 0
   b. 0.3333
   c. 0.5
   d. 0.6667
   e. 1
4. Pick a capital stock projection method in cell F10. The first option calculates the capital stock based on the net GDP, damage to capital, depreciation, and investment. The second option projects capital stock using a logistic curve fit of historical global capital stock.

   Capital Stock (options for projecting capital stock).

   a. Endogenous; capital stock is projected based on investment, depreciation, damage from climate change. Investment is based on the previous year's GDP.
   b. Logistic; capital stock is projected based on historical data using a logistic function. Capital damage cannot be used with this option.
5. Pick an abatement cost method in cell F12. Abatement is the cost incurred to mitigate carbon emissions. The first option is the DICE method. The second option adds the estimated cost of photovoltaic cells to produce carbon free energy. The PV part estimates the cost of photovoltaics at the utility scale. This is often significantly higher than the DICE estimate. It is assumed that the DICE estimate represents other infrastructure required, like power transmission lines, or car charging stations.

   Abatement Cost

   a. DICE; the abatement costs from DICE.
   b. DICE + PV Cost; CAGE uses projected photovoltaic costs based on cumulative production which is added to the DICE abatement cost.
6. Choose whether or not to include a maximum temperature increase constraint in cell F14. The first is option is "On". In this case the carbon prices are chosen to maximize economic utility subject to a maximum temperature increase limit. The maximum temperature increase is chosen in the next step. In the second option, "Off", the carbon prices are chosen to maximize economic utility only.

7. Choose a maximum temperature increase in cell F16. If 1.5°C is chosen, the carbon prices cannot be optimized to limit the temperature increase to 1.5°C except at the lower 95% confidence limit of climate sensitivity. All options come out to about 1.84 for the mean climate sensitivity and 2.38°C for the upper 95% confidence limit climate sensitivity. This is a caused by the carbon emissions already emitted. This means that using this climate model, we still could not meet the 1.5°C limit for mean and upper 95% confidence climate sensitivities even if we stopped emitting CO2 today. For the upper 95% confidence limit climate sensitivity even the 2°C is not possible. The is summarized in the following table. Climate sensitivity is discussed below.

Max Temperature	1.5°C	2.0°C	2.5°C
Climate Sensitivity			
LCL = 2.3	Yes	Yes	Yes
Mean = 3.5	No	Yes	Yes
UCL = 4.7	No	No	Yes

8. Choose whether or not to optimize the carbon price in cell F18. If this is Off, then the base carbon prices are used. If this is On, then carbon prices are optimized to maximize economic utility. If the temperature maximum constraint is on, then these are subject to temperature increase limitations of 1.5°C, 2.0°C or 2.5°C. In the cases where a maximum temperature constraint is not possible, then backstop carbon prices are used. CAGE doesn't actually optimize the carbon prices in real time. It would be too slow and optimization can be finicky. Rather it has a library of optimized carbon prices for all the combinations, and just loads those.
9. Choose the climate sensitivity of 2.3°C, 3.5°C or 4.7°C in cell F20. Climate sensitivity is the long-term equilibrium temperature increase for a doubling of CO_2 in the atmosphere. DICE uses 3.2, CAGE uses 3.5, the mean of the confidence limits or the lower and upper 95% confidence limits of 2.3°C and 4.7°C.
10. The graphs are updated “live” as inputs are changed. For comparisons, click on the Copy Lines button in cell F25 and then change the inputs. Up to 10 lines, representing the live line plus latest 9 copied lines, can be plotted on each graph. The lines are identified by a case code which consists of 9 digits representing all the choices made above. The code for the current case is shown in cell F22. To use different descriptions of the lines, enter new descriptions in cells F38:F47. The values for the individual data points for all the graphs are shown in cells E37:KQ239.

The output is available as series of graphs which are updated as new input choices are made. The following graphs are available.

1. Net GDP, trillions 2011 $/year
2. Capital Stock, trillions 2011 $
3. Cost of abatement, trillion 2011 $/yr
4. Atmospheric Temperature Increase, degrees C
5. Carbon Price, 2011 $/ton CO2

6. GDP Growth %/year
7. Emissions control rate (fraction of total)
8. Energy Use, tWh/year
9. Carbon Dioxide Emissions, GTons/year
10. Damage fraction
11. PV added to stock, GW/year
12. Per Capita Consumption, 2011 $/year
13. Consumption, trillion 2011 $/year
14. Abated Energy Use, tWh/year
15. PV Cost, 2011 $/kW
16. Capital Damage, fraction of output/year
17. Depreciated PV Deployed, GW

Automatic. The Auto mode is really only useful to copy a point from a graph on the Results tab to Main tab for further analysis. The Results tab shows graphs of 13 particular parameters for all the possible input cases. These graphs can be a bit messy, but they give an overall view of the range of outputs. If a point is of particular interest, it is possible to load this case into the Main tab. Hovering the mouse pointer the point will show the series number and point number. Enter these, and the climate sensitivity number (top graph=1, middles graph=2, and bottom graph=3) in the cells in the lower right below the graphs and click on the Load button. The graphs for this case will then be show on the Main tab. It will be in the Auto mode. If the graphs are of interest, click on the Copy Lines button and switch back to manual in cell F21.

The inputs for a particular point you can be seen by looking at three things. First, the climate sensitivity is indicated by which graph the point is on (top, middle or bottom). Next, hover the mouse pointer over the point of interest to show the optimization used (Base, Optimized, Optimized with Tmax=2.5, Optimized with Tmax=2.0, or Optimized with Tmax=1.5). Finally, the horizontal axis shows the rest of the inputs for each point. On the temperature graph row 23 shows the TPF, row 24 shows the damage curve, row 25 shows how capital is projected including the damage fraction applied to capital, and row 26 shows how abatement is calculated.

Graphs are available for the following parameters.

1. Peak Temperature Increase, Degrees C
2. Year of Peak Temperature Increase = Year of Maximum Damage
3. Utility
4. Maximum Annual Cost of Abatement, trillion 2011$
5. Maximum Annual Cost of Abatement, percent of gross GDP
6. Maximum Annual Primary Energy Use, tWh
7. Year of Maximum Energy Use
8. Maximum Damage, percent
9. Maximum CO2 Emission Price, 2011$
10. Year of Maximum CO2 Emission Price

11. Maximum PV Annual Production, GW (if all PV)
12. Year of Maximum PV Annual Production
13. First Year of 100% Abatement (may be same as graph number 10)

Graph 11 assumes that all new renewable capacity will be PV. In fact, some will be wind turbines, some may other renewables, and some may be nuclear (if that is considered renewable). It may be better to think of as the maximum increase in renewable energy generating capacity.

While these graphs show all the data points and allow individual points to be easily investigated further in the Main tab, they are messy and of limited value in making overall conclusions.

Box and Whisker Plots. Another way to look at the overall results is to use box and whisker plots. These plots show statistical information about a dataset. The box shows the second and third quartiles of the dataset. The line in the middle is the median. An X shows the mean. The whiskers show the range of the dataset excluding outliers. In Excel, outliers are defined as being outside the box by more than 1.5 times the length of the box. Individual outliers are shown as solid circular points.

The Results Box & Whisker tab shows the results for the following parameters.

1. Maximum Temperature Increase, °C
2. Maximum Annual Abatement Cost, % of Gross GDP
3. Year of First 100% Abatement (zero carbon emissions)
4. Maximum Damage Fraction
5. Minimum Annual Net GDP growth, %
6. Number of Years with Negative Net GDP Growth

These graphs are helpful in developing overall conclusions.

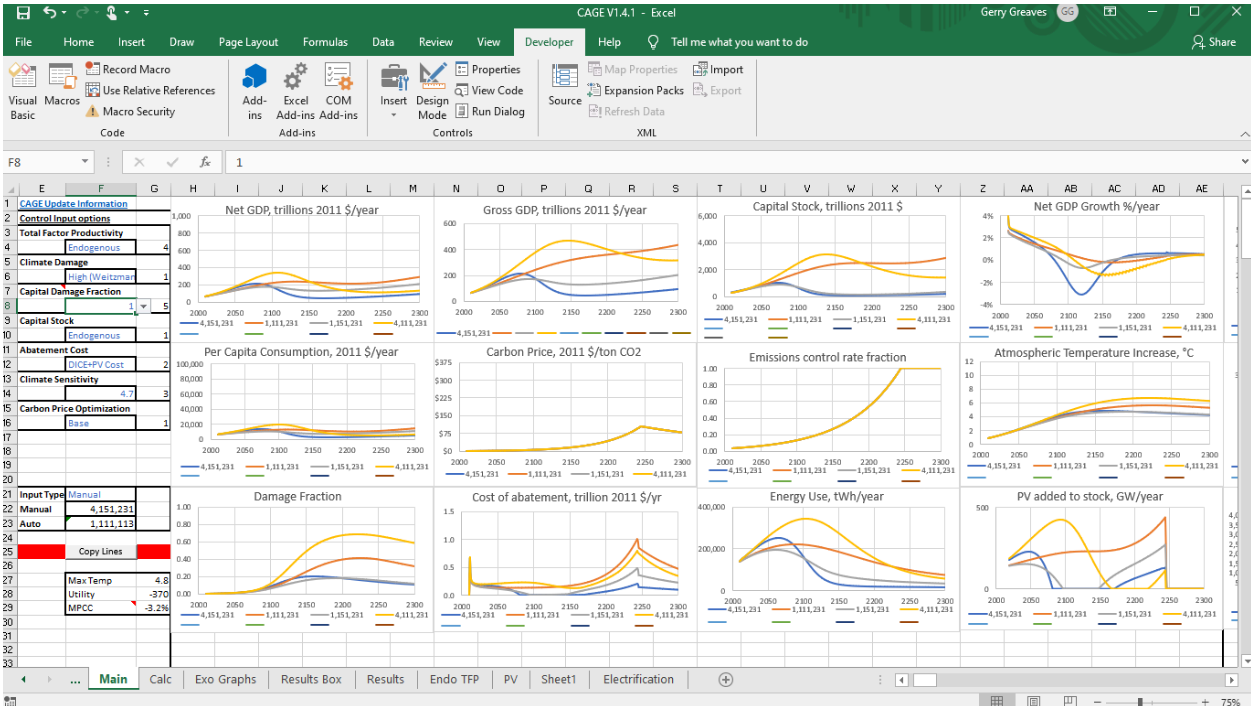
Results.

Future Work

Conclusions

References

Working Paper



	E	F	G
1	CAGE Update Information		
2	Control Input options		
3	Total Factor Productivity		
4		Endogenous	4
5	Climate Damage		
6		High (Weitzman	1
7	Capital Damage Fraction		
8		1	5
9	Capital Stock		
10		Endogenous	1
11	Abatement Cost		
12		DICE+PV Cost	2
13	Climate Sensitivity		
14		4.7	3
15	Carbon Price Optimization		
16		Base	1
21	Input Type	Manual	
22	Manual	4,151,231	
23	Auto	1,111,113	
25		Copy Lines	
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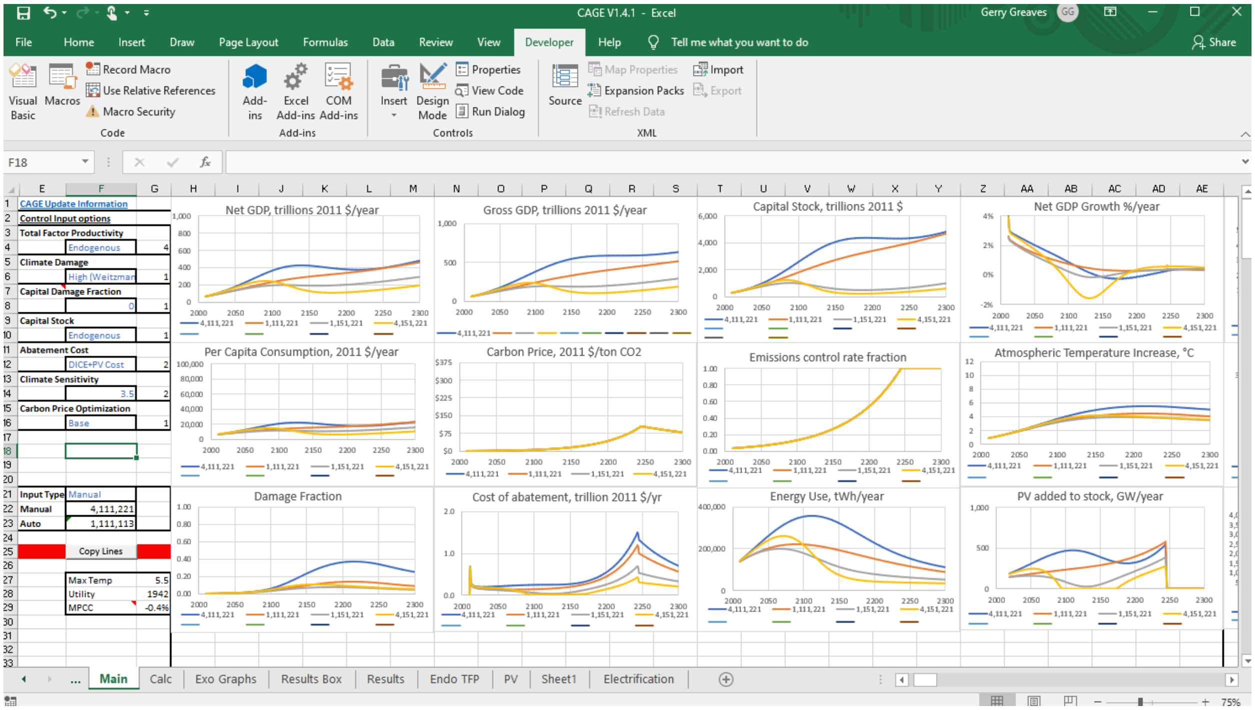
29		MPCC	-3.2%



	E	F	G
1	CAGE Update Information		
2	Control Input options		
3	Total Factor Productivity		
4		Endogenous	4
5	Climate Damage		
6		High (Weitzman	1
7	Capital Damage Fraction		
8		0	1
9	Capital Stock		
10		Endogenous	1
11	Abatement Cost		
12		DICE+PV Cost	2
13	Climate Sensitivity		
14		3.5	2
15	Carbon Price Optimization		
16		Base	1
21	Input Type	Manual	
22	Manual	4,111,221	
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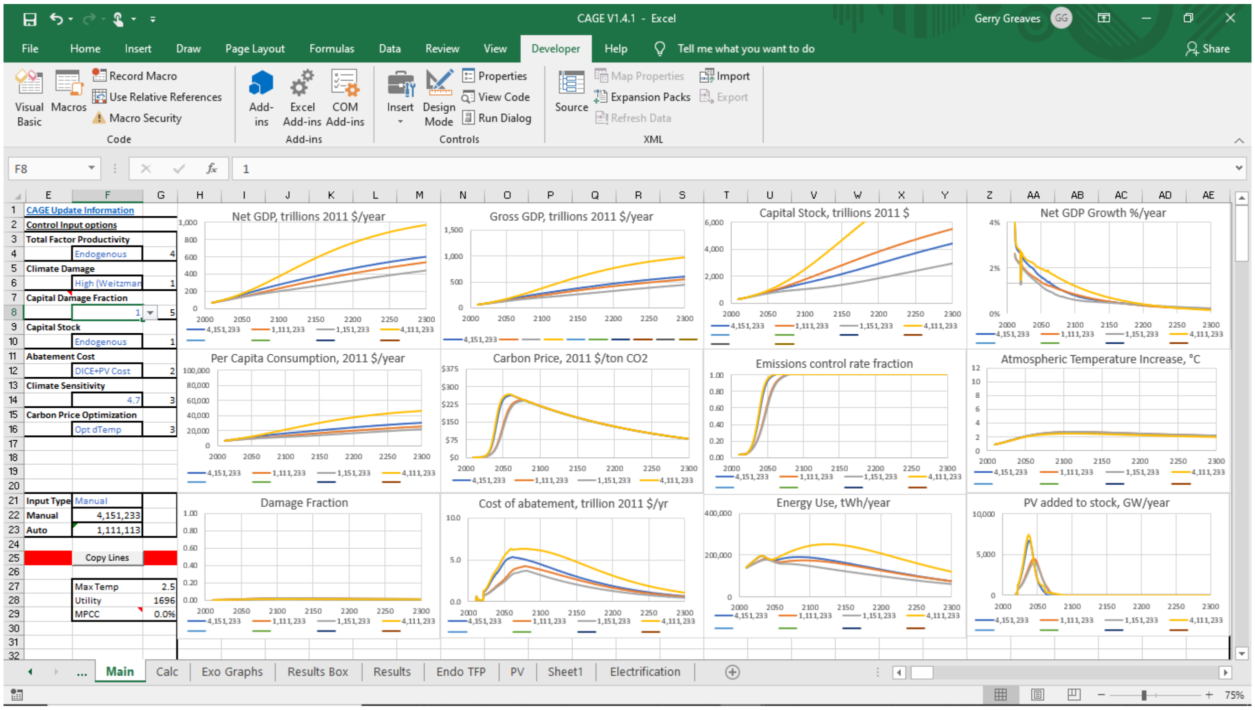
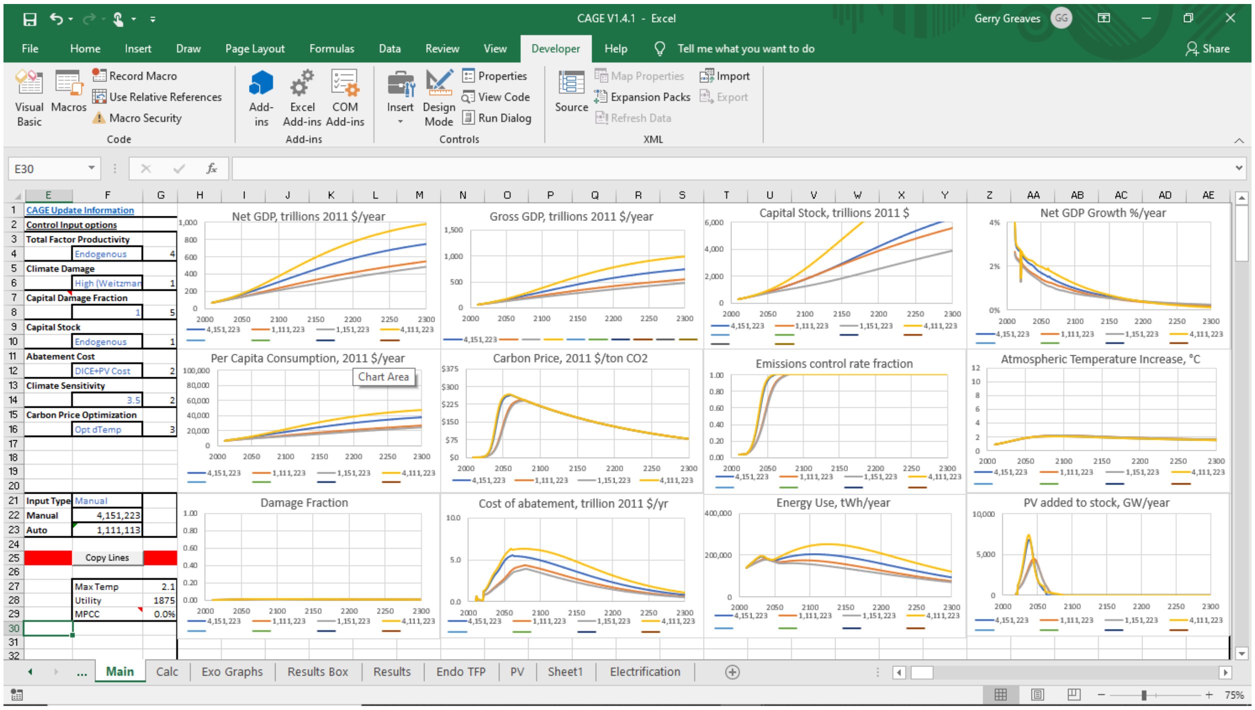
25		Copy Lines	
27		Max Temp	5.5
28		Utility	1942
29		MPCC	-0.4%

Working Paper



	E	F	G
1	CAGE Update Information		
2	Control Input options		
3	Total Factor Productivity		
4		Endogenous	4
5	Climate Damage		
6		High (Weitzman	1
7	Capital Damage Fraction		
8		1	5
9	Capital Stock		
10		Endogenous	1
11	Abatement Cost		
12		DICE+PV Cost	2
13	Climate Sensitivity		
14		4.7	3
15	Carbon Price Optimization		
16		Opt dTemp	3
21	Input Type	Manual	
22	Manual	4,151,233	
23	Auto	1,111,113	
25		Copy Lines	
27		Max Temp	2.5
28		Utility	1696
29		MPCC	0.0%



	E	F	G
1	CAGE Update Information		
2	Control Input options		
3	Total Factor Productivity		
4		Endogenous	4
5	Climate Damage		
6		High (Weitzman	1
7	Capital Damage Fraction		
8		1	5
9	Capital Stock		
10		Endogenous	1
11	Abatement Cost		
12		DICE+PV Cost	2
13	Climate Sensitivity		
14		3.5	2
15	Carbon Price Optimization		
16		Opt dTemp	3
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