Roll No…………………….. **Total No. of Printed Pages: 1**

Total No. of Questions: [09]

B. Tech. (CSE) (Semester – 4th)
DISCRETE MATHEMATICS
Subject Code: BMATH1401
Paper ID: [18111115]

Time: 03 Hours **Maximum Marks: 60**

Instruction for candidates:

1. Section A is compulsory. It consists of 10 parts of two marks each.
2. Section B consist of 5 questions of 5 marks each. The student has to attempt any 4 questions out of it.
3. Section C consist of 3 questions of 10 marks each. The student has to attempt any 2 questions.

Section – A **(2 marks each)**

Q1. Attempt the following:

a. Define isomorphic graphs.
b. Give the planar representation of K_4.
c. What is the sum of the entries in a column of incidence matrix of an undirected graph?
d. Prove that in a group G inverse of an element is unique.
e. Give example of an integral domain.
f. Simplify the Boolean expression $(x+y).(x+z).\overline{(\bar{x}.y)}$.
g. In how many ways can 10 identical balloons be distributed among 4 children if each child receives at least two balloons.
h. Give a recursive definition of the sequence $\{a_n\}$, $n = 1, 2, 3...$, if $a_n = 4n - 2$.
i. Let $f: R \rightarrow R$ be given by $f(x) = x^3 - 2$. Prove that f is one-one.
j. If $A_i = \{1, 2, 3...., i\}$ for $i = 1, 2, 3\ldots$, then what is $\cup_{i=1}^{\infty} A_i$?

Section – B **(5 marks each)**

Q2. Let A = $\{a, b, c\}$ and $\rho(A)$ denotes its power set. Let $\subseteq$ be the inclusion relation on the elements of $\rho(A)$. Prove that $\langle\rho(A), \subseteq\rangle$ is partial order relation and draw its Hasse diagram.

Q3. Obtain the principal disjunctive normal form of $P \vee (\neg P \rightarrow (Q \vee (\neg Q \rightarrow R)))$.

Q4. Simplify the following Boolean expression and draw its switch circuit and logic gate:

$r.s.t + r.s.\bar{t} + r.\bar{s}.t + \bar{r}.s.t.$

Q5. Prove that there is always a Hamiltonian path in a directed complete graph.

Q6. Prove by induction $\frac{1}{1.2.3} + \frac{1}{2.3.4} + \frac{1}{3.4.5} + \ldots + \frac{1}{n(n+1)(n+2)} = \frac{n(n+3)}{4(n+1)(n+2)}$

Section – C **(10 marks each)**

Q7. a) For all sets A, X and Y show that $A \times (X \cap Y) = (A \times X) \cap (A \times Y)$.

b) Prove the following equivalence by using truth tables:

$$P \rightarrow (Q \vee R) \Leftrightarrow (P \rightarrow Q) \vee (P \rightarrow R).$$

Q8. a) Prove that every subgroup of a cyclic group is cyclic.

b) State and prove cancellation laws for Boolean algebra.

Q9. a) Determine the number of edges in a graph with 6 nodes, 2 of degree 4 and 4 of degree
2. Draw two such graphs.

b) State and prove Euler's formula for connected planar graphs.