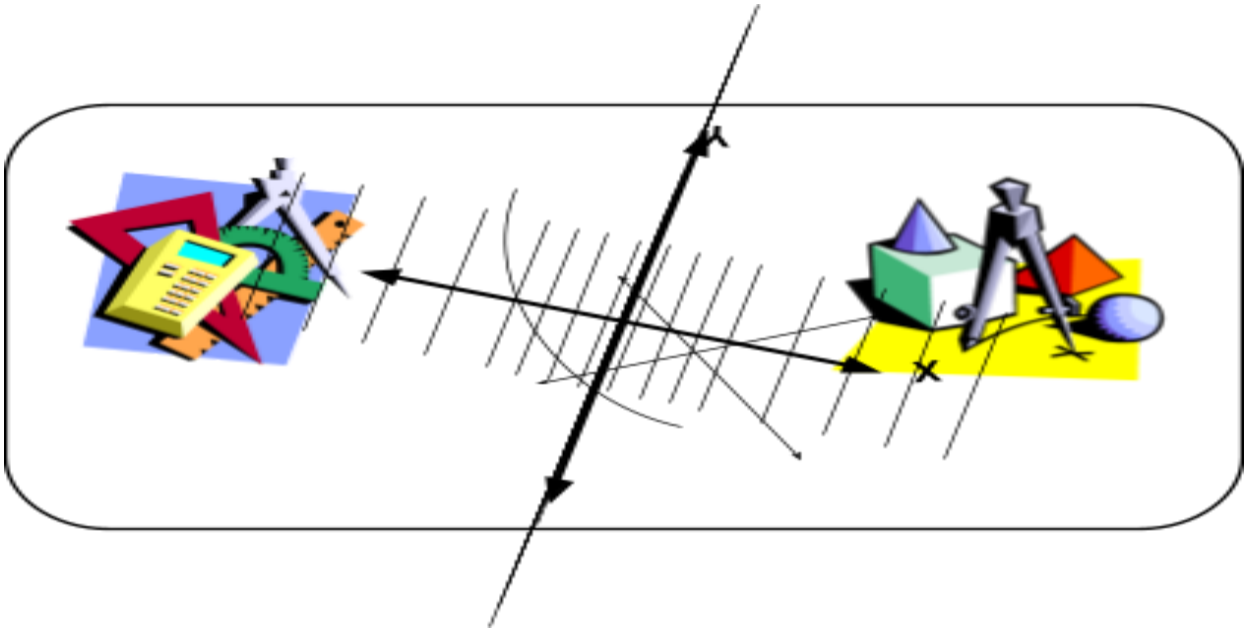
(Effective Alternative Secondary Education)

# MATHEMATICS III



## *MODULE 8*

*Geometry of Shape and Size*

BUREAU OF SECONDARY EDUCATION
Department of Education
DepEd Complex, Meralco Avenue, Pasig City

# Module 8

## Geometry of Shape and Size



### What this module is about

This module is about volume of solids. The volume of a solid is the number of cubic units contained in the solid. If measures are given in centimeter, the volume is stated in cubic cm, written as $cm^3$.



### What you are expected to learn

This module is designed for you to:

1. define volume of solids.
2. find the volume of solids such as:
   - cube
   - prism (rectangular, triangular)
   - pyramid (square, rectangular, triangular)
   - cylinder
   - cone
   - sphere
3. solve problems involving volume of solids.



### How much do you know

Find the volume of each solid:

1. a cube with side (s) = 2.4 m
2. a cylinder with h = 20 cm, r = 22 cm.

3. a rectangular prism with l = 25 cm, w = 17 cm, h = 30 cm

4. a square pyramid with s = 5 m, h = 6 m.

5. a cone with r = 2 cm, h = 6 cm.

6. a triangular prism with height 10 cm, base (a right triangle with sides 3, 4 and 5 cm and the right angle between shorter sides).

7. a ball with radius of 17 cm (use  = 3.14)

8. a triangular pyramid with b = 4 cm, h = 8.2 cm (altitude of the base), h = 7 cm (height of the pyramid).

9. a rectangular pyramid with l = 6 cm, w = 4.3 cm, h = 8 cm (height of the pyramid)

10. a cylindrical tank is 5.3 meters high. If the radius of its base is 2.8 meters, what is its volume?

11. Find the volume of a rectangular prism which is 46 cm long, 37 cm wide and 25 cm high.

12. Find the volume of a pyramid with a square base if the length of the sides of the base is 2.4 m and the height of the triangular face is 3.5 m.

13. cube with edge of $6\frac{2}{3}$ cm.

14. cylinder with radius of base 8.7 cm and height 12 cm.

15. rectangular prism with base 8 m by 10 m by 15 m.



*What you will do*

## Lesson 1

## Finding the Volume of a Cube, Prism and Pyramid

One problem with rooms that have high ceilings is that they are hard to heat and cool. The amount of air in a room determines the heating or cooling power needed. To find the amount of air in a room, you need to find the volume of the room.

In finding volume of solids, you have to consider the area of a face and height of the solid. If the base is triangular, you have to make use of the area of a triangle, if rectangular, make use of the area of a rectangle and so on.

The next examples will help you to understand more about volume or the amount of space in three – dimensional figures.

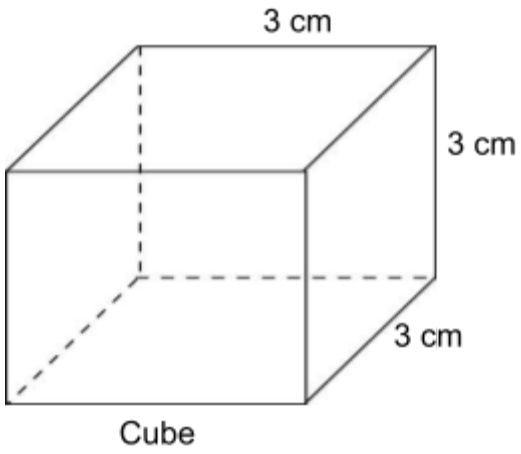
**Volume of a cube**

> The volume V of a cube with edge e is the cube of e. That is,
>
> $V = e^3$.

**Example:**

Find the volume of a cube with edge ( *e* ) of 3 cm.

Figure:



Cube

Solution:

$$V = e^3$$

Substituting e by 3 cm:

$$V = 3^3$$

$$V = 27 \text{ cm}^3$$

**Volume of Prism**

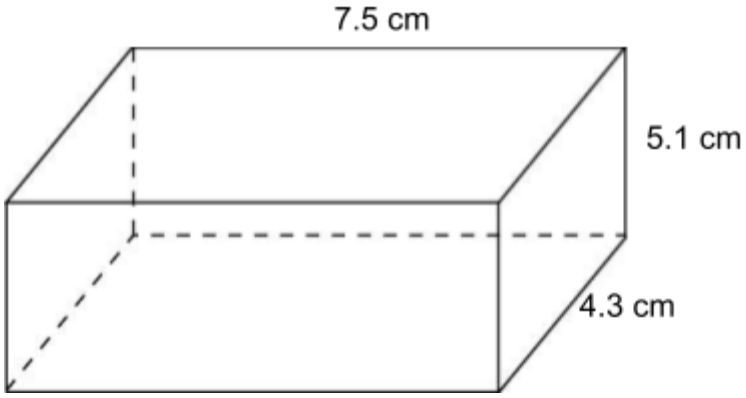
The volume V of a rectangular prism is the product of its altitude h, the length l and the width w of the base. That is,

$$V = lwh.$$

**Example:**

Find the volume of a rectangular prism whose length is 7.5 cm, width is 4.3 cm and thickness is 5.1 cm.

Figure:




Solution:

$V = lwh$

$V = (7.5\text{ cm})(4.3\text{ cm})(5.1\text{ cm})$

$V = 164.475\text{ cm}^3$

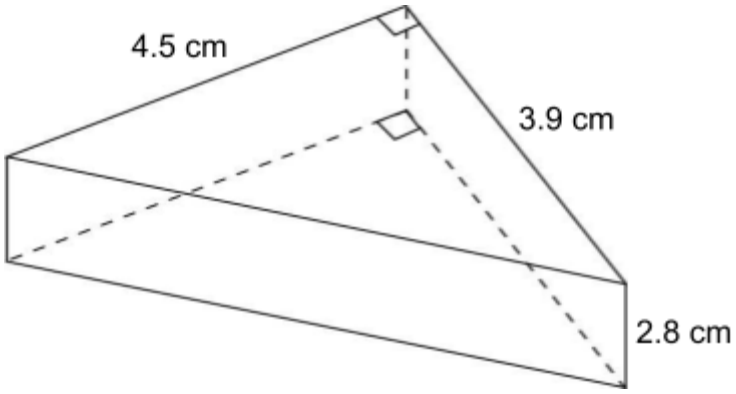
The volume of a prism can also be expressed in terms of area of the base, B.

The volume V of a prism is the product of its altitude h and area B of the base. That is,

$$V = Bh.$$

**Example:**

Find the volume of a triangular prism whose dimensions is given in the figure below.




Solution:

Let B = area of the triangular base

$$B = \frac{1}{2}\,bh$$

$$= \frac{1}{2}\,(4.5\text{ cm})(3.9\text{ cm})$$

$$B = 8.775\text{ cm}^2$$

Finding the volume of the prism:

$$V = Bh$$

$$= 8.775\text{ cm}^2\,(2.8\text{ cm})$$

= 24.57 $cm^3$

**Volume of Pyramids**

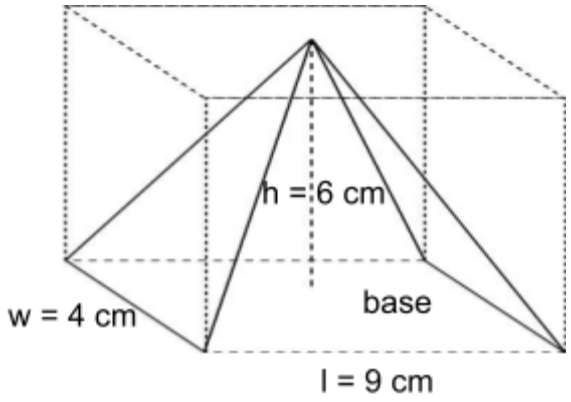
Consider a pyramid and a prism having equal altitudes and bases with equal areas. If the pyramid is filled with water or sand and its contents poured into a prism, only a third of the prism will be filled. Thus the volume of a pyramid is $\frac{1}{3}$ the volume of the prism.

> The volume V of a pyramid is one third the product of its altitude h and the area B of its base. That is,
>
> V = EMBED Equation.3 Bh.

**Example:**

1. Find the volume of the rectangular pyramid with the given dimensions.

Figure:




Solution:

Let B = the area of the rectangular base

B = lw

= (9 cm)(4 cm)

$= 36 \text{ cm}^2$

Finding the volume V:

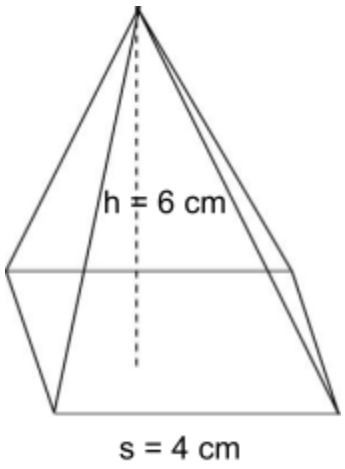
$$V = \frac{1}{3} Bh$$

$$= \frac{1}{3} (36 \text{ cm}^2) (6 \text{ cm})$$

$$= 72 \text{ cm}^3$$

2. Find the volume of a square pyramid with a side of the base as 4 cm and the height of a pyramid as 6 cm.

Figure:




Solution:

Let B = area of the square base

$= s^2$

$= (4 \text{ cm})^2$

$B = 16 \text{ cm}^2$

Finding the volume of the pyramid:

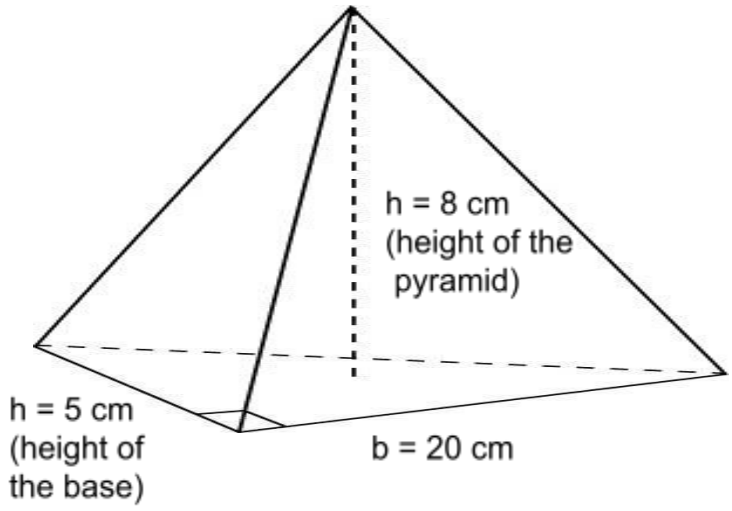
$V = \frac{1}{3} Bh$

$= \frac{1}{3} (16 \text{ cm}^2)(6 \text{ cm})$

$= 32 \text{ cm}^3$

3. Find the volume of a triangular pyramid with the given dimensions.

Figure:




Solution:

Let B = area of the triangular base

$B = \frac{1}{2} bh$

$= \frac{1}{2} (20 \text{ cm})(5 \text{ cm})$

$B = 50\ cm^2$

Finding the volume of the pyramid:

$V = Bh$

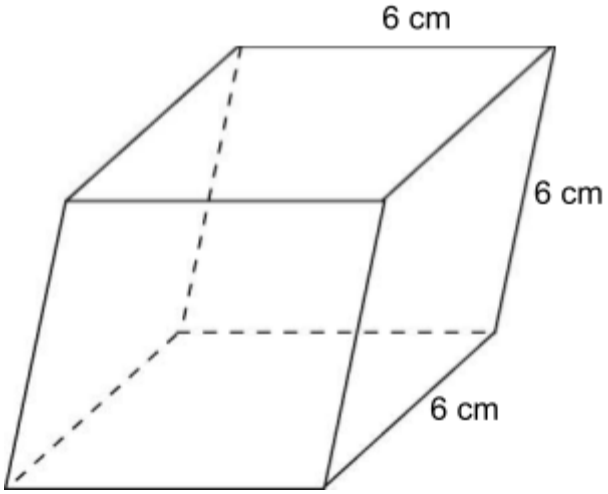
$= (50\ cm^2)(8cm)$

$V = 400\ cm^3$

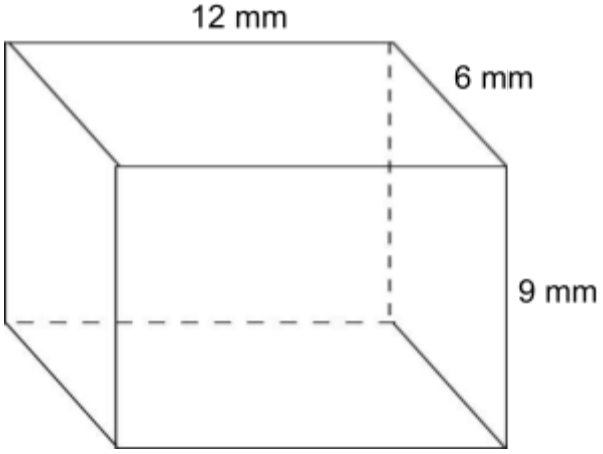
## Try this out

Find the volume of each solid:

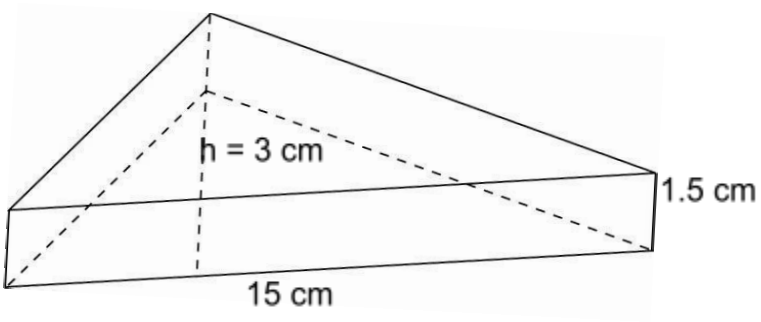
1.




2.

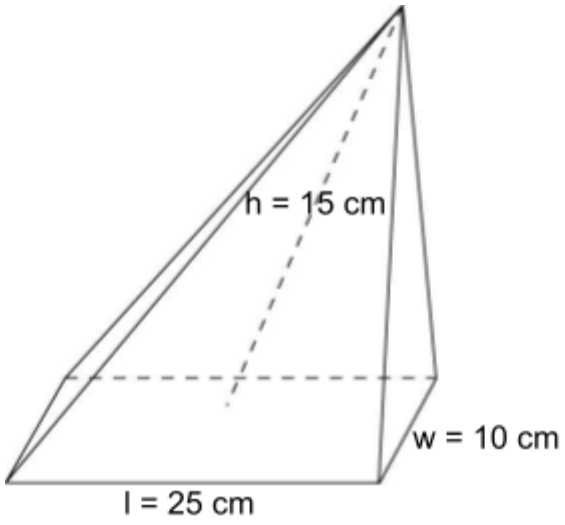


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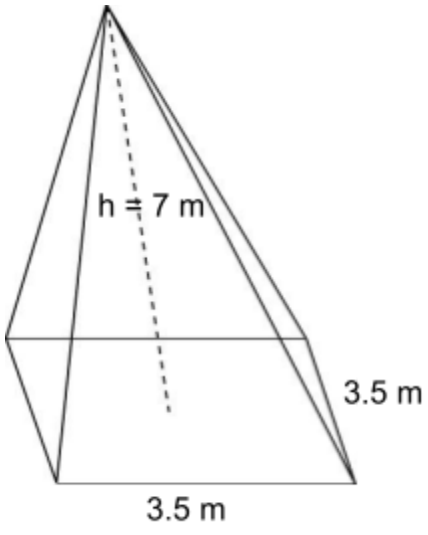



Find the volume of each pyramid:

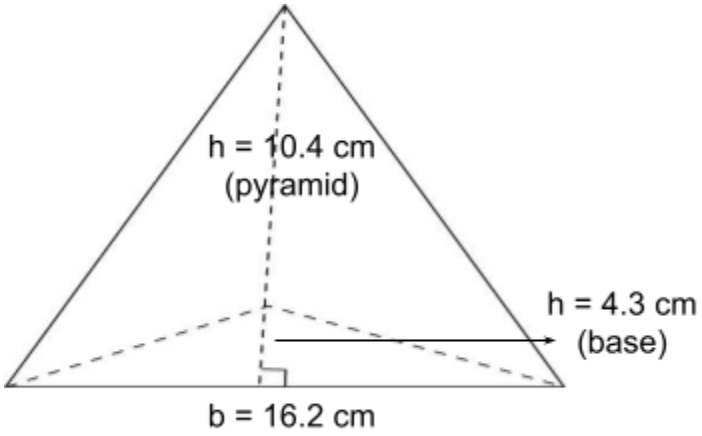
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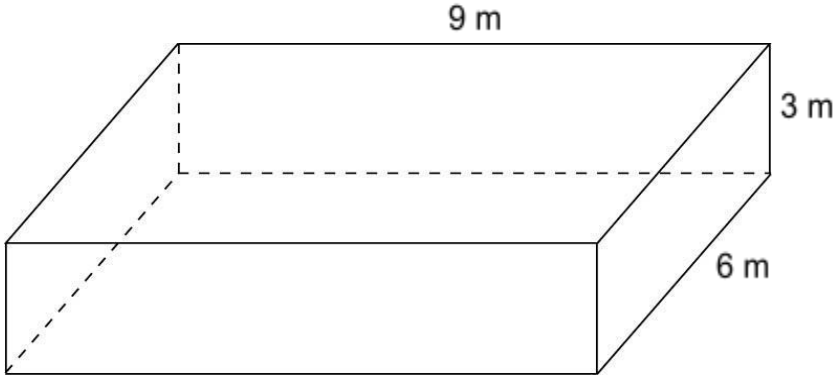

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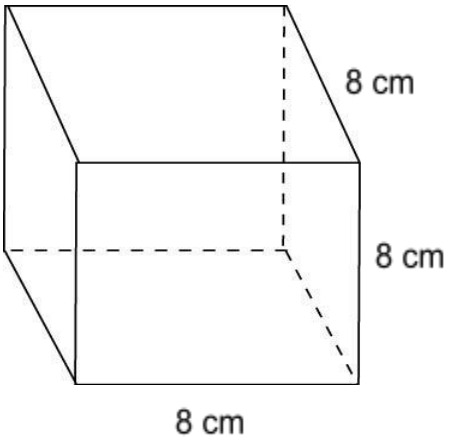
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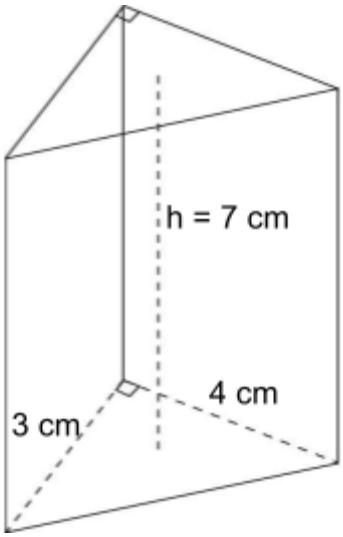

7. What is the volume of a cardboard box that is 9 m long, 6 m wide, and 3 m high?



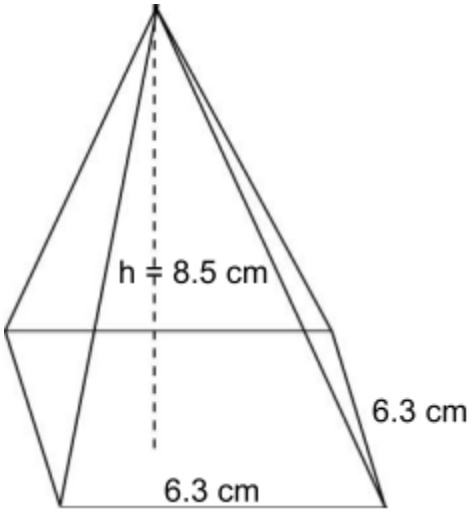

8. Find the volume of a cube with side of 8 cm.



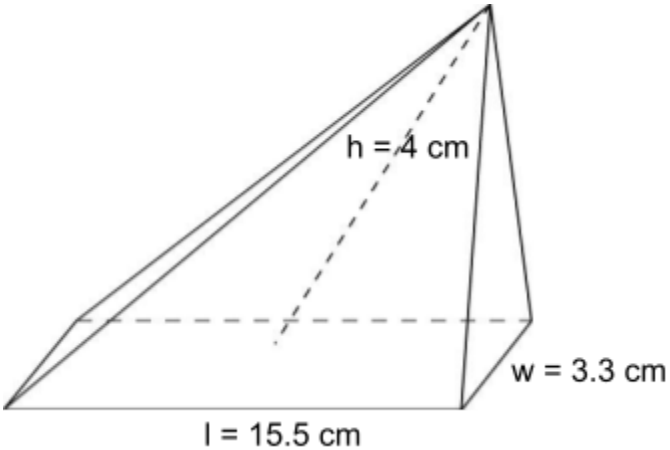
9. Find the volume of a triangular chocolate box.



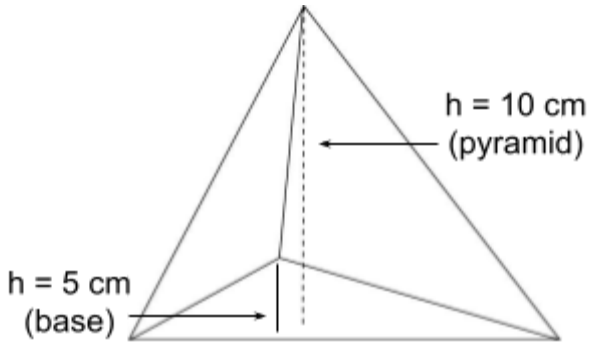

10. Find the volume of a camping tent in a square pyramid shape with a side of the base as 5 cm and the height of a triangle as 7 cm.




11. A pyramid has a rectangular base whose length and width are 15.5 cm and 3.3 cm respectively. The height of the pyramid is 4 cm. Find its volume.



12. Find the volume of a tetra pack juice drink in triangular pyramid shape with the given dimensions.

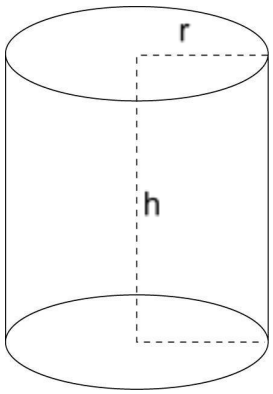



## Lesson 2

## Finding the Volume of a Cylinder, Cone and Sphere

A cylinder has 2 congruent circular bases. The volume of a cylinder is just like finding the volume of a prism.

Figure:



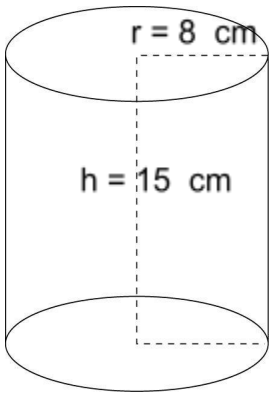

**Volume of a Cylinder**

The volume V of a circular cylinder is the product of the altitude h and the area B of the base. That is,

$V = Bh$ or $V =$ EMBED Equation.3 $r^2h$.

**Example:**

Find the volume of a cylinder which has a radius of 8 cm and a height of 15 cm. (Use 3.14 for $\pi$ )




Solution:

Let B = area of the circular base

$= \pi r^2$

$= (3.14)(8\text{ cm})$

$B = 200.96\text{ cm}^2$

Finding the volume of the cylinder:

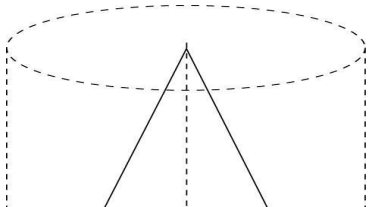
$V = Bh$

$= (200.96\text{ cm}^2)(15\text{ cm})$

$= 3014.4\text{ cm}^3$

**Volume of a Cone**

If a cone is filled with water or sand, and then its content is poured into the cylinder (the cone and cylinder have equal areas) only a third of the cylinder will be filled. This shows that the volume of a cone is $\frac{1}{3}$ that of the cylinder.



h

r

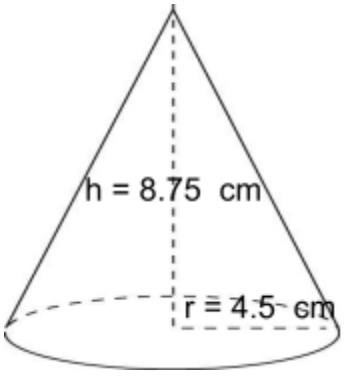
The volume V of a circular cone is one third the product of the altitude h and the area B of the base. That is,

V = EMBED Equation.3 Bh or V = EMBED Equation.3 EMBED Equation.3 $r^2h$

**Example:**

Find the volume of a cone if the radius of its base is 4.5 cm and its height is 8.75 cm (Use $\pi$ = 3.14)

Solution:




$V = \frac{1}{3} \pi r^2 h$

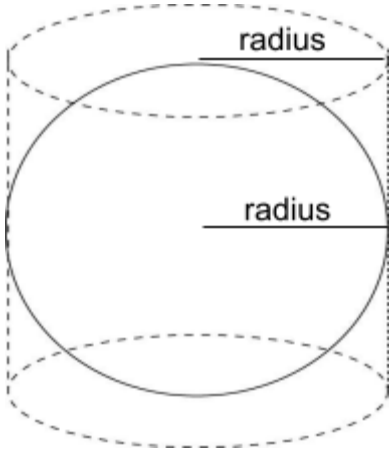
$= \frac{1}{3} (3.14)(4.5 \text{ cm})^2(8.75 \text{ cm})$

$V = 185.46 \text{ cm}^3$

**Volume of a Sphere**

Fill a cylinder with water. Push the sphere into the cylinder and determine the amount of water displaced. About $\frac{2}{3}$ of the water will be displaced, so the volume of the sphere is $\frac{2}{3}$ that of the cylinder.

Figure:




In the figure, the height of the cylinder is equal to the diameter of the sphere, the volume of the cylinder will now be equal to $2\pi r^3$. Since the volume of the sphere is $\frac{2}{3}$ that of the cylinder and the height of the cylinder = 2r, then $V = \frac{2}{3}(2\pi r^3) = \frac{4}{3}\pi r^3$.

| The volume V of a sphere = EMBED Equation.3 EMBED Equation.3 $r^3$ |
|---|

**Example:**

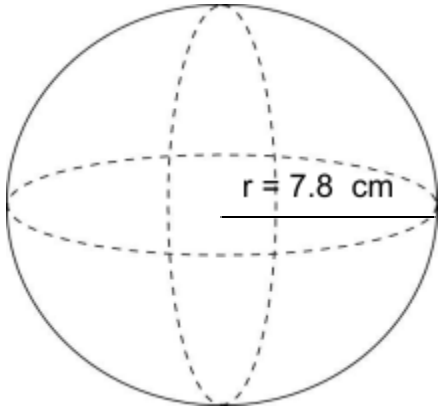
What is the volume of a ball with radius equal to7.8 cm?

Solution:

$$V = \frac{4}{3}\pi r^3$$

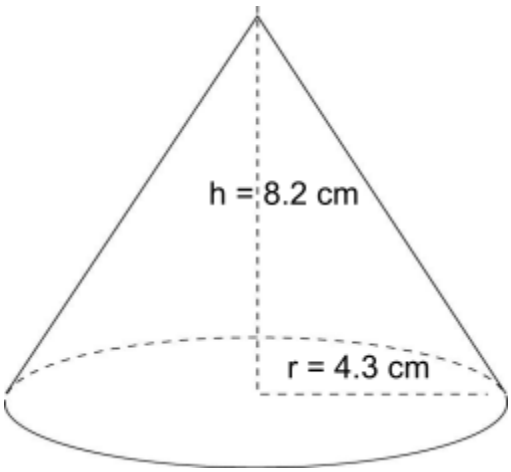
$$= \frac{4}{3}(3.14)(7.8\text{ cm})^3$$

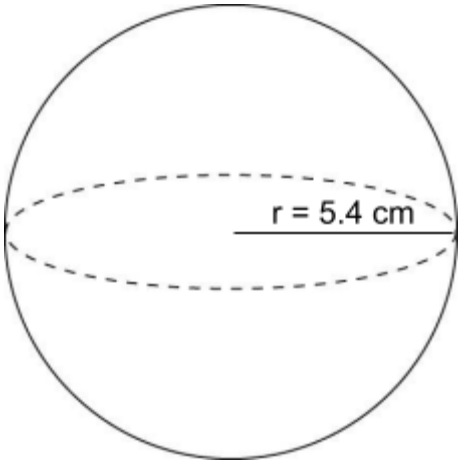
$$= 1{,}986.79\text{ cm}^3$$

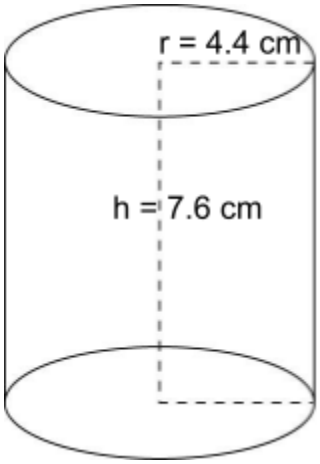


## Try this out

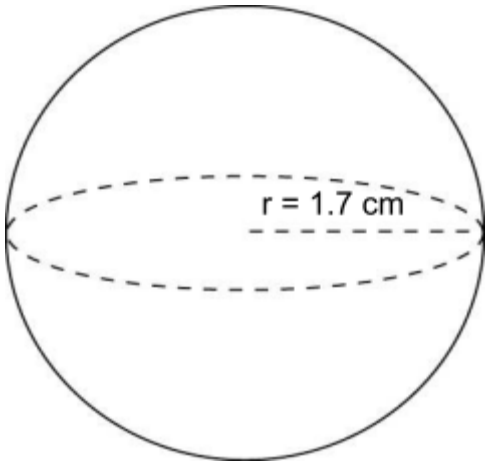
Find the volume of each solid.

1. 

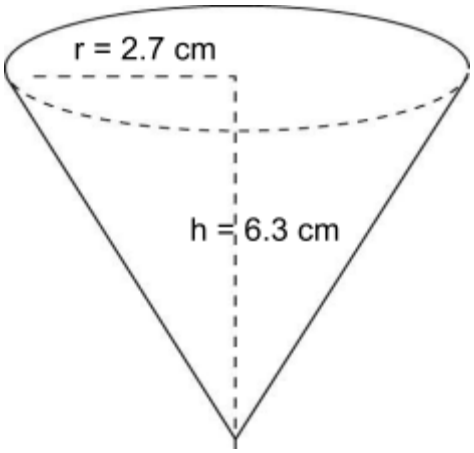

2. 


3. 

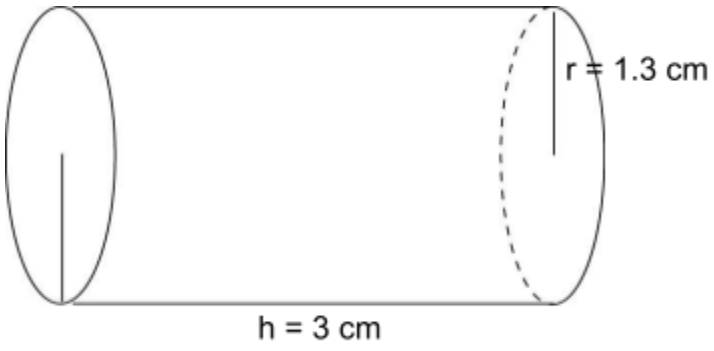
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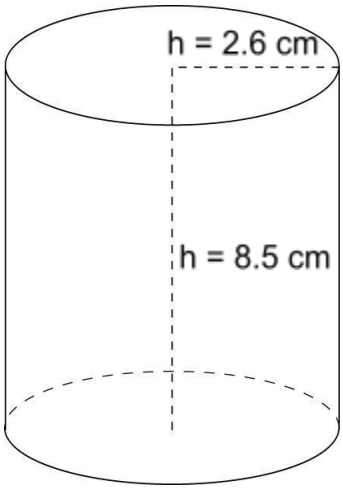

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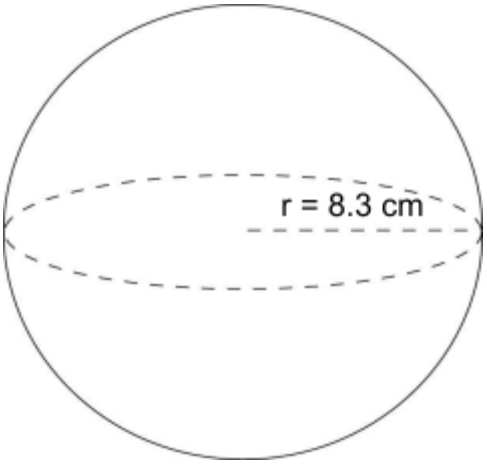

6.



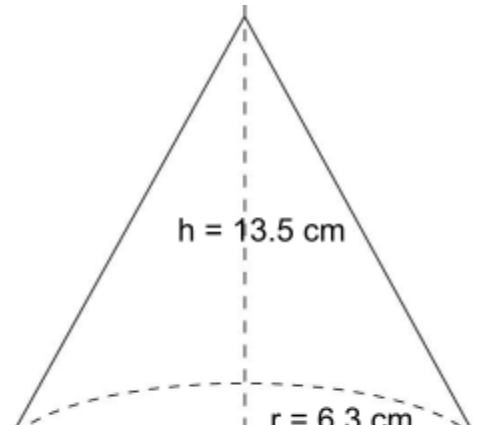
7. A cylindrical water tank is 6.2 meters high. If the radius of its base is 1.8 m, what is its volume.



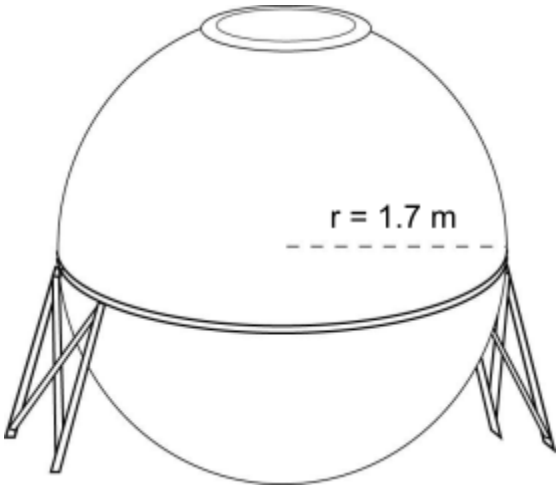

8. The radius of a ball is 5.2 cm. What is its volume?



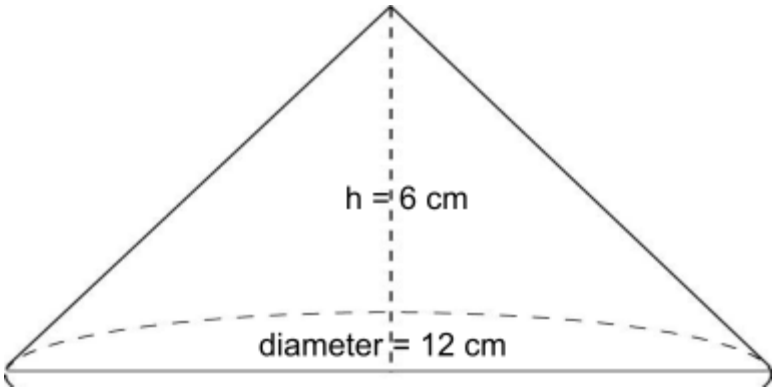

9. Find the volume of a conic solid whose radius is 6.3 cm and its height is 13.5 cm.



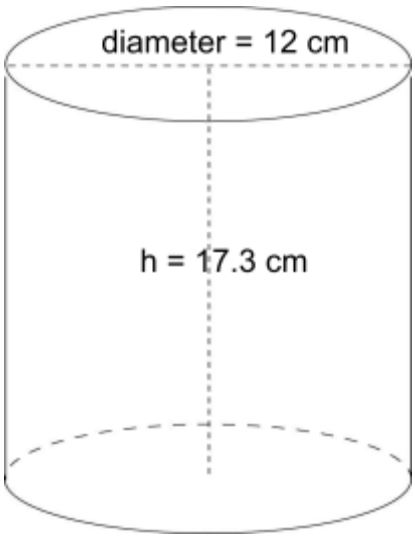
10. Find the volume of a spherical tank whose radius is 1.7 meters.




11. A cone with a diameter of 12 cm and height of 6 cm. Find its volume.



12. A can of milk has a diameter of 12 cm and a height of 17.3 cm. Find its volume.






*Let's summarize*

The volume of a three dimensional figure is the amount of space it encloses.

The volume V of a cube with edge e is the cube of e. That is,

$$V = e^3.$$

The volume V of a rectangular prism is the product of its altitude h, the length l and the width w of the base. That is,

$$V = lwh.$$

The volume of a prism can be expressed in terms of area of the base, B.

The volume V of a prism is the product of its altitude h and area B of the base. That is,

$$V = Bh.$$

The volume V of a pyramid is one third the product of its altitude h and the area B of its base. That is,

$$V = \frac{1}{3} Bh.$$

The volume V of a circular cylinder is the product of the altitude h and the area B of the base. That is,

$$V = Bh \text{ or } V = \pi r^2 h.$$

The volume V of a circular cone is one third the product of the altitude h and the area B of the base. That is,

$$V = \frac{1}{3} Bh \text{ or } V = \frac{1}{3} \pi r^2 h$$

The volume V of a sphere = $\frac{4}{3} \pi r^3$



*What have you learned*

Find the volume of each solid:

1. A cube with edge ( e) = 6.3 cm.

2. A cylinder with h = 15 cm, r = 7.1 cm.

3. A rectangular prism with l = 18 cm, w = 7 cm, h = 5 cm.

4. A square pyramid with s = 8.5 cm, h = 6 cm.

5. A cone with r = 3.8 cm, h = 7.2 cm.

6. A triangular prism with height 16 cm, base ( a right triangle with sides 3, 4 and 5 cm and the right angle between shorter sides).

7. A ball with radius of 13 cm.

8. A triangular pyramid with b = 5 cm, h = 7.2 cm (altitude of the base), h = 8 cm (height of the pyramid).

9. A rectangular pyramid with l = 9 cm, w = 6.3 cm, h = 8 cm (height of the pyramid).

10. A cylindrical tank is 5.4 m high. If the radius of its base is 4.9 m, what is its volume?

11. Find the volume of a rectangular prism which is 42 cm long, 38 cm wide and 22 cm high.

12. Find the volume of a pyramid with a square base if the length of the sides of the base is 3.6 m and a height of 1.8 m.

13. Cube with edge 10.5 cm.

14. Cylinder with radius of base 9.7 cm and height of 12 cm.

15. Rectangular prism with base 12 m by 14.6 m and height of 9.1 m.




How much do you know

1. 13.82 $m^3$
2. 30,395.20 $cm^3$ or 9,680 ∏ $cm^3$
3. 12,750 $cm^3$
4. 50 $m^3$
5. 25.12 $cm^3$ or ∏ $cm^3$
6. 60 $cm^3$
7. 20,569 $cm^3$
8. 38.27 $cm^3$
9. 68.8 $cm^3$
10. 130.47 $m^3$ or 41.55∏ $m^3$
11. 42,550 $cm^3$
12. 6.31 $m^3$
13. 296.30 $cm^3$
14. 2,851.99 $cm^3$ or 908.28∏ $cm^3$
15. 1,200 $m^3$

Try this out
Lesson 1

1. 216 $cm^3$
2. 648 $mm^3$
3. 33.75 $cm^3$
4. 1,250 $cm^3$
5. 28.58 $m^3$
6. 120.74 $cm^3$
7. 162 $m^3$
8. 512 $cm^3$
9. 42 $cm^3$

10. 112.46 $cm^3$
11. 68.2 $cm^3$
12. 125 $cm^3$ (?)

Lesson 2

1. 158.69 $cm^3$ or 50.54 ∏$cm^3$
2. 659.25 $cm^3$ or 209.952 ∏ $cm^3$
3. 462.01 $cm^3$ or 147.136 ∏ $cm^3$
4. 20.57 $cm^3$ or 6.55067 ∏ $cm^3$
5. 48.07 $cm^3$ or 15.309 ∏ $cm^3$
6. 15.92 $m^3$ or 5.07 ∏ $cm^3$
7. 180.42 $cm^3$ or 57.46 ∏ $cm^3$
8. 2,393.88 $cm^3$ or 762.38267 ∏ $cm^3$
9. 560.82 $cm^3$ or 178.605∏ $cm^3$
10. 20.57 $m^3$ or 6.55067 ∏ $m^3$
11. 226.08 $cm^3$ 72 ∏ $cm^3$
12. 1,955.59 $cm^3$ or 622.8 ∏ $cm^3$

What have you learned

1. 250.047 $cm^3$
2. 2.374.311 $cm^3$
3. 630 $cm^3$
4. 144.5 $cm^3$
5. 108.82 $cm^3$
6. 96 $cm^3$
7. 9,198.11 $cm^3$
8. 48 $cm^3$
9. 151.2 $cm^3$
10. 407.11356 $m^3$
11. 35,112 $cm^3$

12. 7.776 $m^3$
13. 1,157.625 $cm^3$
14. 3,545.3112 $cm^3$
15. 1,594.32 $m^3$