(Effective Alternative Secondary Education)

MATHEMATICS III
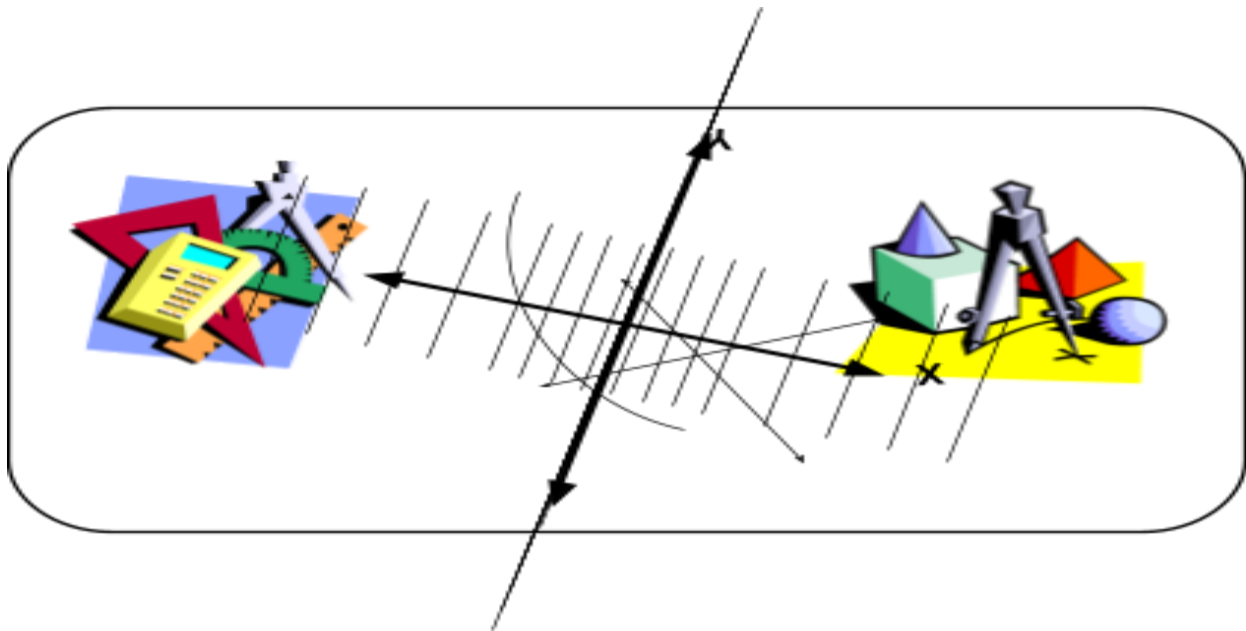
MODULE 8

Geometry of Shape and Size

Geometry of Shape and Size

What this module is about

This module is about volume of solids. The volume of a solid is the number of cubic units contained in the solid. If measures are given in centimeter, the volume is stated in cubic cm, written as cm^3.

What you are expected to learn

This module is designed for you to:

1. define volume of solids.

2. find the volume of solids such as:

- cube
- prism (rectangular, triangular)
- pyramid (square, rectangular, triangular)
- cylinder
- cone
- sphere

3. solve problems involving volume of solids.

How much do you know

Find the volume of each solid:

1. a cube with side (s) = 2.4 m

2. a cylinder with h = 20 cm, r = 22 cm.

3. a rectangular prism with l = 25 cm, w = 17 cm, h = 30 cm

4. a square pyramid with s = 5 m, h = 6 m.

5. a cone with r = 2 cm, h = 6 cm.

6. a triangular prism with height 10 cm, base (a right triangle with sides 3, 4 and 5 cm and the right angle between shorter sides).

7. a ball with radius of 17 cm (use = 3.14)

8. a triangular pyramid with b = 4 cm, h = 8.2 cm (altitude of the base), h = 7 cm (height of the pyramid).

9. a rectangular pyramid with l = 6 cm, w = 4.3 cm, h = 8 cm (height of the pyramid)

10. a cylindrical tank is 5.3 meters high. If the radius of its base is 2.8 meters, what is its volume?

11. Find the volume of a rectangular prism which is 46 cm long, 37 cm wide and 25 cm high.

12. Find the volume of a pyramid with a square base if the length of the sides of the base is 2.4 m and the height of the triangular face is 3.5 m.

13. cube with edge of $6 \frac{2}{3}$ cm.

14. cylinder with radius of base 8.7 cm and height 12 cm.

15. rectangular prism with base 8 m by 10 m by 15 m.

What you will do

Lesson 1

Finding the Volume of a Cube, Prism and Pyramid

One problem with rooms that have high ceilings is that they are hard to heat and cool. The amount of air in a room determines the heating or cooling power needed. To find the amount of air in a room, you need to find the volume of the room.

In finding volume of solids, you have to consider the area of a face and height of the solid. If the base is triangular, you have to make use of the area of a triangle, if rectangular, make use of the area of a rectangle and so on.

The next examples will help you to understand more about volume or the amount of space in three – dimensional figures.
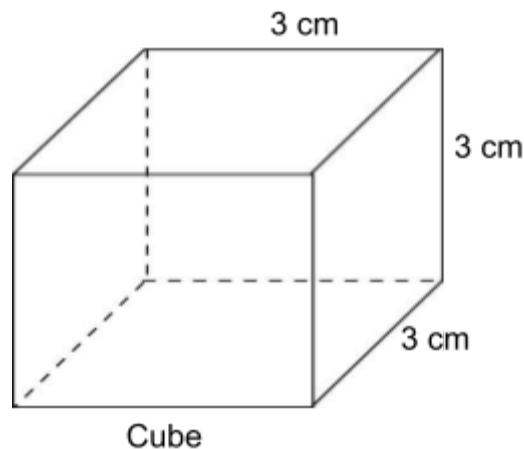
Volume of a cube

> The volume V of a cube with edge e is the cube of e. That is,
> $$V = e^3.$$

Example:

Find the volume of a cube with edge (*e*) of 3 cm.

Figure:

3 cm

3 cm

3 cm

Cube

Solution:

$$V = e^3$$

Substituting e by 3 cm:

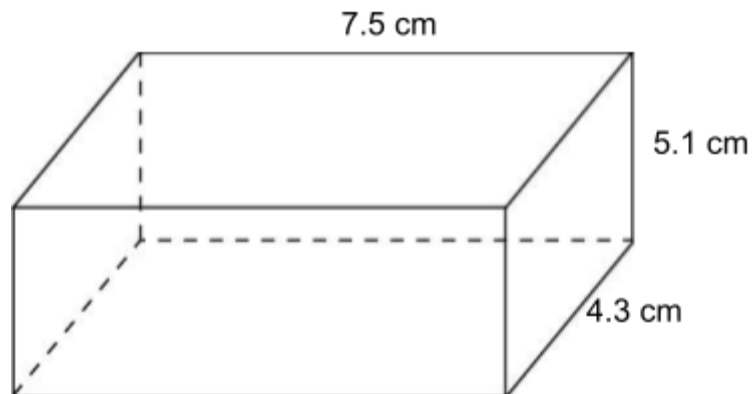$$V = 3^3$$

$$V = 27 \text{ cm}^3$$

Volume of Prism

> The volume V of a rectangular prism is the product of its altitude h, the length l and the width w of the base. That is,
>
> $$V = lwh.$$

Example:

Find the volume of a rectangular prism whose length is 7.5 cm, width is 4.3 cm and thickness is 5.1 cm.

Figure:

Solution:

$$V = lwh$$

$$V = (7.5\ cm)(4.3\ cm)(5.1\ cm)$$

$$V = 164.475\ cm^3$$

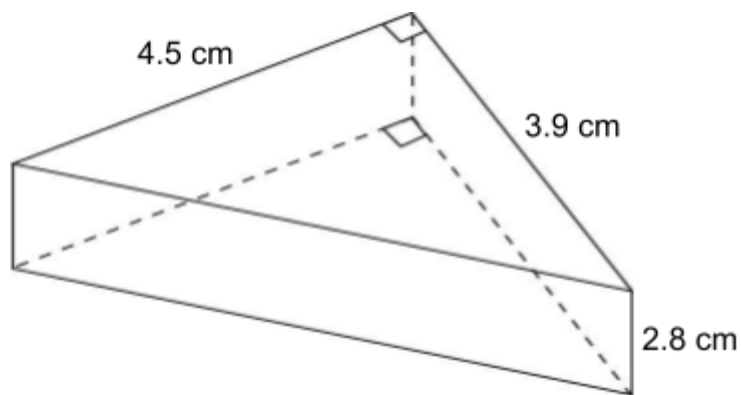The volume of a prism can also be expressed in terms of area of the base, B.

> The volume V of a prism is the product of its altitude h and area B of the base. That is,
>
> $$V = Bh.$$

Example:

Find the volume of a triangular prism whose dimensions is given in the figure below.

4.5 cm

3.9 cm

2.8 cm

Solution:

Let B = area of the triangular base

$$B = \frac{1}{2} bh$$

$$= \frac{1}{2} (4.5 \text{ cm})(3.9 \text{ cm})$$

$$B = 8.775 \text{ cm}^2$$

Finding the volume of the prism:

$$V = Bh$$

$$= 8.775 \text{ cm}^2 (2.8 \text{ cm})$$

$$= 24.57 \text{ cm}^3$$

Volume of Pyramids

Consider a pyramid and a prism having equal altitudes and bases with equal areas. If the pyramid is filled with water or sand and its contents poured into a prism, only a third of the prism will be filled. Thus the volume of a pyramid is $\frac{1}{3}$ the volume of the prism.
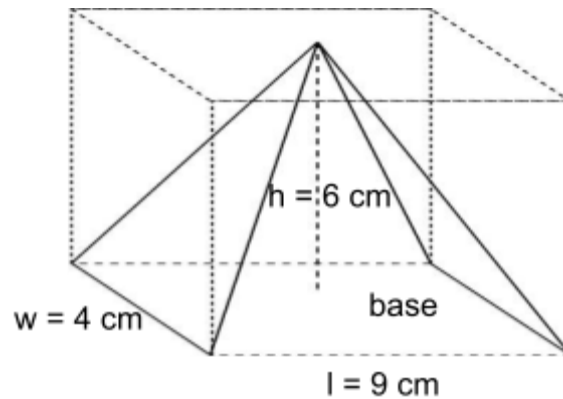
> The volume V of a pyramid is one third the product of its altitude h and the area B of its base. That is,
> $$V = \text{EMBED Equation.3 } Bh.$$

Example:

1. Find the volume of the rectangular pyramid with the given dimensions.

Figure:

h = 6 cm

base

w = 4 cm

l = 9 cm

Solution:

Let B = the area of the rectangular base

B = lw

= (9 cm)(4 cm)

$$= 36 \text{ cm}^2$$

Finding the volume V:

$$V = \frac{1}{3} Bh$$

$$= \frac{1}{3} (36 \text{ cm}^2) (6 \text{ cm})$$

$$= 72 \text{ cm}^3$$

2. Find the volume of a square pyramid with a side of the base as 4 cm and the height of a pyramid as 6 cm.

Figure:

h = 6 cm

s = 4 cm
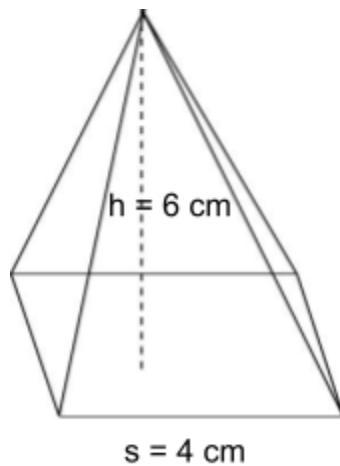
Solution:

Let B = area of the square base

$$= s^2$$

$$= (4 \text{ cm})^2$$

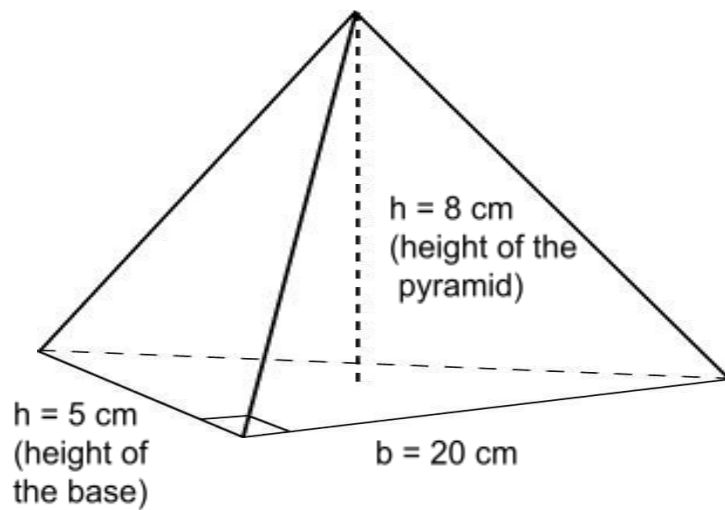$$B = 16 \text{ cm}^2$$

Finding the volume of the pyramid:

$$V = \frac{1}{3}Bh$$

$$= \frac{1}{3}(16 \text{ cm}^2)(6 \text{ cm})$$

$$= 32 \text{ cm}^3$$

3. Find the volume of a triangular pyramid with the given dimensions.

Figure:

h = 8 cm
(height of the pyramid)

h = 5 cm
(height of the base)

b = 20 cm

Solution:

Let B = area of the triangular base

$$B = \frac{1}{2}bh$$

$$= \frac{1}{2}(20 \text{ cm})(5 \text{ cm})$$
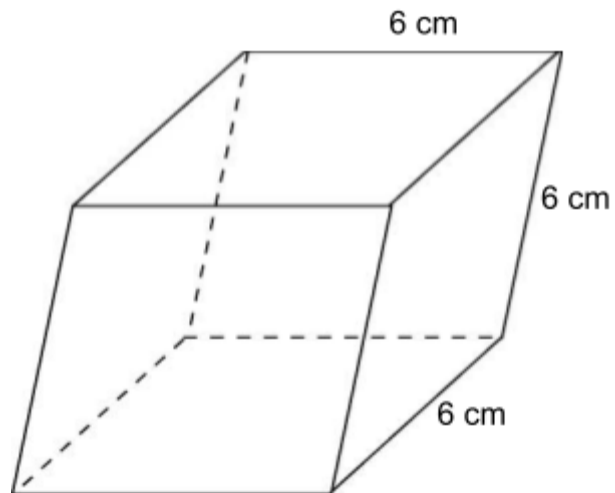
B = 50 cm^2

Finding the volume of the pyramid:

V = Bh

= (50 cm^2)(8cm)

V = 400 cm^3

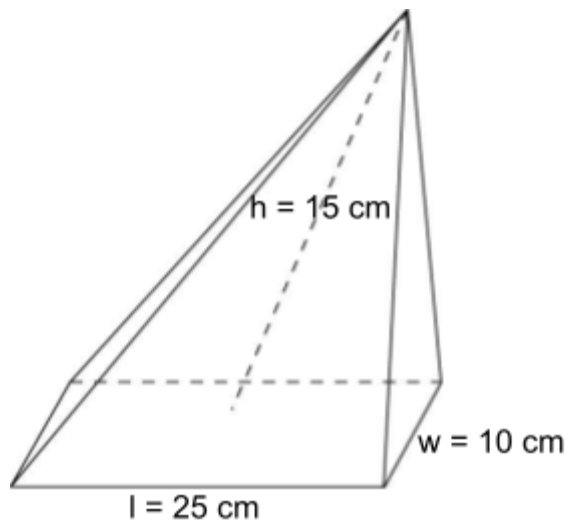## Try this out

Find the volume of each solid:
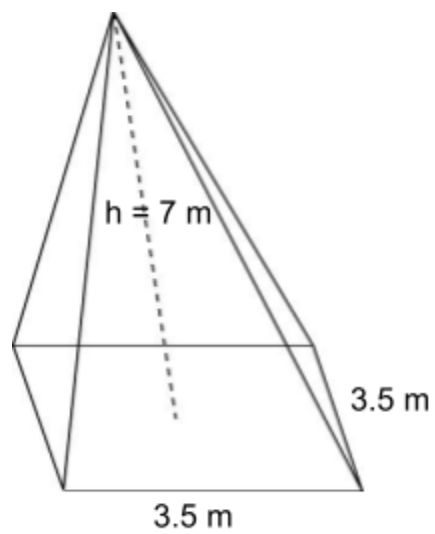
1.

6 cm

6 cm

6 cm

2.

12 mm

6 mm

9 mm

3.
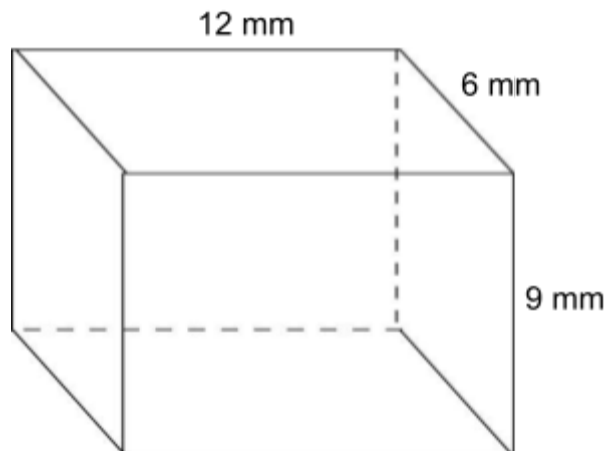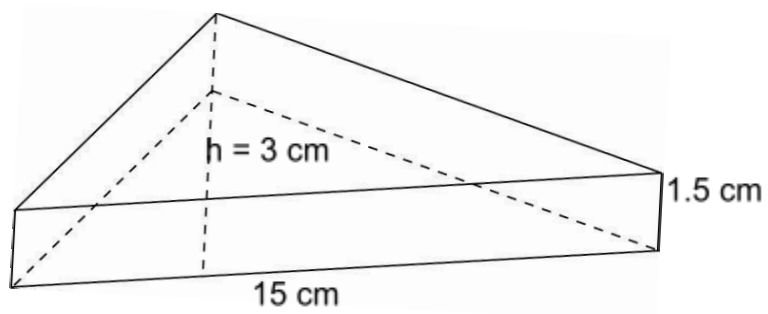
Find the volume of each pyramid:

4.
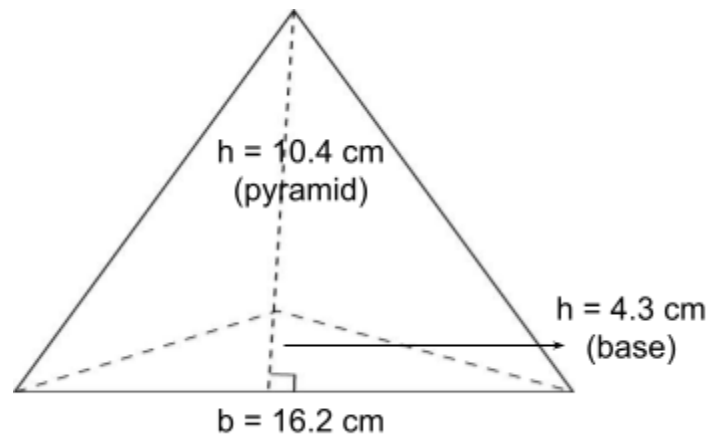

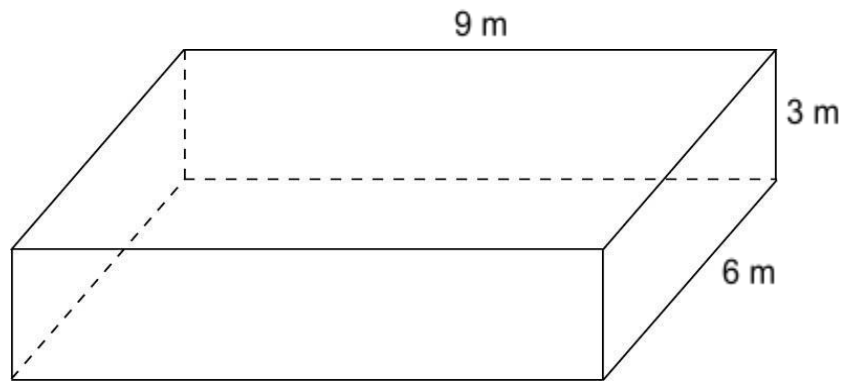5.

6.

h = 10.4 cm
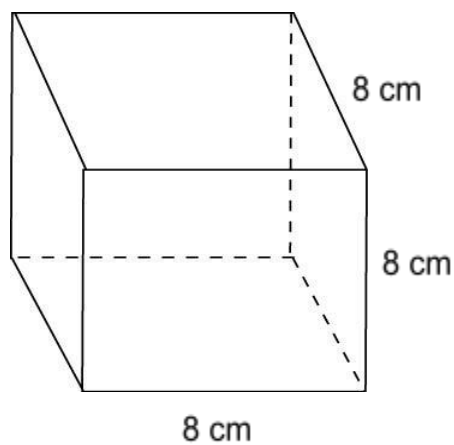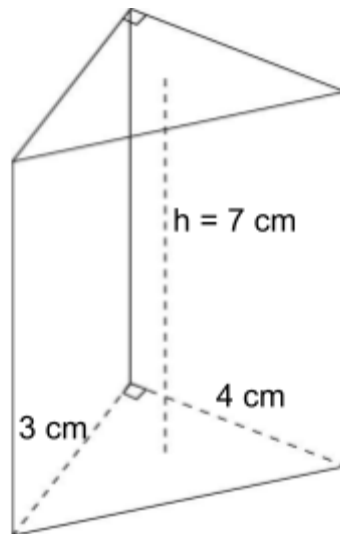(pyramid)

h = 4.3 cm
(base)

b = 16.2 cm

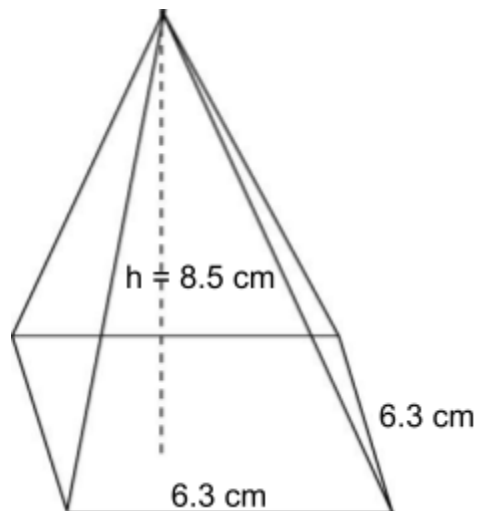7. What is the volume of a cardboard box that is 9 m long, 6 m wide, and 3 m high?

9 m

3 m

6 m

8. Find the volume of a cube with side of 8 cm.

8 cm

8 cm

8 cm

9. Find the volume of a triangular chocolate box.

h = 7 cm

4 cm

3 cm

10. Find the volume of a camping tent in a square pyramid shape with a side of the base as 5 cm and the height of a triangle as 7 cm.

h ≑ 8.5 cm

6.3 cm

6.3 cm

11. A pyramid has a rectangular base whose length and width are 15.5 cm and 3.3 cm respectively. The height of the pyramid is 4 cm. Find its volume.

h = 4 cm

w = 3.3 cm

l = 15.5 cm
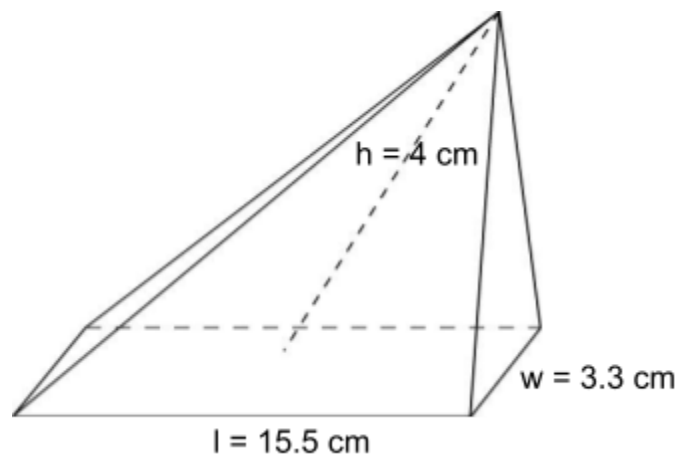
12. Find the volume of a tetra pack juice drink in triangular pyramid shape with the given dimensions.

h = 10 cm
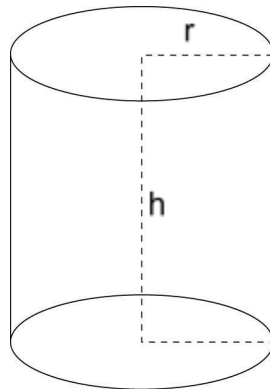(pyramid)

h = 5 cm
(base)
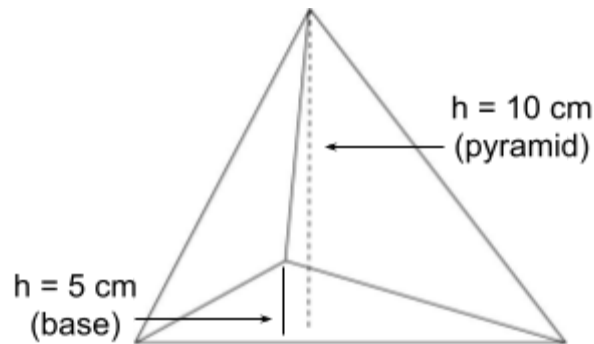
Lesson 2

Finding the Volume of a Cylinder, Cone and Sphere

A cylinder has 2 congruent circular bases. The volume of a cylinder is just like finding the volume of a prism.
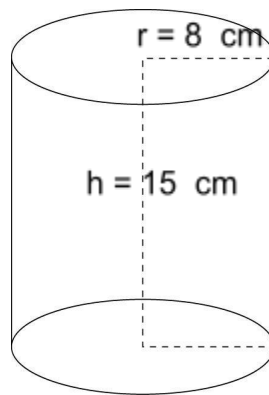
Figure:

r

h

Volume of a Cylinder

The volume V of a circular cylinder is the product of the altitude h and the area B of the base. That is,

V = Bh or V = EMBED Equation.3 r²h.

Example:

Find the volume of a cylinder which has a radius of 8 cm and a height of 15 cm. (Use 3.14 for π)

Solution:

Let B = area of the circular base

$= \pi r^2$

= (3.14)(8 cm)

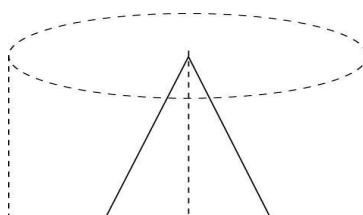B = 200.96 cm²

Finding the volume of the cylinder:

V = Bh

= (200.96 cm²)(15 cm)

= 3014.4 cm³

Volume of a Cone

If a cone is filled with water or sand, and then its content is poured into the cylinder (the cone and cylinder have equal areas) only a third of the cylinder will be filled. This shows that the volume of a cone is $\frac{1}{3}$ that of the cylinder.

h

r

The volume V of a circular cone is one third the
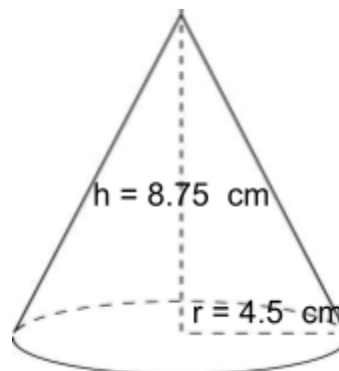product of the altitude h and the area B of the base. That is,

V = EMBED Equation.3 Bh or V = EMBED Equation.3
EMBED Equation.3 r²h

Example:

Find the volume of a cone if the radius of its base is 4.5 cm and its height is 8.75 cm (Use π = 3.14)

Solution:

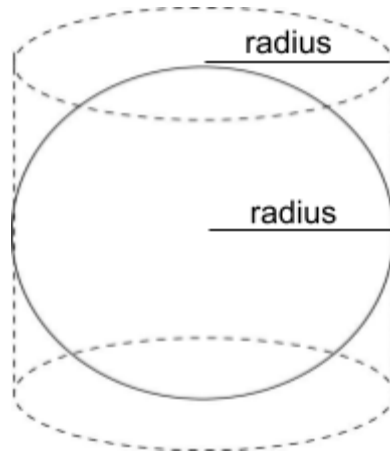$V = \dfrac{1}{3} \pi r^2 h$

$= \dfrac{1}{3}$ (3.14)(4.5 cm)²(8.75 cm)

V = 185.46 cm³

h = 8.75 cm

r = 4.5 cm

Volume of a Sphere

Fill a cylinder with water. Push the sphere into the cylinder and determine the amount of water displaced. About $\frac{2}{3}$ of the water will be displaced, so the volume of the sphere is $\frac{2}{3}$ that of the cylinder.

Figure:

In the figure, the height of the cylinder is equal to the diameter of the sphere, the volume of the cylinder will now be equal to $2\pi r^3$. Since the volume of the sphere is $\frac{2}{3}$ that of the cylinder and the height of the cylinder = 2r, then
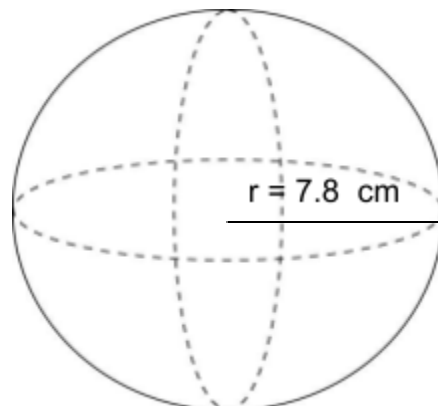
$$V = \frac{2}{3}(2\pi r^3) = \frac{4}{3}\pi r^3.$$

> The volume V of a sphere = EMBED
> Equation.3   EMBED Equation.3 r^3

Example:

What is the volume of a ball with radius equal to7.8 cm?

Solution:

$$V = \frac{4}{3}\pi r^3$$

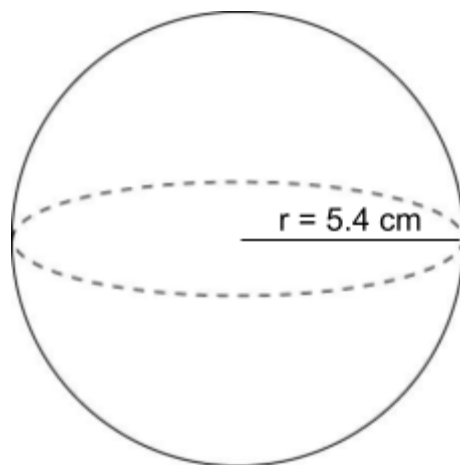$$= \frac{4}{3}(3.14)(7.8 \text{ cm})^3$$

$$= 1,986.79 \text{ cm}^3$$

r = 7.8 cm

Try this out

Find the volume of each solid.

1.

h = 8.2 cm

r = 4.3 cm

2.

r = 5.4 cm

3.

r = 4.4 cm

h = 7.6 cm

4.

r = 1.7 cm

5.

r = 2.7 cm

h = 6.3 cm

6.

r = 1.3 cm

h = 3 cm
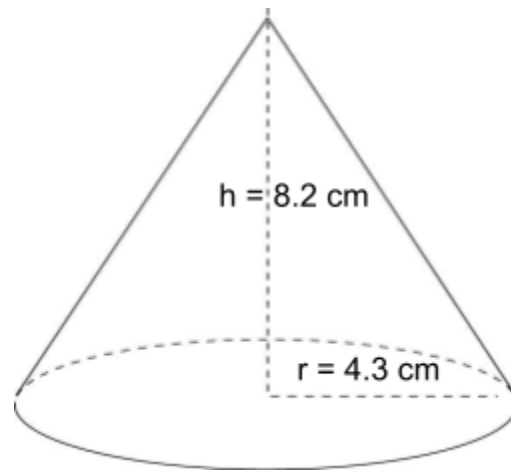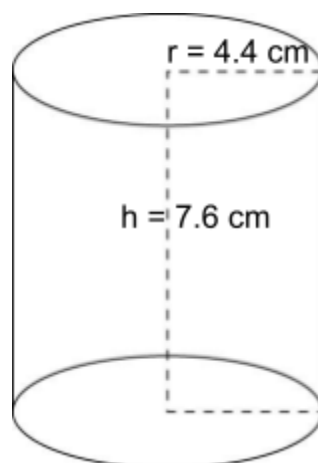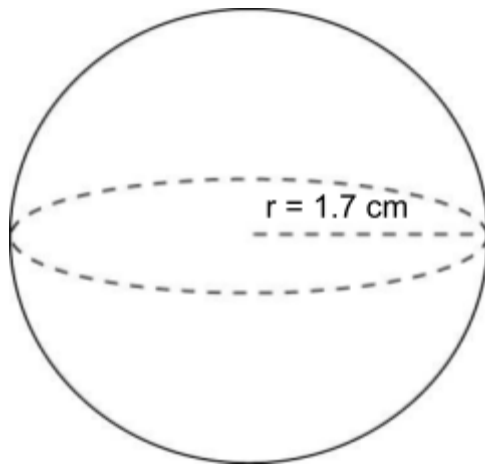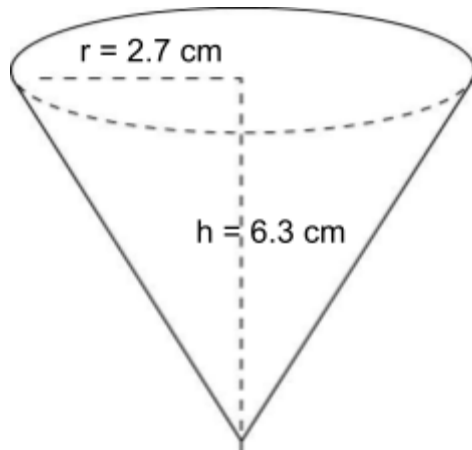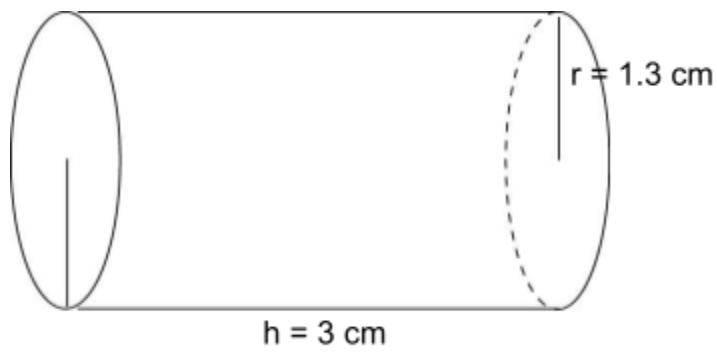
7. A cylindrical water tank is 6.2 meters high. If the radius of its base is 1.8 m, what is its volume.

h = 2.6 cm

h = 8.5 cm
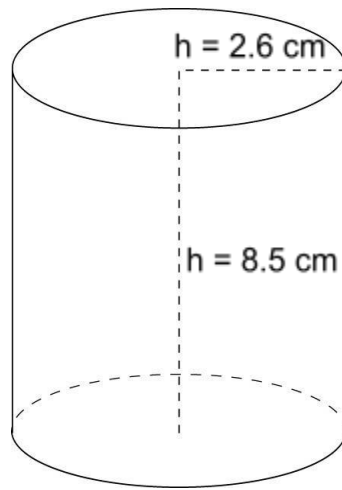
8. The radius of a ball is 5.2 cm. What is its volume?

r = 8.3 cm
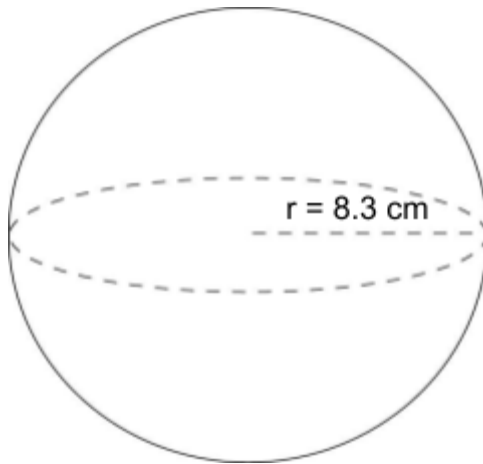
9. Find the volume of a conic solid whose radius is 6.3 cm and its height is 13.5 cm.

h = 13.5 cm

r = 6.3 cm
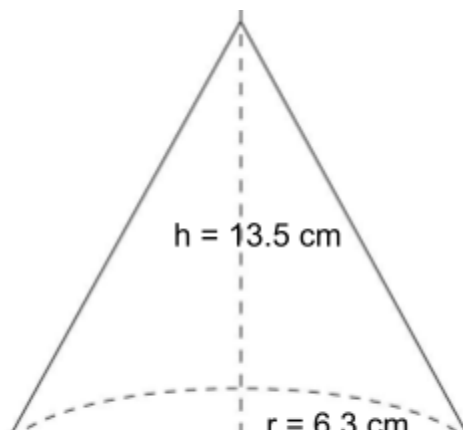
10. Find the volume of a spherical tank whose radius is 1.7 meters.

r = 1.7 m
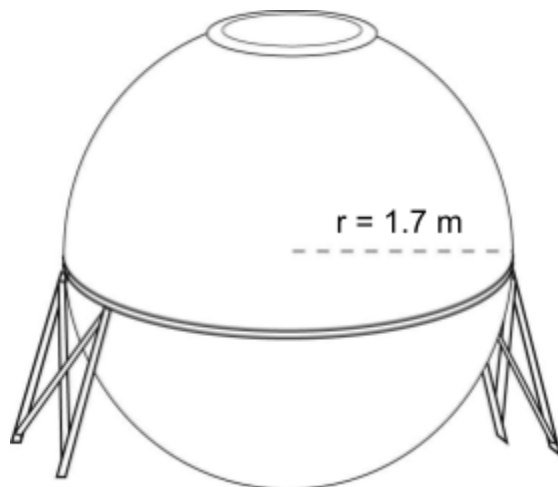
11. A cone with a diameter of 12 cm and height of 6 cm. Find its volume.

h = 6 cm
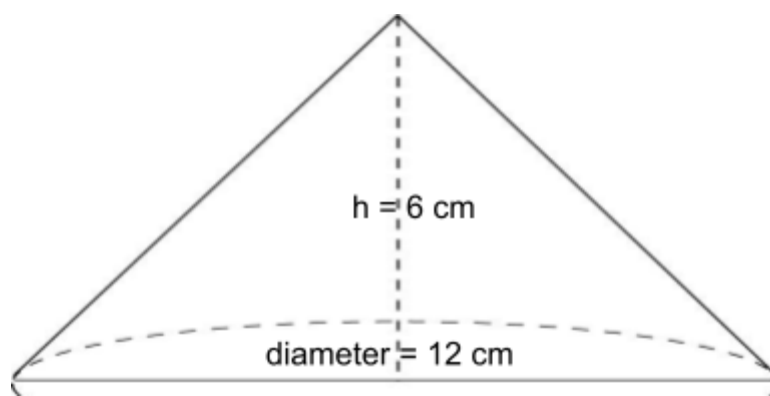
diameter = 12 cm

12. A can of milk has a diameter of 12 cm and a height of 17.3 cm. Find its volume.

diameter = 12 cm

h = 17.3 cm

Let's summarize

The volume of a three dimensional figure is the amount of space it encloses.

The volume V of a cube with edge e is the cube of e. That is,

$$V = e^3.$$

The volume V of a rectangular prism is the product of its altitude h, the length l and the width w of the base. That is,

$$V = lwh.$$

The volume of a prism can be expressed in terms of area of the base, B.

The volume V of a prism is the product of its altitude h and area B of the base. That is,

$$V = Bh.$$

The volume V of a pyramid is one third the product of its altitude h and the area B of its base. That is,

$$V = \frac{1}{3} Bh.$$

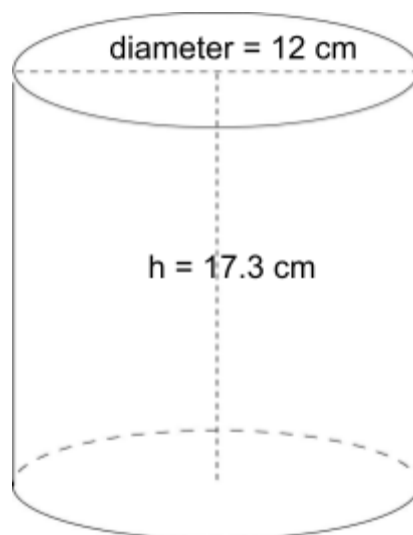The volume V of a circular cylinder is the product of the altitude h and the area B of the base. That is,

$$V = Bh \text{ or } V = \pi r^2 h.$$

The volume V of a circular cone is one third the product of the altitude h and the area B of the base. That is,

$$V = \frac{1}{3} Bh \text{ or } V = \frac{1}{3} \pi r^2 h$$

The volume V of a sphere = $\frac{4}{3} \pi r^3$

What have you learned

Find the volume of each solid:

1. A cube with edge (e) = 6.3 cm.

2. A cylinder with h = 15 cm, r = 7.1 cm.

3. A rectangular prism with l = 18 cm, w = 7 cm, h = 5 cm.

4. A square pyramid with s = 8.5 cm, h = 6 cm.

5. A cone with r = 3.8 cm, h = 7.2 cm.

6. A triangular prism with height 16 cm, base (a right triangle with sides 3, 4 and 5 cm and the right angle between shorter sides).

7. A ball with radius of 13 cm.

8. A triangular pyramid with b = 5 cm, h = 7.2 cm (altitude of the base), h = 8 cm (height of the pyramid).

9. A rectangular pyramid with l = 9 cm, w = 6.3 cm, h = 8 cm (height of the pyramid).

10. A cylindrical tank is 5.4 m high. If the radius of its base is 4.9 m, what is its volume?

11. Find the volume of a rectangular prism which is 42 cm long, 38 cm wide and 22 cm high.

12. Find the volume of a pyramid with a square base if the length of the sides of the base is 3.6 m and a height of 1.8 m.

13. Cube with edge 10.5 cm.

14. Cylinder with radius of base 9.7 cm and height of 12 cm.

15. Rectangular prism with base 12 m by 14.6 m and height of 9.1 m.

How much do you know

1. 13.82 m^3
2. 30,395.20 cm^3 or 9,680 $\sqcap$ cm^3
3. 12,750 cm^3
4. 50 m^3
5. 25.12 cm^3 or $\sqcap$ cm^3
6. 60 cm^3
7. 20,569 cm^3
8. 38.27 cm^3
9. 68.8 cm^3
10. 130.47 m^3 or 41.55$\sqcap$ m^3
11. 42,550 cm^3
12. 6.31 m^3
13. 296.30 cm^3
14. 2,851.99 cm^3 or 908.28$\sqcap$ cm^3
15. 1,200 m^3

Try this out
Lesson 1

1. 216 cm^3
2. 648 mm^3
3. 33.75 cm^3
4. 1,250 cm^3
5. 28.58 m^3
6. 120.74 cm^3
7. 162 m^3
8. 512 cm^3
9. 42 cm^3

10. 112.46 cm^3

11. 68.2 cm^3

12. 125 cm^3 (?)

Lesson 2

1. 158.69 cm^3 or 50.54 πcm^3

2. 659.25 cm^3 or 209.952 π cm^3

3. 462.01 cm^3 or 147.136 π cm^3

4. 20.57 cm^3 or 6.55067 π cm^3

5. 48.07 cm^3 or 15.309 π cm^3

6. 15.92 m^3 or 5.07 π cm^3

7. 180.42 cm^3 or 57.46 π cm^3

8. 2,393.88 cm^3 or 762.38267 π cm^3

9. 560.82 cm^3 or 178.605π cm^3

10. 20.57 m^3 or 6.55067 π m^3

11. 226.08 cm^3 72 π cm^3

12. 1,955.59 cm^3 or 622.8 π cm^3

What have you learned

1. 250.047 cm^3

2. 2.374.311 cm^3

3. 630 cm^3

4. 144.5 cm^3

5. 108.82 cm^3

6. 96 cm^3

7. 9,198.11 cm^3

8. 48 cm^3

9. 151.2 cm^3

10. 407.11356 m^3

11. 35,112 cm^3

12. 7.776 m^3

13. $1{,}157.625 \text{ cm}^3$

14. $3{,}545.3112 \text{ cm}^3$

15. $1{,}594.32 \text{ m}^3$