Whiteboard: 1/60, f4.5, iso 640, EV+1.3, Auto Focus, H4n **INPUT** @ **95%** gain.

BBB 1/60, f6.7, iso 640, **WB Use Grey Card**, Manual Audio at level 11, focal length ½ way to ∞. H4n **INPUT** @ **95%** gain.

BBB Check Horizontal Level & Billy's Lights!! Middle line at bottom 1/3 of desk & remember Billy and Bo chair locations!

From Power to Work using an Integral – Example

Mr.p: Good morning. The net power delivered to an object is described by the equation net power equals 4.00 times time squared plus time in watts. Bobby, please determine the net work done on the object from 0 to 4.00 seconds.

$$P_{net} = \left[4.00t^2 + t\right] watts$$

- # Bobby: [♪ Flipping Physics ♪] Okay. Well. We know the equation for net power and we are looking for the net work done on the object from zero to 4.00 seconds. … We have a couple of equations for work. The general equation is work equals the integral with respect to x-position from x-position initial to x-position final of force in the x-direction. … The equation for work done by a constant force equals the dot product of force and displacement or force times displacement times cosine theta. … But I do not see how any of those help us because the net power equation is with respect to time, not position.

- Billy: … Actually, we know power equals the derivative of work with respect to time.
- Bo: … And power equals the dot product of force

and velocity or force times velocity times cosine theta.

- Bobby: Yeah. …
- **Mr.p: (Remember, any derivative can be rearranged to form an integral, because, class, another term for an integral is an …)**
- BBB: Antiderivative.
- Bobby: Right. We did this before. Starting with power equals the derivative of work with respect to time, multiply both sides by dt. … Then take the integral of both sides. … The left-hand side is with respect to work, so the limits are from work initial to work final. … The right-hand side is with respect

to time, so the limits are from time initial to time final. … The integral of just dW is just work with the same limits. … That equals work final minus work initial or change in work. … However, work by definition, is over a change in position or a change in time. So, we get work equals the integral from time initial to time final of power with respect to time.

- Billy: And now we can plug in everything we know. We get the net work from zero to 4 seconds equals the integral from zero to 4 seconds of the quantity 4 times time squared plus time all with respect to time. … That integral equals, well, 2 plus 1 is 3, so 4 times time cubed divided by 3 minus … 1 plus 1 is 2, so time squared over 2.

- Bo: Don’t forget the limits.
- Billy: Right. From zero to 4 seconds. … Now we can substitute in numbers for our limits.
- **Mr.p: (Actually Billy, I’d like to substitute in the variables for time final and time initial into our equation. I’d like to have it written down this way one more time because this is still pretty new for some of you. After this lesson I am fine with you not showing this single step. … Billy, please keep going.)**
- Billy: ***Now*** we can substitute in numbers for our

limits. We get … 4 times 4 cubed all over 3 minus … 4 squared all over 2 minus … well we substitute zero in for all each time initial, so minus zero. …. And that equals {calc}

- Bo: 280 over 3.
- **Mr.p: (Actually Bo, in physics we do not usually leave our answers in fractions because we need to show the correct number of sig figs.)**
- Bo: Sure. Then it is 93.3 repeating or 93.3 with 3 sig figs.
- **Mr.p: (And what are the units.)**
- Bo: Watts.

- Bobby and Billy: … Joules.
- Bo: Oh, right, joules. We started with power, so I got a bit befuddled.
- Billy: Befuddled.
- Bo: Yeah. Befuddled. You got beef with that.
- Billy: No, I do not have any beef with befuddled. Though it does leave me a skosh bewildered and perhaps even perplexed.
- Bo: Good.
- Bobby: You both are a modicum hellaciously aggravating.
- Billy and Bo: … Uh?

Mr.p: {confused} Okay. Very nice solution. … And lastly, because net work equals change in kinetic energy, we know the change in kinetic energy from 0 to 4 seconds of the object equals positive 93.3 joules. In other words, because the net work done on the object is positive, the net work done on the object is causing a positive change in the kinetic energy of the object; it is increasing the speed of the object. … Thank you very much for learning with me today, I enjoyed learning with you.

$$W = \vec{F} \bullet \Delta\vec{r} = F\,\Delta r \cos\theta$$

$$P = \frac{dW}{dt} = \vec{F} \bullet \vec{v} = Fv \cos\theta$$

$$P=\frac{dW}{dt}\Rightarrow dW=P\,dt\Rightarrow\int_{W_i}^{W_f}dW=\int_{t_i}^{t_f}P\,dt\Rightarrow W\Big]_{W_i}^{W_f}=W_f-W_i=\Delta W=\int_{t_i}^{t_f}P\,dt\Rightarrow W=\int_{t_i}^{t_f}P\,dt$$

$$W_{0\to 4\sec}=\int_0^4\left(4t^2+t\right)dt=\left[\frac{4t^3}{3}+\frac{t^2}{2}\right]_0^4=\left[\left(\frac{4t_f^{\ 3}}{3}+\frac{t_f^{\ 2}}{2}\right)-\left(\frac{4t_i^{\ 3}}{3}+\frac{t_i^{\ 2}}{2}\right)\right]_0^4=\left(\frac{(4)(4)^3}{3}+\frac{(4)^2}{2}\right)-0$$

$$\Rightarrow W_{0\to 4\sec}=\frac{280}{3}=93.\bar{3}\approx 93.3J$$