SECOND TERM E-LEARNING NOTE

SUBJECT: FURTHER MATHEMATICS

CLASS: SS1

SCHEME OF WORK

WEEK	TOPIC
1	Arithmetic Progression (AP)
2	Geometric Progression (GP)
3	Linear inequalities in one variable
4	Inequalities in two variables (Graph of inequalities)
5	Introduction to the concept of functions.
6	Review of half term work.
7	Functions (one – to – one, onto, composite and inverse functions)
8	Trigonometric ratio: Graph of Sine, Cosine and tangent of angles, deviation of trigonometric ratio of special angles (30^0, 45^0 and 60^0). Application of trigonometric ratios.
9	Logical reasoning: Simple True and False statement, Negation, Converse and Contra positive of statement,
10	Logical reasoning continues: Compound statement, connectives and their symbols, conditional statements and symbols.
11	Revision of Second Term's lesson
12	Examination

REFERECES

- FutherMaths Project 1 and 2 by TuttuhAdegun (main text).
- Additional Mathematics by Godman
- Further Mathematics by E. Egbe et al.

WEEK ONE **DATE……………**

TOPIC: SEQUENCE & SERIES

CONTENT

- Sequence and series
- Arithmetic Progression (AP)
- Arithmetic Mean
- Sum of terms in an AP

Sequence & Series

A sequence is a pattern of numbers arranged in a particular order. Each of the number in the sequence is called a term. The terms are related to one another according to a well defined rule.

Consider the sequence 1, 4, 7, 10, 13 …., 1 is the first term,(T_1) 4 is the second term(T_2), 7 is the third term (T_3).

The sum of the terms in a sequence is regarded as series. The series of the above sequence is

1 + 4 + 7 + 10 + 13 = 35

The nth term of a Sequence

The nth term of a sequence whose rule is stated may be represented by $T_n$so that T_1, T_2, T_3etc represent the first term, second term, third term … etc respectively.

Consider the sequence 5, 9, 13, 17, 21 ……..

$T_1 = 5 + 4(0)$

$T_2 = 5 + 4(1)$

$T_3 = 5 + 4(2)$
$T_4 = 5 + 4(3)$
$T_n = 5 + 4(n - 1)$
$T_n = 5 + 4n - 4 \quad = 4n + 1$
when n = 30
$T_{30} = 4(30) + 1$
$T_{30} = 121$

Find the nth term of these sequences:
(i) 3, 5, 7, 9 …… $2n + 1$
(ii) 0, 1, 4, 9 ……… $(n-1)^2$
(iii) 1/3, 3/4, 1, 7/6 ……………… $\frac{2n - 1}{n + 2}$

Examples
Write down the first four terms of the sequence whose general term is given by:
(i) $T_n = \frac{n+1}{3n+2}$ (ii) $T_n = 5 \times (^1/_2)^{n-2}$

Solution
i. $T_n = \frac{n+1}{3n + 2}$

$T_1 = \frac{1 + 1}{3(1) + 2} = {}^2/_5$

$T_2 = \frac{2 + 1}{3(2) + 2} = {}^3/_8$

$T_3 = \frac{3 + 1}{3(3) + 2} = {}^4/_{11}$

$T_4 = \frac{4 + 1}{3(4) + 2} = {}^5/_{14}$

The sequence is ${}^2/_5, {}^3/_8, {}^4/_{11}, {}^5/_{14}$ ……..

(ii) $T_n = 5 \times (1/2)^{n-2}$
$T_1 = 5 \times (1/2)^{1-2} = 5(^1/_2)^{-1} = 5(2^{-1})^{-1} = 5 \times 2 = 10$
$T_2 = 5 \times (1/2)^{2-2} = 5(^1/_2)^0 = 5 \times 1 = 5$
$T_3 = 5 \times (1/2)^{3-2} = 5 \times (^1/_2) = {}^5/_2$
$T_4 = 5 \times (1/2)^{4-2} = 5(^1/_2)^2 = {}^5/_4$
The sequence is 10, 5, ${}^5/_2$, ${}^5/_4$ ………

Evaluation
Find the first term of the sequence whose general term is given by
(i) $50 - (½)^n$ (ii) $2 + {}^3/_2{}^{(n+1)}$

Arithmetic Progression (A.P) or Linear Sequence
An arithmetic progression (A.P) is generated by adding or subtracting a constant number to a preceding term to get a term. This constant number is called the common difference designated by the letter d. The first term is designated by a.

Ex: A.P	d (common difference)	a (first term)
6½, 5, 3½, 2 ….	-1½	6½
-2, $-^3/_4$, ½, 1 ¾	1¼	-2

T_1	T_2	T_3	T_4	T_5
a	a + d	a + 2d	a + 3d	a + 4d

So for any A.P, the nth term ($T_n = U_n$) is given by
$T_n = U_n = a + (n - 1)d.$ $T_n = U_n$ = nth term
a = first term
d = common difference
n = no of terms

Examples
1. What is the 10th term of the sequence 10, 6, 2, -4 …..
2. Find the term of the A.P 3½, 7, 10½ ….. Which is 77.
3. The fist term of an A.P is 3 and the 8th term is 31. Find the common difference.

Solution

(1.) The A.P = 10, 6, 2, -4

$a = 10$, $d = 6 – 10 = - 4$, $n = 10$

$T_n = a + (n – 1) d$

$T_{10} = 10 + (10 – 1) (-4)$

$T_{10} = 10 +9(-4) = 10 – 36$

$T_{10} = -26$.

(2.) A.P = 3 ½, 7, 10½ ……………… 77

$a = 3½$, $d = 7 – 3½ = 3½$, $n =?$ $T_n = 77$

$T_n = a + (n-1)d$

$77 = 3½ + (n-1)3½$

$77 = 3½ + 3½n - 3½$

$77 = 3½ n$

$n = {}^{77}/_{3½} = {}^{77}/{}^{7}/_{2}$

$n = {}^{77} \times {}^{2}/_{7} = 22$

(3) $a = 3$, $T_8 = 31$, $d = ?$ $n = 8$

$T_n = a + (n-1) d$

$31 = 3 + (8-1) d$

$31 – 3 = 7d$

$d = {}^{28}/_{7} = 4$

Evaluation

(i) Find the 15th term of the A.P 5, 2, -1, -4 …………

(ii) Find the term of the A.P 1, 6, 11, 16…. which is 66.

Arithmetic Mean

If a, b, c are three consecutive terms of an A.P, then the common difference, d, equals

$b – a = c – b$ = common difference.

$b + b = a +c$

$2b = a + c$

$b = ½(a +c)$

Examples

(i) Insert four arithmetic means between -5 and 10.

(ii) The 8th term of a linear sequence is 18 and the 12th term is 26. Find the first term, the common difference and the 20th term.

Solution

(i) Let the sequence be -5, a, b, c, d, 10.

$a = -5$, $T6 = 10$, $n =6$.

$T_n = a + (n-1) d$

$10 = -5 + (6 – 1) d$

$15 = 5d$

$d = {}^{15}/_{5} = 3$

$a = -5 + 3 = -2$

$b = -2 + 3 = 1$

$c = 1 + 3 = 4$

$d = 4 + 3 = 7$

The numbers will be -5, -2, 1, 4, 7, 10.

(ii) $T_8 = a + 7d = 18$, $T_{12} = a +11d = 26$

$a + 7d = 18$ ……………….. (i)

$a + 11d =26$ ……………….. (ii)

Subtract (i) from (ii)

$4d = 8$

$d = 2$

Substitute for d = 2 in (i)

$a + 7 (2) = 18$

$a = 18 – 14$

$a = 4$

$T_{20} = a + (n – 1) d = a + 19d$

$T_{20} = 4 + (20 – 1) 2$

$= 4 + 19 \times 2$

$T_{20} = 42$

Evaluation

(1) Given that 4, p, q, 13 are consecutive terms of an A.P, find the values of p and q.

(2) The sum of the 4th and 6th terms of an A.P is 42. The sum of the 3rd and 9th terms of the progression is 52. Find the first term, the common difference and the twentieth term of the progression.

Sum of terms in an A.P

To find an expression for the sum of n terms of a linear sequence, Let S_n be the sum, then

$S_n = a + (a + d) + (a + 2d) + \ldots\ldots + T_n$ …………. (i)
Also
$S_n = T_n + (T_n - d) + (T_n - 2d) + \ldots\ldots\ a$ ………. (ii)
Adding (1) and (2)
$2S_n = (a + T_n) + (a + T_n) + (a + T_n) + \ldots\ldots + (a + T_n)$
$\therefore 2S_n = n(a + T_n)$
$\therefore S_n = {}^n/_2 (a + T_n)$
But $T_n = a + (n-1)d$
$S_n = {}^n/_2 (2a + (n-1)d)$

Examples

(1) Find the sum of the first 25 terms of the A.P 3, 10, 17 ……….
(2) Find the sum of the first eight terms of a linear sequence whose first term is 6 and last term is 46.
(3) The sum of the first ten terms of an arithmetic progression is 255. Find the sum of the next twenty term of the A.P if the sum of the first twenty terms is 1010.

Solution

1. A.P = 3, 10, 17 …………..
$a = 3, d = 7, n = 25$
$S_n = {}^n/_2 (2a + (n-1)d)$
$= {}^{25}/_2 (2 \times 3 + (25 - 1)7)$
$S_n = {}^{25}/_2 (6 + 24 \times 7)$
$S_{25} = {}^{25}/_2 \times 174 = 2175$

2. A.P , $a = 6$, $T_n = 46$, $n = 8$
$S_n = {}^n/_2 (a + T_n)$
$= {}^8/_2 (6 + 46)$
$S_n = 4(52) = 208$.

3. $S_{10} = {}^{10}/_2 (2a + (10 - 1)d) = 255$
$S_{20} = {}^{20}/_2 (2a + (20 - 1)d) = 1010$
$5(2a + 9d) = 255$
$10(2a + 19d) = 1010$

$2a + 9d = 51$ …………….(i)
$2a + 19d = 101$ ……………… (ii)
Subtract (i) from (ii)
$10d = 50$
$d = 5$
Substitute for d = 5 in (i)
$2a + 9 \times 5 = 51$
$2a = 51 - 45$
$2a = 6$
$a = 3$
Sum of the next 20 terms = $S_{30} - S_{10}$
$S_{30} = {}^{30}/_2 (2 \times 3 + (30 - 1)5)$
$= 15(6 + 29 \times 5)$
$S_{30} = 2265$
$S_{30} - S_{10} = 2265 - 255$
$= 2010$

<u>Evaluation</u>:
The sum of the first ten term of a linear sequence is -60 and the sum of the first fifteen term of the sequence is -165. Find the 18^{th} term of the sequence.

<u>General Evaluation</u>

1. The sum of the first four terms of a linear sequence (A.P) is 26 and that of the next four terms is 74. Find the values of (i) the first term (ii) the common difference.
2. Calculate the (i) common difference (ii) the 20^{th} term of the arithmetic progression; 100, 96, 92, 88, 86...
3. Solve the equation: $\log_4(x^2 + 6x + 11) = ½$
4. Express $\frac{1}{3\sqrt{5} + 5\sqrt{3}}$ in the form $m\sqrt{5} + n\sqrt{3}$ where m and n are rational numbers

<u>Reading Assignment</u>: *Further Mathematics Project Book 1(New third edition).Chapter 28 -33 & 36 – 37*

<u>Weekend Assignment</u>

1. Find T_9 of the sequence -1, 2, 5, 8 …………… A. 21 B. 22 C. 23 D. 24
2. The 10^{th} term of an A.P is 68 and the common difference is 7, find the first term of the sequence.
A. 3 B. 5 C. 7 D. 9
3. Find the sum of the first twelve term of the sequence 2, 5, 8, 11... A. 202 B. 212 C.222 D. 232
4. What is the general term of the sequence 31, 26, 21, 16, 11…………
A. $1 + 4n$ B. $3 \times 2^{n-1}$ C. $36 - 5n$ D. $5(½)^{n-2}$
5. Find the sum of n terms of the A.P 3 + 6 +9 + 12 + ……….

A. $\frac{3n(n+1)}{2}$ B. $5n + \frac{3}{2}(n+1)$ C. $\frac{n + 3(n-1)}{2}$ D. $\frac{2n}{3} + n(n-3)$

Theory

1. The first three terms of an A.P are x, 2x+1, 4x+1, find x and the sum of the first 18 terms.
2. The sum of the first twenty –one terms of an A.P is 28, and the sum of the first twenty-eight terms is 21. Find which terms of the sequence is o and also the sum of the term proceeding it.

WEEK TWO **DATE……………**

TOPIC: SEQUENCE AND SERIES (II)

CONTENT

- Nth term of a G.P
- Geometric Mean
- Sum of n terms of a G.P
- Sum to infinity of a G.P

Nth term of a G.P

A Geometric Progression is a sequence generated by multiplying or dividing a preceding term by a constant number to get a term. This constant number is called common ratio designated by letter r.

Examples:	r		a
4, 8, 16, 32, ……………..	$^8/_4 = 2$		4
8, 4, 2, 1, ½	$^4/_8 = ½$		8
3,-1, $1^1/_3$, $^{-1}/_9$	$^{-1}/_3 = ^{-1}/_3$	3	

For any G.P, the nth term is given by

$T_n = ar^{n-1}$

T_n = nth term

a = first term

r = common ratio

n = number of terms

Examples:

1. Find the 9^{th} term of the sequence G.P 2, -10, 50 …………
2. Find the number of term of the G.P 27, 81, 243 ………… 3^{20}
3. If 7, x, y, 189 are in G.P, find x and y

Solution

(1) G.P = 2, -10, 50 ………………

a = 2, r = -5, n = 9, T_9 = ?

$T_n = ar^{n-1}$

$T_9 = 2(-5)^{9-1}$

= 2 x 390625

$T_9 = 781250$

(2) G.P = 27, 81, 243 ………3^{20}

a = 27, r = 3, n =?, $T_n = 3^{20}$

T_n = arn-1

$3^{20} = 27(3)^{n-1}$

$3^{20} = 3^3(3)^{n-1}$

$3^{20} = 3^{3+n-1}$

$3^{20} = 3^{2+n}$

2+n = 20

n = 20 – 2 = 18

(3) The G.P = 7, x, y, 189.

a = 7, n = 4, $T_4 = 189$

$T_n = ar^{n-1} = 189$

$7(r)^{4-1} = 189$

$r^3 = {}^{189}/_7 = 27 = 3^3$

r = 3

$T_2 = x = ar = 7x\ 3 = 21$

$T_3 = y\ ar^2 = 7x3x3 = 63$

Evaluation

(1) Find T_9 of the G.P 5, 2½, 1¼, $^5/_8$ …………..

(2) If 3, p, q, 24 are consecutive term of an exponential sequence, find the values of p and q.

Geometric Mean

Suppose x, y, z are consecutive terms of a geometric progression, then the common ratio r can be written as:

r = y/x = z/y

∴y/x = z/y

y2 = xz

$y = \sqrt{xz}$

y is the geometric mean of x and z.

Examples:

(1) Insert two geometric mean between 12 and 324.

(2) The 2nd term of an exponential sequence is 9 while the 4th term is 81. Find the common ratio and the first term of the G.P

Solution

(1) Let the G.P = 12, x, y, 324.

$a = 12, T_4 = 324, n = 4$

$T_n = ar^{n-1}$

$324 = 12(r)^{4-1}$

$r^3 = {}^{324}/_{12} = 27 = 3^3$

$r = 3$

$x = T^2 = ar = 12 \times 3 = 36$

$y = T_3 = ar^2 = 12 \times 3 \times 3 = 108$

The geometric means are 36 and 108

2) $T_2 = 9, T_4 = 81$

$T_4 = ar^3 = 81$ ……………………(i)

$T_2 = ar = 9$ …………………… (ii)

Divide (i) by (ii)

$\frac{ar^3}{ar^1} = 81/9$

$r^2 = 9 \Rightarrow r = \pm\sqrt{9} = \pm\sqrt{3}$

$ar = 9$

$a(\pm 3) = 9$

$a = \frac{9}{\pm 3} = \pm 3$

The first term = ± 3, the common ratio = ± 3

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Evaluation

(1) Insert two geometric mean between -3 and $-^8/_9$.

(2) The 4^{th} term of a G.P in 75 and the 6^{th} term is 192. Find the common ratio and the first term of the G.P

Sum of n terms of a G.P

The sum of n terms of a G.P whose first term is a and whose common ratio is r is given by

$S_n = a + ar + ar^2 + \ldots\ldots\ldots\ldots\ldots\ldots ar^{n-1} \quad \ldots\ldots\ldots\ldots\ldots\ldots(i)$

$r S_n = ar + ar^2 + ar^3 + \ldots\ldots\ldots\ldots ar^n \ldots\ldots\ldots\ldots (ii)$

Subtracting (2) from (1)

$S_n - rS_n = a - ar^n$

$S_n (1- r) = a(1 - r^n)$

$S_n = \frac{a(1 - r^n)}{1 - r}$ if $r < 1$

$S_n = \frac{a(r^n - 1)}{r - 1}$ if $r > 1$

Examples:

1. The third term of a G.P is 63 and the fifth term is 567. Find the sum of the first six terms of the progression.
2. Find the sum of first 6 terms of the G.P 18, 6, 2 ………..

Solution

1. $T_3 = 63$, $T_5 = 567$

$T_5 = ar^4 = 567 \ldots\ldots\ldots\ldots\ldots (i)$

$T_3 = ar^2 = 63 \ldots\ldots\ldots\ldots\ldots (ii)$

Divide (i) by (ii)

$\frac{ar^4}{ar^2} = \frac{567}{63}$

$r^2 = 9$

$r = 3$

Substitute for r = 3 in (ii)

$a (3)^2 = 63$

$a = {}^{63}/_9 = 7$

$S_6 = a \frac{(r^n - 1)}{r - 1}$

$= 7 \frac{(3^6 - 1)}{3 - 1}$

$= 7\frac{(729 - 1)}{2}$

$S_6 = 2548$

2. G.P = 18, 6, 2 …….

$a = 18$, $r = {}^6/_{18} = {}^1/_3$, $n = 6$, S_6?

$S_n = \frac{a (1 - r^n)}{1 - r}$

$= \frac{18(1- ({}^1/_3)^6)}{1-{}^1/_3{}^2/_3} = 18(1- {}^1/_{729})$

$S_6 = \frac{18 \times 3 \times 728}{2 \times 729}$

$S_6 = 26.9$

Sum to Infinity

The sum of the n terms as n approaches infinity is called the sum to infinity of the series and is designated S_∞

Thus:

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$S_\infty = \frac{a}{1-r}$ if r<1

$S_\infty = \frac{a}{r-1}$ if r>1

Examples:

Find the sum to infinity of the sequence 1, ¼, $^1/_{16}$, $^1/_{64}$.

Solution

a = 1, r = ¼

$S_\infty = \frac{1}{1-\frac{1}{4}} = \frac{1}{\frac{3}{4}}$

$S_\infty = {}^4/_3$

Evaluation

1. The second and fourth terms of a G.P are 21 and 189. Find the sum of the first seven terms.
2. Find the sum to infinity of $1+{}^1/_3 + {}^1/_9 + {}^1/_{27}$ …………

GENERAL EVALUATION

1. Find the (a)sum of the first 8 terms (b)sum to infinity of the series: -5 , 5/2, -5/4 , 5/8…….
2. The sum of the first two terms of a G.P is 2½ and the sum of the first four terms is $3^{11}/18$. Find the G.P if r > 0.
3. Solve the following exponential equations (a) $2^{2x} - 6(2^x) + 8 = 0$ (b) $2^{2x+1} - 5(2^x) + 2 = 0$

READING ASSIGNMENT: *Further Mathematics Project Book 1(New third edition).Chapter 33-36 & 37-45*

WEEKEND ASSIGNMENT

1. The sum to infinity of a G.P is 60. If the first term of the series 12, find its second term of the series 12, find its second term. A. 9.6 B. 6.9 C. 12.6 D. 8.6
2. A G.P has 6 terms. If the 3rd and 4th terms are 28 and -56 respectively, find the sum of the G. P. A. 471 B. -471 C. – 147 D. -741
3. Find the sum of the G.P 2 + 6 + 18 + 54 + …………1458. A. 8216 B. 6218 C.1682 D. 2186
4. The 8th term of a G.P is -7/32. Find its common ratio if its first term is 28. A. ½ B. -½ C. $-^2/_3$ D. $^3/_2$
5. Given the geometric progression 5, 10, 20, 40, 80 …………….. find its nth term. A. $2(5^{n+2})$ B. $5(2^{n+1})$ C. $5(2^{n-1})$ D. $2(5^{n-1})$

THEORY

1. The fifth term of a G.P is greater than the fourth term by 13½, and the fourth term is greater than the third by 9. Find (i) the common ratio (ii) the first term
2. The sum of the first two terms of an exponential sequence is 135 and the sum of the third and the fourth terms is 60. Given that the common ratio is positive, calculate
(i) the common (ii) the limit of the sum of the first n terms as n becomes large
(iii) the least number of terms for which the sum exceeds 240

WEEK THREE **DATE……………**

TOPIC:LINEAR INEQUALITIES

CONTENT

- ❖ Linear & Analytical Solutions of Linear Inequalities in One Variable
- ❖ Quadratic Inequalities in One Variables
- ❖ Absolute Values

LINEAR INEQUALITIES IN ONE VARIABLE

Most of the rules for solving a linear inequalities in one variable are similar to those for solving a linear equation in one variable with exception of the rules on multiplication and division by negative number which reverses the sense of the inequality

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EXAMPLE : Find the solution set of each of the following inequalities and represent them graphically
(a) 2x – 3 < x + 7 (b) 3x + 4 > 1 – 2x

Solution

(a) 2x – 3 < x + 7
Adding 3 to both sides
2x < x + 10
Subtracting x from both sides
X < 10

(b) 3x + 4 > 1 – 2x
Subtracting 4 from both sides
3x > - 3 – 2x
Adding 2x to both sides
5x > -3
Dividing both sides by 5
x > -3/5

QUADRATIC INEQUALITIES IN ONE VARIABLE

To find the solution sets, of the quadratic inequalities of the form, $ax^2 + bx + c \geq 0$ or $ax^2 + bx + c \leq 0$. Note the following

1) If a>0 and b>0 then a.b>0
or a<0 and b<0 then a.b>0

2) If a<0 and b>0 then a.b< 0
Or a>0 and b<0 then a.b< 0

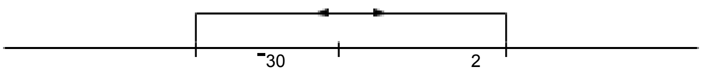
Worked examples

1) Find the solution set of $x^2 + x – 6 > 0$

Solution

$x^2 + x – 6 > 0$
(x – 2)(x + 3} > 0
x – 2> 0 or x + 3<0
x >2 or x < -3
x – 2 < 0 or x +3>0
x < 2 or x > -3
-3 < x < 2

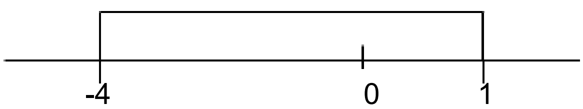
-3 < x < 2



2) Show graphically the solution Set of the inequality $x^2 + 3x – 4 \leq 0$

Solution

$X^2 + 3x – 4 \leq 0$
$X^2 + 3x – 4 = 0$
. (x – 1)(x + 4) ≤ 0
x – 1 ≤ 0 or x + 4 ≥ 0
.x ≤ 1 or x ≥ -4
X – 1 ≥ 0 or x + 4 ≤ 0
X ≥ 1 or x ≤ -4
X ≤ 1 or x ≥ -4
- 4 ≤ x ≤ 1



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Evaluation

Find the solution set of the inequalities

a) $x^2 + 5x – 14 < 0$

b) $2 – 3x – 9x^2 > 0$

c) $1 – x^2 \leq 0$

Quadratic Inequality curve

We recall that th graph of $f(x) = ax^2 + bx + c$ is a parabola if $D \geq 0$, the parabola crosses the axis at two distinct points, this fact can be used to solve the inequality $ax^2 + bx + c \geq 0$ or $ax^2 + bx + c \leq 0$

Worked examples

1) Determine the solution set of the inequality $x^2 – x – 10 < 2$

$X^2 – x – 10 – 2 < 0$

$X^2 – x – 12 < 0$

$(x + 3)(x – 4) < 0$

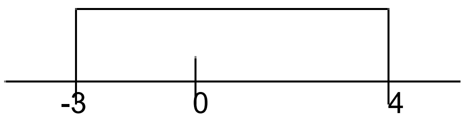
$x + 3 < 0$ or $x – 4 > 0$

$x < -3$ or $x > 4$

$x + 3 > 0$ or $x – 4 < 0$

$x > -3$ or $x < 4$

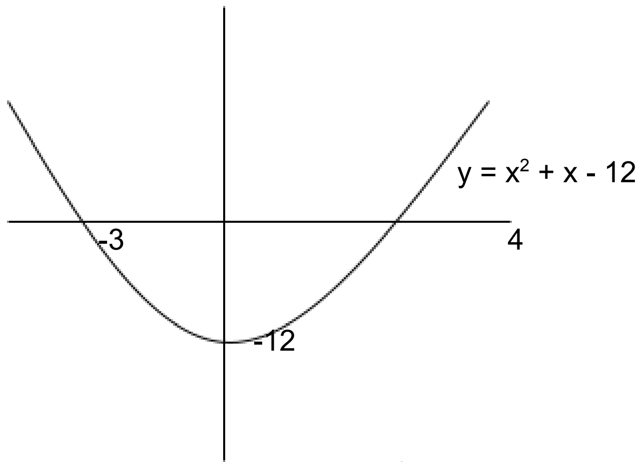
$- 3 < x < 4$



Using Parabolic curves

Coordination of points at which the curve cuts the axis $(x + 3)(x – 4) = 0$

$X = -3$, $x = 4$



2) Find the solution of the inequality $x^2 – 2x – 3 \geq 0$

Solution

$x^2 – 2x – 3 \geq 0$

$(x + 1)(x – 3) \geq 0$

$x + 1 \geq 0$ or $x – 3 \geq 0$

$x \geq -1$ or $x \geq 3$

$(x + 1) \leq 0$ or $(x – 3) \leq 0$

$x \leq -1$ or $x \leq 3$

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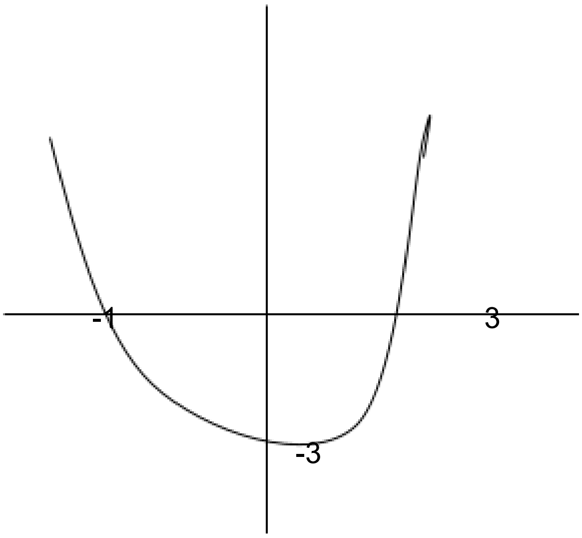
Solution set $-1 \leq x \leq 3$



b) $9 - x^2 \geq 0$

$3^2 - x^2 \geq 0$

$(3 - x)(3 + x) \geq 0$

$3 - x \geq 0$ or $3 + x \geq 0$

$- x \geq -3$ or $x \geq -3$

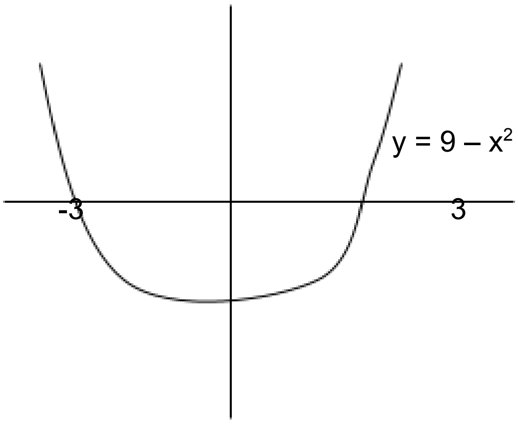
$x \leq 3$ or $x \geq -3$

$(3 - x) \leq 0$ or $(3 + x) \leq 0$

$-x \leq -3$ or $x \leq - 3$

$x \geq 3$ or $x \leq - 3$

Solution set $-3 \leq x \leq 3$



ABSOLUTE VALUES

If a number x is positive or negative the absolute value of x is denoted as $|x|$. The absolute value of a number is the magnitude of the number regardless of the sign.

Worked examples

1) $|2x - 3| \geq 4$

$2x - 3 \geq 4$

$2x \geq 4 + 3$

$2x \geq 7$

$.x \geq 7/2$

$.x \geq 3½$

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OR

$-(2x - 3) \geq 4$
$-2x + 3 \geq 4$
$-2x \geq 4 - 3$
$-2x \geq 1$
$.x \leq -½$

2) Find the solution set of the inequality $|x - 2| < |x + 3|$

Solution

$|x - 2| < |x + 3|$
$(x - 2)^2 < (x + 3)^2 \equiv x^2 - 4x + 4 < x^2 + 6x + 9$
$-4x - 6x < 9 - 4$
$-10x < 5$
$x > -5/10$
$x > -½$

Evaluation

Find the solution set of the inequality

a) $|2x - 1| > 3$
b) $|x - 3| - |x - 1| < 0$
c) $|x - 3| \leq |x - 2|$

General Evaluation

1) Find the range of values of x for which $7x - 12 \geq x^2$
2) For what values of x is $2x^2 - 11x + 12$ positive?
3) Find the values of x satisfying: $|3x - 2| \geq 3|x - 1|$
4) The 2nd term of an exponential sequence is 9 while the 4th term is 81. Find the common ratio, the first term and
The sum of the first five terms of the sequence.
5) Find the value of the constant k for which the equation $2x^2 + (k + 3)x + 2k = 0$ has equal roots.

Reading Assignment: F/maths Project 1 pg 104 - 111

WEEKEND ASSIGNMENT

1) Find the range of x for which $|2x - 1| > 3$
(a) $1 < x < 3/2$ (b) $-3/2 < x < -1$ (c) $-3/2 < x < 1$ (d) $x > 3/2$ and $x < -1$
2) Find the range of the value that satisfies the inequality $x^2 + 3x - 18 < 0$
(a) $-3 < x < 6$ (b) $-3 > x < 6$ (c) $-6 > x > 3$ (d) $-6 > x < 3$ (e) $-6 < x < 3$
3) Find the range of values of x for which $2x^2 - 5x + 2 \geq 0$
(a) $-2 < x < -½$ (b) $½ < x < 2$ (c) $x < -½$ or $x \geq -2$ (d) $x \leq ½$ or $x \geq 2$
4) Find the range of values of y which satisfies the inequality $2y - 1 < 3$ and $2 - y \leq 5$
(a) $-3 \leq y \leq 1$ (b) $-2 \leq y \leq 3$ (c) $-3 \leq y \leq 4$ (d) $-3 \leq y \leq 2$
5) Find the range of values of x for which $1/x + 3 < 2x$ is satisfy
(a) $-3 < x < 5/2$ (b) $x < -3$ and $x > -5/2$ (c) $x < 1$ and $x < ½$

THEORY

1) Find the range of values of x for which $1 < x^2 - x + 1 < 7$

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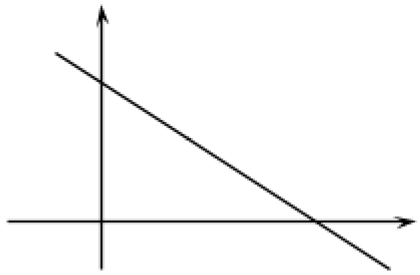
2) Find the values of x satisfying $|x - 5| - |x - 3| \geq 0$
3) Given that a and b are two real numbers, show that $a^2 + b^2 \geq 2ab$.

WEEK FOUR **DATE……………**
TOPIC: LINEAR INEQUALITY (PART TWO)
CONTENT

- ❖ Linear Inequalities in Two Variables by Graphical Method.
- ❖ Graphical Solution of Simultaneous Linear Inequalities in Two Variables.
- ❖ Linear Programming

GRAPHICAL SOLUTION OF INEQUALIIES IN TWO VARIABLES
A straight line has the general equation ax+by+c=0, where a,b and c are real numbers.

The line ax + by + c =0 partitions the x-y plane into two regions



Worked Examples
1) Show the region representing $2x + y + 1 > 0$
Solution
$2x + y + 1 > 0$
Steps
1. make y the subject of the inequality
2. convert the inequality into a line equation
3. obtain x and y co-ordinates of the line
4. draw the line and shade the required by the inequality

$2x + y + 1 > 0$
$y > -2x - 1$
When $x = 0$, $y = -2(0) - 1$
$y = -1$ (0, 1)
When $y = 0$, $0 > -2x - 1$
$1 > -2x$
$x = -½$ (-½ ,0)

2) Show the region represented by $x - 2y + 3 \leq 0$
Solution
$x - 2y + 3 \leq 0$

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2y = - 3 – x → y = -3/-2 - x/-2 → y = 3/2 + x/2 or $y = \frac{3 + x}{2}$

When x = 0

$y = \frac{3 + 0}{2} = \frac{3}{2}$ (0, 3/2)

When y = 0

$0 = \frac{3 + x}{2}$

x = -3 (-3, 0)

Evaluation

Show the region which represents the following inequality

a) 2x – 3y + 1 ≤ 0 b) x – 4y + 7 ≥0

SIMULTANEOUS INEQUALITIES

The set of simultaneous inequalities in two variables can be found from the intersection of the areas representing the inequalities.

Worked Examples

1) Show graphically the region R which satisfies the set of inequalities

2x + 2y ≤ 2, x + 2y≤ 16, x ≥ 0, y ≥ 0.

Solution

2x + 2y ≤ 2

2y ≤ 2 – 2x

$y \leq \frac{2 - 2x}{2}$

$y \leq \frac{2}{2} - \frac{2x}{2}$

y ≤ 1 - x

When x = 0, y = 1 – 0 =1 (0, 1)

When y =0, 0 = 1 – x

1= -x point (1, 0).

X + 2y ≤ 16

2y ≤ 16 – x, $y \leq \frac{16 - x}{2}$

When x = 0

$y = \frac{16 - 0}{2} = \frac{16}{2} = 8$ (0, 8)

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When y = 0, 0 = $\frac{16 - x}{2}$

16 – x = 0

x = - 16

x = 16 (16, 0).

x ≥ 0, x = 0 , y ≥ 0, y = 0

2) Show graphically, the region which satisfies the set of inequalities

4x + y ≤ 15, 8x – y ≥ 9, x ≥ 0, y ≥0.

Solution

4x + y ≥ 15, y ≤ 15 – 4x

When x = 0

. y = 15 – 4(0)

. y = 15 (0, 15)

When y = 0

0 = 15 – 4x

$\frac{-15}{4} = \frac{-4}{-4}$

X = 15/4 (3¾, 0)

8x – y ≥ 9

. –y ≥ 9 – 8x, y ≥ 8x – 9

When x = 0, y = 8(0) – 9

. y = -9 (0, -9)

When y = 0, 0 = 8x – 9

9 = 8x, x = 9/8 = 1⅛ (1⅛, 0)

. x ≥ 0 or x = 0, y ≥ 0 or x = 0

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Evaluation
Show the regions which represent the set of solution of
1) $2y \le x + 8$, $x + 2y + 4 \ge 0$, $x \le 2y + 12$
2) $y \ge 0$, $x + 2y \le 4$, $-x + 2y \le 11$, $-2x + 5y \le 10$

LINEAR PROGRAMMING

The linear function $z = ax + by$ is called the objective function while the given set of the inequalities are called the constraint linear programming attempts to maximize or minimize an objective function under the set of given constraints.

Example 1

A caterer can make two types of drinks A and B. A litre of A contains 2gramme of orange juice and 3gramme of pineapple juice. A litre of B contains 4gramme of orange juice and 5gramme of pineapple juice. There are not more than 16gramme of orange juice and 21gramme of pineapple juice.
The caterer can make a profit of 1ok on 1gramme of A and 15k on 1gramme of B. Assuming that the caterer makes *x* litres of A and *y* litres of B.
(a) Write all the inequalities connecting *x* and *y*.
(b) Show by shading the required region satisfying the inequalities in (a)
(c) Find the quantity of each type of drink a caterer must make if she is to maximize profit.

Solution

(a) $2x+4y \le 16$, $3x + 5y \le 21$, $x \ge 0$, $y \ge 0$
(b) $2x + 4y \le 16$, $4y = 16 - 2x$, $y = \frac{16-2x}{4}$

When $x = 0$
$y = \frac{16 - 2(0)}{4} = \frac{16}{4} = 4$ (0, 4)

When $y = 0$, $0 = \frac{16 - 2x}{4}$

$16 - 2x = 0$, $\frac{16}{2} = \frac{2x}{2}$ $x = 8$ (8, 0)

$3x + 5y \le 21$
$\frac{5y}{5} = \frac{21 - 3x}{5}$
$y = \frac{21-3x}{5}$

When $x = 0$, $y = \frac{21-0}{5} = \frac{21}{5}$ (0, 21/5)

When $y = 0$, $0 = \frac{21 - 3x}{5}$

$21 - 3x = 0$
$3x = -21$, $x = \frac{-21}{3} = 7$ (7,0)

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(c) 7 litres of A and none of B.

Example 2.
A fashion designer makes two types of dresses X and Y by making use of two types of materials P and Q. The quantity of material used for each unit of dress in m^2, and the profit on each dress in N are as shown in the following table.

	P	Q	Profit
X	3	2	2
Y	4	5	3
Quantity available	18	19	

(a) Assuming that the designer makes *x* unit of X and *y* and units of Y. write down the four inequalities connecting *x* and *y.*
(b) Find how many of each type of dresses the fashion designer should make in order to maximize Profit.
Solution
The quantity of material P used in making *x* units of dress X and *y* units of dress Y is $3x + 4y$, since the quantity of material P available is $18m^{2.}$
$3x + 4y \le 18$
Similarly for material Q
$2x + 5y \le 19$
Also, $x \ge 0$, $y \ge 0$
$3x + 4y = 18$,
$4x = 18 - 3x$
$y = \frac{18 - 3x}{4}$

When x = 0, $y = \frac{18 - 3(0)}{4} = \frac{18}{4} = 4\frac{1}{2}$ (0, 4½)

When y = 0, $18 - 3x = 0$
$3x = -18$
. $x = \frac{-18}{3} = 6$ (6, 0)
$2x + 5y = 19 \rightarrow 5y = 19 - 2x, \rightarrow y = \frac{19 - 2x}{5}$
When x = 0, $y = \frac{19 - 2(0)}{5} = \frac{19}{5}$ (0, 19/5)
When y= 0, $19 - 2x = 0$
$2x = -19$
$x = \frac{-19}{2} = \frac{19}{2}$ (19/2, 0)

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Let z be the profit, then z = 2x + 3y at the point C (2, 3)
z = 2(2) + 3(3)
z = 4+ 9 = 13
Hence the fashion designer should make 2 dresses of type X and 3 dresses of type Y in order to make a maximum profit of N13.00

Evaluation
A petty trader sells two types of detergents A and B. a dm^3 of A contains 2gm of Omo detergent and 5gm of Surf detergent. A dm^3 of B contains 3gm of omo detergent and 2gm of surf detergent. Altogether she has at most 26g of omo detergent and 32g of surf detergent, the trader makes a profit of 2k per gm on A and 1k per gm on B. If the trader sells x dm^3 of A and y dm^3 of B

1. Write down all the inequalities connecting x and y.
2. Indicate by shading the region R satisfying all the inequalities in (a)
3. Determine the values of x and y which maximises the traders profit.

Solution

	Omo	Surf	Profit
A	2	5	2
B	3	2	1
Total	26	32	

$x \geq 0$, $y \geq 0$, $2x + 3y \leq 26$, $5x + 2y \leq 32$
2x + 3y = 26,
3y = 26 - 2x, $y = \frac{26 - 2x}{3}$
When x = 0, $y = \frac{26 - 2(0)}{3} = \frac{26}{3} = 8\frac{2}{3}$ $(0, 8\frac{2}{3})$
When y = 0, 26 – 2x = 0

- 2x = - 26, $x = \frac{-26}{-2} = 13$ (13, 0)
$5x + 2y \leq 32$
2y = 32 – 5x, $y = \frac{32 - 5x}{2}$
When x = 0, y = 32 - 5(0) = 32/2 = 16 (0, 16)
When y = 0, 32 – 5x = 0
-5x = -32, $x = -32/-5 = \frac{32}{5}$ (32/5, 0)

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The corner points are A(0, 8.6) , B(4, 6), C(6.4,0) , D(0,0)
Profit Z = 2x + y
At A, Z = 2(0 + 8.6) = 8.6
At B, Z = 2(4 + 6) = 14
At C, Z = 2(6.4 + 0) = 12.8
At D, Z = 2(0 + 0) = 0
Hence, the trader should sell 4 of detergent A and 6 of detergent B to make a profit of 14k.

Evaluation
1) Show graphically the region represented by the inequalities (a) $y \geq 4x^2 + 11x – 3$ (b) $y \geq 6x^2 – x – 2$
2) Show graphically the region R which satisfies the set of inequalities: $2x + 3y \leq 26$, $x + 2y \leq 16$, $x \geq 0$, $y \leq 0$.

General Evaluation
1. show the region R which satisfies the following simultaneous inequalities
$y + x \leq 3$, $y+ x \geq 1$, $y - x \leq 1$, $x \geq 0$, $y \geq 0$.
2. Show the region R which satisfies simultaneously $2x + y \leq 7$, $3x – 4y \geq - 6$, $x \geq 0$, $y \geq 0$.
3. $3x^2 + 7x – 3 = 0$ solve using formula method
4. Using completing the square and formula method solve $3x^2 – 12x + 10 = 0$
5. Solve the following exponential equations (a) $2^{2x} - 6(2^x) + 8 = 0$ (b) $2^{2x+1} - 5 (2^x) + 2 = 0$
6. Janet buys p sweet and q marbles. The sweets cost ₦5 each and the marbles cost ₦6 each. Janet has ₦90.
She wants to share the sweets with her friends, so she needs at least 5sweets, she needs more than 4 marbles
To be able to join in the game. (a) Write down three inequalities connecting p and q (b) Draw the graph to show
Their inequalities (c) What is the highest number of sweets she can buy? (d) What is the highest number of marbles she can buy?

Reading Assignment: F/maths Project 1 pg 113 – 119 Exercise 8c Q1, 16 and 17

WEEKEND ASSIGNMENT
1) Find the range of x for which $|2x - 1| > 3$
(a) $1< x < 3/2$ b) $-3/2 < x < -1$ c) $-3/2 < x < 1$ d) $x > 3/2$ and $x < -1$
2) Find the range of the value that satisfies the inequality $x^2 + 3x – 18 < 0$
(a) $-3 < x < 6$ (b)$-3 > x <6$ (c)$-6 >x >3$ (d)$-6 >x < 3$ (e)$-6 < x <3$
3) Find the range of values of x for which $2x^2 – 5x + 2 \geq 0$
(a) $-2 < x < -½$ (b) $½ < x < 2$ (c) $x < -½$ or $x \geq -2$ (d) $x \leq ½$ or $x \geq 2$
4) Find the range of values of y which satisfies the inequality $2y – 1 < 3$ and $2 – y \leq 5$
(a) $– 3 \leq y \leq 1$ (b) $– 2 \leq y \leq 3$ (c) $-3\leq y \leq 4$ (d) $-3 \leq y \leq 2$
5) Find the range of values of x for which $1/x + 3 < 2x$ is satisfy
(a) $– 3 < x < 5/2$ (b) $x < -3$ and $x > -5/2$ (c) $x < 1$ and $x < ½$

THEORY
1) Illustrate graphically the set P of all points (x, y) which satisfy simultaneously the following inequalities:
$2y \leq x + 8$, $x + 2y + 4 \geq 0$, $3x \leq 2y + 12$. Using your diagram, calculate on the set P the maximum values
of (i) x (ii) y (iii) 12x + 5y
2) Determine the values of x satisfying $|x + 3| \geq 8$

WEEK FIVE **DATE……………**
MAPPING AND FUNCTIONS
- Concept of mapping and function
- Domain, Co-domain of function

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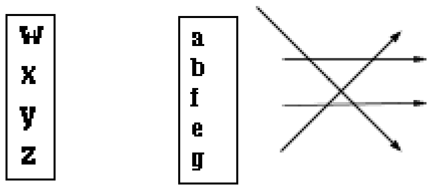
- Types of mapping.

MAPPING

Definition, Concept, Example and evaluation.

Definition: This is the rule which assign an element x in set A to another unique element y in set B. The set A is called the Domain while set B is the Co- domain



Set A = {w, x, y, z} → Domain
Set B = {e, f, g, h, i} → Co-domain

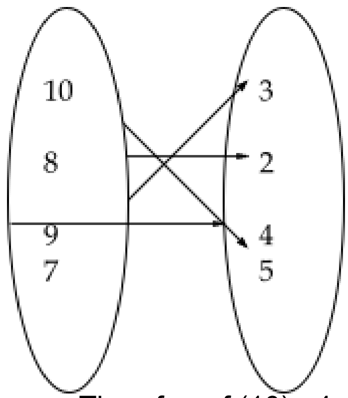
Image: This is the unique element in set B produced by an element in set A.
Range: This is the collection of all the images of the elements of the domain.

Using the diagram above:
f(w)= g, f(x)= b, f(y)=f, f(z)=a
a, b, f and g are the images of elements a,b,c and d respectively.
Range = {a, b, f, g,}

The rule which associates each element in set A to a unique element in set B is denoted by any of the following notations: f : A → B or f: A→ B

FUNCTION: A function is a mapping whose co-domain is the set of numbers.

X F Y →



Therefore, f (10) =4, f (9) =3 e.t.c

Example 1: Given $f(x) = 3x^2 + 2$, find the values of (a) f (4) (b) f (-3) (c) f (-1/2)

SOLUTION:

$F(x) = 3x^2 + 2$

(a) F(4), i.e. x=4

$F(4) = 3(4^2) + 2 = 3(16) + 2$
$= 48 + 2$
$= 50$

(b) $F(-3) = 3(-3)^2 + 2$
$= 3(9) + 2 = 27 + 2$
$= 29$

(c) $F(-1/2) = 3(-1/2)^2 + 2$

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$= 3(1/4) + 2 = \frac{3}{4} + 2$

$= 11/4.$

Example 2: Determine the domain D of the mapping, $g: x \rightarrow 2x^2 - 1$, if R= { 1,7,17} is the range and g is defined on D.

SOLUTION:

$g(x) = 2x^2 - 1$, R = {1, 7,17}

To find the domain, when g(x) = 1,

$1 = 2x^2 - 1$

$1+1 = 2x^2$

$x^2 = 2/2$

$x=1$

When g(x) = 7,

$7 = 2x^2 - 1$

$7+1 = 2x^2$

$8 = 2x^2$

$x^2 = 4$, $x = 2$

When g(x) = 17,

$17 = x^2 - 1$

$17+1 = 2x^2$

$18 = x^2$

$x^2 = 9$, $x = 3$

Domain D = {1, 2, 3}

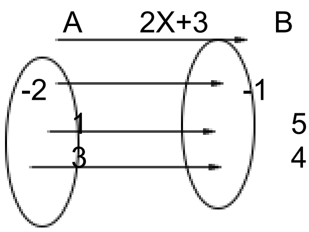
EVALUATION

1. Given $f(x) = x^2 + 4x + 3$ find the values of.
   (a) f (2) (b) f(½) (c) f(-3)

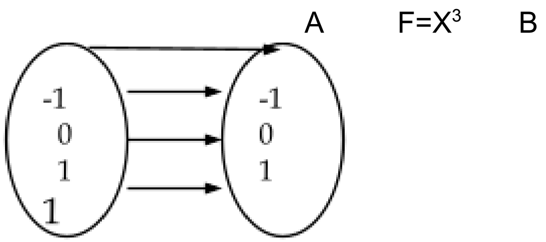
2. Given that f(x) = ax + b and that f (2) = 7, f (3) = 12. Find a and b.

TYPES OF MAPPING

One-One mapping: A mapping is one-one if different elements in the domain have different images in the co- domain. If $x_1 = x_{2 \text{ then}}$ $f(x_1) = f(x_2)$



Onto Mapping: A mapping is onto if every element of the co- domain is at least an image of elements in the domain. E.g Let A = {-1, 0, 1} f: A → A be a mapping defined by $f(x) = x^3$.



The mapping is onto and one-one.

NB: In an onto mapping, the range is the same as the co- domain.

Identity Mapping: This is a mapping which takes an element onto itself. If f: x→ x is a mapping such that f(x) = x for all x € X.

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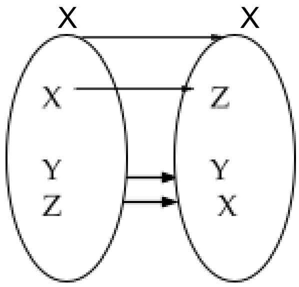
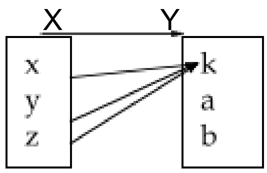


The mapping is one –one and onto. It has a unique property that the domain, the co-domain and the range are equal.

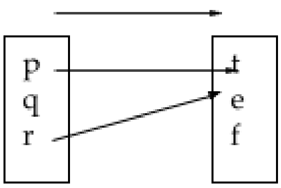
Constant Mapping: This is the mapping which assigns every element in the domain to the same image in the co- domain.



The range of a constant mapping consists of only one element.

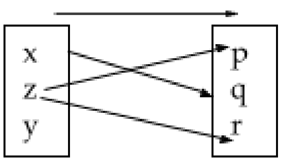
Important Notes:

1.
X F Y



The relation F above is not a mapping because element q in X has no image in Y.

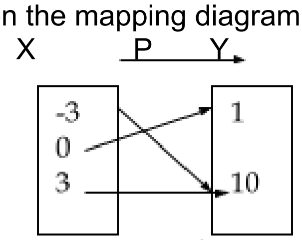
2.
A g B



The relation is not a mapping because element z in the domain has two images in the co –domain.

EVALUATION

1. Given the mapping diagram below:



(a). Determine the rule of the mapping
(b) Is the mapping one- one? Is it onto?
(c) What is the range of the mapping?

GENERAL EVALUATION

1. Solve the system of equation; $2^{x+y} = 32$, $3^{3y-x} = 27$
2. Given $h(x) = x^3 - 6x^2 - 3x + 5$ find the values of.
   (a) h (-2) (b) h (-½) (c) h(3)
3. given that g(x) = 2p - q and that g (2) = 20, g (-3) = 15. Find p and q.

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4. Given the functions $h(y) = 3y^2 - y + 5$, $p(y) = 6y^3 + 7y^2 + 7y + 15$. Simplify, as far as possible, the expressions

(a) $3h(y) - p(y)$ (b) $h(y)\,p(y)$ (c) $h(y)/p(y)$

READING ASSIGNMENT: Read Mapping, Further Mathematics Project 2, and page 25- 35.

WEEKEND ASSIGNMENT

1. If every element in the domain have different image I the co-domain, such type of mapping is called -------
   (a) Constant mapping (b) onto mapping (c) one- to – one mapping
2. A mapping f is called ------ if every element of the co-domain is an image of at least one element in the domain
   (a) Constant mapping (b) onto mapping (c) one- to – one mapping
3. Given $f(y) = p^x$ and $f(3) = 81$, determine the value of x.
   A -4 B 27 C 4
4. The rule that assign an element to two or non-empty set is (a) logic (b) set (c) mapping
5. If f is a function defined by $f(x) = 2x^2 - 3$, find $f(-3)$.
   A. -15 B 18 C. 15

THEORY

1. Determine the domain D of the mapping $f: x \to 2x - 2$, if c = { -3, -1, 5 } is range and f is defined on D
2. Given that $h: x \to x^2 + 2x - 3$ is a mapping defined on the set A = { -1, 0, 1, 2}. Find the range of h.

WEEK SIX **DATE……………**

Revision of half term work

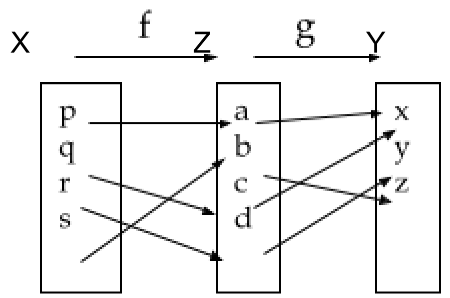
WEEK SEVEN **DATE……………**

CONTENT:

Composite Mapping and Inverse Mapping

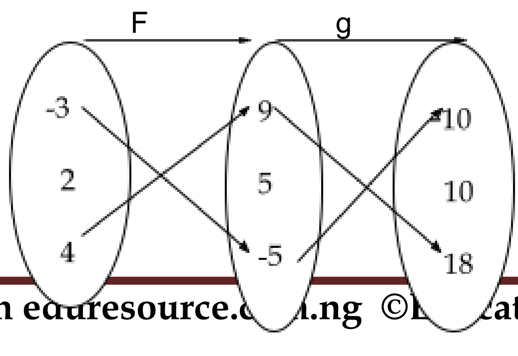
COMPOSITE MAPPING:

A mapping is composite when the co- domain of the first mapping is the domain of the second mapping. Consider the mapping f; X→ Z and g: Z→Y



The mapping f takes an element in X and produces an image in Z, and then the mapping g takes an element in Z and produces an image in Y. It can be denoted by gf or gof.

Example 1. The mappings f and g are defined by the diagram below:



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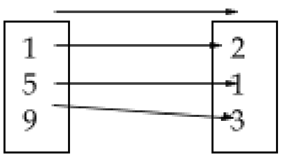
Name: ______________________________ Class: ______________

Determine
(a) f(-3) +f(4)   (b) f(2) +g(-5)   (c) g[f(-3)]   (d) g[(-3)] +g[f(4)]

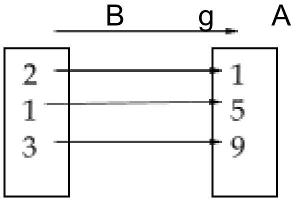
SOLUTION:
(a) F(-3) + F(4) = -5 + 9 = 4
(b) F(2) + F(-5) = 5 +(-10) = -5
(c) G[f(-3)] = g(-5) = -10
(d) G[f(-3)] + g[f(4)] = g(-5) + g(9) = -10 + 18 = 8

Example 2: The functions f and g on the set of real numbers are defined by f(x) = 3x-1 and g(x) = 5x+2 respectively. Find (a) F [g(x)] (b) g [f(x)] (c) 2f(x) – g(x)
SOLUTION:
(a) f [g(x)] = f(5x+2),   5x+2 will represent x in f(x)
f (5x +2) = 3 (5x+2) – 1
= 15x +5
(b) g [f(x)] = g (3x-1)
= g (3x-1) = 5(3x-1) + 2
= 15x -5 +2
= 15x-3
(c) 2f(x) – g(x) = 2(3x-1) – (5x+2)
= 6x -2 -5x -2
= x-4

INVERSE MAPPING:
A function has an inverse if it's both one- one and onto. Consider the function f(x) = x-3 on the set p={ -1, 5, 9} into set
Q = {-2,1,3 }
A F B



If we reverse the function, that is making range of F to be the domain of the inverse function.
Therefore, B g A



g represents the inverse function of f i.e. f^{-1}, to obtain function f^{-1}, we follow the procedure below:

$$f(x) = \frac{x-3}{2}$$

$$y = \frac{x-3}{2}$$

Make x the subject of the formula

$$2y = x-3$$

$$x = 2y + 3$$

Then, $f^{-1}(x) = 2x+3$.

Example: The function f is defined on the ser of real numbers by $f(x) = \frac{2x-1}{3x+2}$, ($x \neq -2/3$)

Determine (a) $f^{-1}(x)$ (b) $f^{-1}(-2)$ (c) determine the largest domain of $f^{-1}(x)$

SOLUTION:

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Name: ______________________________ Class: ______________

(a) $F(x) = \frac{2x-1}{3x+2}$

$f^{-1}(x)$, $y = \frac{2x-1}{3x+2}$

$(3x+2)y = 2x-1$

$3xy + 2y = 2x-1$

$3xy - 2x = -1-2y$

$x(3y-2) = -1-2y$

$x = \frac{-1-2y}{3y-2}$

$f^{-1} = \frac{-1-2x}{3x-2}$

(b) $f^{-1}(-2)$ i.e. $x = -2$ in $f^{-1}(x)$

$f^{-1}(-2) = \frac{-1-2(-2)}{3(-2)-2}$

$= \frac{-1+4}{-6-2}$

$= \frac{3}{-8}$

(c) The largest domain of $f^{-1}(x)$ is all real values of x except 2/3

EVALUATION

1. Given $g(x) = x^3$ and $h(x) = 4x + 1$
(a) find the value of $g(2) + h(2)$
(b) find the value of $h[g(2)]$
(c) find the value of $3g(-1) - 4h(-1)$

2. A function $g(x) = \sqrt{(x-2)}$, $x \geq 2$, find $g^{-1}(x)$ and $g^{-1}(4)$

GENERAL EVALUATION

1. Determine the values of p and q if (x -1) and (x + 2) are factors of $2x^3 + px^2 - x + q$
2. If $f(x) = 6x^3 + 13x^2 + 2x - 5$, shows that $f(-1) = 0$
3. Given that $f(x) = \frac{x^2}{x^2 + 2}$ and $g(x) = \sqrt{(x-2)}$, $x \neq 2$ find
(a) $f^{-1}(x)$
(b) $g^{-1}(x)$
(c) F(g(x)
(d) The value of x for which f (g(x) is not undefined.

READING ASSIGNMENT: Read Mapping, Further Mathematics Project 2, and page 32- 41.

WEEKEND ASSIGNMENT

1. Given that $f(x) = x^2 + 4x + 3$, for what values of x is $f(x) = f(x+1)$.
   A. $-\frac{11}{2}$ B $-\frac{5}{2}$ C $-\frac{3}{4}$
2. Given $f(x) = x^2 - 1$ and $g(x) = 2x+3$, determine the formula for gf(x)
   A. $2x^2 + 4x + 1$ B. $2x^2 + 1$ C. x+1
3. Given $g(x) = x^n$ and g (3) = 81, determine the value of n.
   A -4 B 27 C 4
4. Given that the image of x under the mapping $f(x) \rightarrow 3x + 2$ is -10. What is the value of x.
   A -4 B -28 C 0
5. If f is a function defined by f(x) = 2x-3, find ff.
   A. 4x-6 B 2x+3 C. 4x +6

THEORY

1. Given the functions $f(x) = 3x^2 - x + 5$, $g(x) = 6x^3 + 7x2+7x+15$. Simplify, as far as possible, the expressions
   (a) 3f(x) - g(x) (b) f(x) g(x) (c) g(x)/f(x)

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2. A relation R is defined by $g(x) = \frac{2}{X-2}$, $x \neq 2$, find $g^{-1}(x)$.

WEEK EIGHT **DATE……………**

TOPIC: TRIGONOMETRIC RATIO

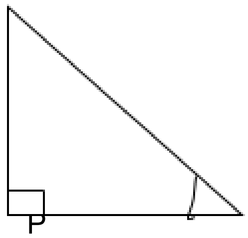
CONTENT

- ❖ Basic trigonometric Ratio
- ❖ Ratio of General Angle
- ❖ Trigonometric Identities

BASIC TRIGONOMETRIC RATIO

The basic trigonometric ratios can be defined in terms of the sides of a right angled triangle.

Q



r
p

R q P

▲PQR in the figure above is a right angle triangle with QPR = ϴ and PRQ = 90˚

We define the three basic ratios as follows:

Cosine of angle ϴ = $\frac{PR}{PQ} = \frac{q}{r}$

Sine of angle ϴ = $\frac{QR}{PQ} = \frac{p}{r}$

Tangent of angle ϴ = $\frac{QR}{PR} = \frac{p}{r}$

The cosine of angle ϴ, sine of angle ϴ and the tangent of angle ϴ will be abbreviated as cosϴ, sinϴ and tanϴ respectively.

Thus:

Cosϴ = $\frac{q}{r}$, sinϴ = $\frac{p}{r}$, tanϴ = $\frac{p}{q}$

Also,

$\frac{\text{Sin}\theta}{\text{Cos}\theta} = \frac{p/r}{q/r} = \frac{p}{q}$ = tanϴ

Tanϴ = $\frac{\text{sin}\theta}{\text{Cos}\theta}$

Reciprocals of Basic Ratios

We define the reciprocals of the three basic ratios as:

Secant of angle ϴ = PQ/PR = r/q = 1 / cosine of angle ϴ.

Cosecant of angle ϴ = PQ / QR = r / q = 1 / sine of angle ϴ

Cotangent of angle ϴ = PR / QR = q / p = 1 / tangent of angle ϴ

The secant of angle ϴ, the cosecant of angle ϴ and the cotangent of angle ϴ are abbreviated secϴ, cosecϴ and cotϴ respectively.

Secϴ = r / q = 1 / cosϴ

Cosecϴ = r / p = 1 / sinϴ

Cotϴ = q / p = 1 / tanϴ = cosϴ / sinϴ

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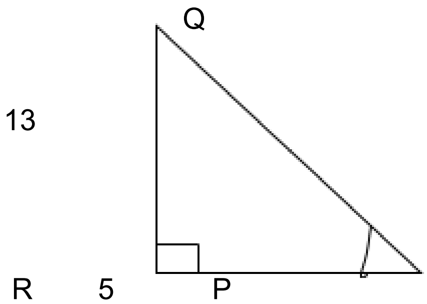
Example 1

Given that sinϴ = 5 / 13 and ϴ is acute, find:

(a) cosϴ
(b) tanϴ
(c) secϴ
(d) cosecϴ
(e) cotϴ

Solution



Use Pythagoras theorem to find PR

$PQ^2 = PR^2 + QR^2$

$13^2 = PR^2 + 5^2$

$PR^2 = 13^2 - 5^2$

$= 169 – 25$

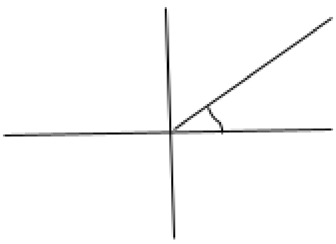
$= 144$

$PR = 12$

Thus, q = 12, r = 13, p = 5.

(a) cosϴ = q / r = 12 /13
(b) tanϴ = p / q = 5 / 12
(c) secϴ = r / q = 13 / 12
(d) cosecϴ = r / p = 13 / 5
(e) cotϴ = q / p = 12 / 5

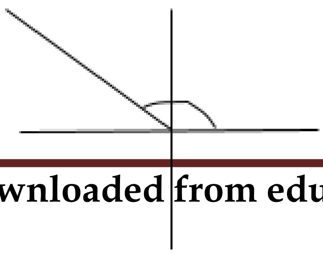
Ratios of General Angles



First Quadrant
Sinϴ = y
Cosϴ= x
Tanϴ = y / x

Example: Use table to evaluate (a) sin37 (b) cos75 (c) tan62
Solution

(a) sin37 = 0.6018
(b) cos75 = 0.2588
(c) tan62 = 1.881



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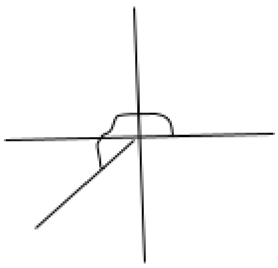
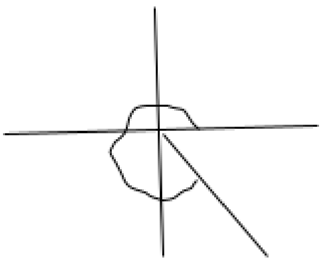
Second Quadrant
Sin (180 - Θ) = sinΘ
Cos (180 - Θ) = -cosΘ
Tan (180 - Θ) = -tanΘ

Example: Use table to evaluate (a) sin143 (b) cos 115 (c) tan 125
Solution
(a) sin143 = sin(180-143) = sin37 = 0.6018
(b) cos115 = -cos(180-115) = -cos65 = -0.4226
(c) tan125 = -tan(180-125) = -tan55 = -1.428



Third Quadrant
Sin (180 + Θ) = - sinΘ
Cos (180 + Θ) = - cosΘ
Tan (180 + Θ) = tanΘ
Example: Use table to evaluate (a) sin220 (b) cos236 (c) tan242
Solution
(a) sin220 = sin (180 + 40) = - sin40 = - 0.6428
(b) cos236 = cos (180 + 56) = - cos56 = - 0.5992
(c) tan242 = tan (180 + 62) = tan 62 = 1.881



Fourth Quadrant
Sin (360 - Θ) = - sinΘ
Cos (360 - Θ) = cosΘ
Tan (360 - Θ) = - tanΘ

Example: Use table to evaluate (a) $sin310^0$ (b) $cos285^0$ (c) $tan334^0$
Solution
(a) $sin310^0$ = - sin (360-310) = - sin50 = - 0.7660
(b) $cos285^0$ = cos (360-285) = cos75 = 0.2588
(c) $tan334^0$ = - tan (360-334) = - tan26 = - 0.4877

Note that:
1. In the first quadrant, all the ratios are positive.
2. In the second quadrant, only sine ratio is positive, while the rest are negative.
3. In the third quadrant, only tangent ratio is positive, while the rest are negative.
4. In the fourth quadrant, only cosine ratio is positive, while the rest are negative.

Evaluation

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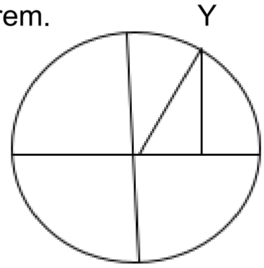
1) Use tables to evaluate the following (a) Sin 162^0 (b) Cos 283^0 (c) Tan 325^0 (d) Cos(- 75) (e)Tan (-100) (f) Sin (-223)

2) Use tables to find the values ϕ between 0^0 and 360^0 which satisfy each of the following.
(a) Sin ϕ = 0.4396 (b) Tan ϕ = - 2.4398 (c) Cos ϕ = 0.8427

TRIGONOMETRIC IDENTITY

Pythagoras theorem. Y

P
1
y
x
O x N



The figure above shows a unit circle. ▲OPN is a right angled with OP = 1, ON= x and PN = y, PON = Ө. From the definition of trigonometric ratios.

$x = \cos\Theta$ …. (1)

$y = \sin\Theta$ …..(2)

From (1) $x^2 = \cos^2\Theta$ …… (3)

From (2) $y^2 = \sin^2\Theta$ …… (4)

Adding equations (3) and (4)

$x^2 + y^2 = \cos^2\Theta + \sin^2\Theta$ …..(5)

Since ▲OPN is a right angled triangle

$ON^2 + NP^2 = OP^2$

$x^2 + y^2 = 1$ …… (6)

Equating equations (5) and (6)

$\text{Cos}^2\Theta + \sin^2\Theta = 1$ …… (7)

Dividing both sides of (7) by $\cos^2\Theta$

$\text{Cos}^2\Theta / \cos^2\Theta + \sin^2\Theta / \cos^2\Theta = 1 / \cos^2\Theta$

$1 + \tan^2\Theta = \sec^2\Theta$ …..(8)

Dividing (7) through by $\sin^2\Theta$

$\text{Cot}^2\Theta + 1 = \text{cosec}^2$

$1 + \cot^2\Theta = \text{cosec}^2\Theta$ …..(9)

Evaluation 1

1) Prove that $\dfrac{(1 - \sin\phi)(1 + \sin\phi)}{\sin^2\phi} = \cot^2\phi$

2) Show that (Sec ϕ - Tan ϕ)(Sec ϕ + Tan ϕ)= 1

Evaluation 2

Find the values of Ө lying between 0 and 360 for each of the following

1) Cos Ө = 0.2874
2) Sin Ө = 0.9361
3) Cos Ө =-0.8271
4) Tan Ө =-2.106

GRAPH OF SINE AND COSINE FOR ANGLES

In the figure below, a circle has been drawn on a Cartesian plane so that its radius, OP, is of length 1unit. Such a circle is called **unit circle.**

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The angle Θ that OP makes with Ox changes according to the position of P on the circumference of the unit circle. Since P is the point (x,y) and /OP/ = 1 unit,

Sin Θ = y/1 = y

Cos Θ = x/1 = x

Hence the values of x and y give a measure of cos Θ and sin Θ respectively.

If the values of Θ are taken from the unit circle, they can used to draw the graph of sin Θ. This is done by plotting values of y against corresponding values of Θ as in figure below.

In the figure above, the vertical dotted lines gives the values of sin Θ corresponding to Θ = 30, 60,90,......., 360.

To draw the graph of cos Θ, use corresponding values of x and Θ. This gives another wave-shaped curve, the graph of cos Θ as in figure below.

As Θ increases beyond 360, both curves begin to repeat themselves as in figures below.

Notice the following:

1) All values of sin Θ and cos Θ lie between +1 and -1.

2) The sine and cosine curves have the same shapes but different starting points.

3) Each curve is symmetrical about its peak (high point) and trough (low point). This means that for any value of sin Θ there are usually two angles between 0 and 360; likewise cos Θ. The only exceptions to this are at the quarter turns, where sinΘ and cosΘ have the values given in table below;

	0	90	180	270	360
SinΘ	0	1	0	-1	0
CosΘ	1	0	-1	0	1

Examples

1) Referring to graph on page 211 of NGM Book 1, (a) Find the value of sin 252, b) solve the equation 5 sin Θ = 4

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Solution

a) On the Ө axis, each small square represents 6. From construction a) on the graph:

Sin 252 = -0.95

b) If 5 sin Ө = 4

Then sin Ө = 4/5 = 0.8

From construction (b) on the graph: when sin Ө = 0.8, Ө = 54 or 126

Graph of tan Ө

Values can be taken from a unit circle to draw a tangent curve. In the figure below, a tangent is drawn to the unit circle OX. A typical radius is drawn and extended to meet the tangent at T. the y – coordinates of T gives a measure of tan Ө, where Ө is the angle that the radius makes with OX.

Note that tan Ө is not defined when Ө =90^0 and 270^0.

Ratio of special Angles (45^0, 30^0 and 60^0)

A. Tan, Sin and Cos of 45^0

Considering the diagram below;

ΔABC is right –angled triangle at B and /AB/ = /BC/ = 1 unit

$/AC/^2 = 1^2 + 1^2 = 2$ (by Pythagoras' theorem)

/AC/ =$\sqrt{2}$

Thus, $\tan 45^0 = 1$

$\sin 45^0 = \frac{1}{\sqrt{2}}$

$\cos 45^0 = \frac{1}{\sqrt{2}}$

B. Tan, Sin and Cos of 30^0 and 60^0

Considering the diagram below;

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ABC is an equilateral triangle with sides of 2 units in length. Line AD is an altitude where /BD/ = /DC/ = 1 unit.

In ΔABD, $/AB/^2 = /AD/^2 + /BD/^2$ (by Pythagoras' theorem)

$2^2 = /AD/^2 + 1^2$

$/AD/^2 = 3$

$/AD/ = \sqrt{3}$ units

Since, $<B = 60^0$

Thus, Tan $60^0 = \sqrt{3}$

Sin $60^0 = \frac{\sqrt{3}}{2}$

Cos $60^0 = ½ = 0.5^0$

Also, $<BAD = 30^0$

Tan $30^0 = 1/\sqrt{3}$

Sin $30^0 = ½ = 0.5^0$

Cos $30^0 = \frac{\sqrt{3}}{2}$

Example: Write the value of each the following in surd form;

1. $\sin 135^0$
2. $\tan 330^0$
3. $\cos 240^0$

Solution

1. $\sin 135^0 = \sin(180 - 135) = \sin 45 = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$
2. $\tan 330^0 = -\tan(360-330) = \tan 30 = 1/\sqrt{3}$
3. Cos2400 = cos (240 -180) = cos60 = - 1/2

Evaluation:

1) Using the same graph used in the above example, find the values of the following

A) sin 24 b) sin 294

2) Use the same graph to find the angles whose sines are as follows:

a) 0.65 b)-0.15

GENERAL EVALUATION

1. Use tables to evaluate each of the following (i) sin310 (ii) tan242 (iii) cos (-243) (iv) sin (-260) (iv) tan (-255)
2. Use tables to find the values of ϴ between 0^0 and 360^0 which satisfy each of the following (i) cos ϴ = -0.4540 (ii) tan ϴ= 1.176 (iii) sin ϴ = -0.9336
3. Using the same axis, a scale of 1cm to represent 30^0 on the ϴ-axis and 2cm to represent 1 unit on the y-axis, draw the graph of the following relations (i) y = sin ϴ (ii) sin ϴ/2
4. Simplify $\frac{3 + \sqrt{2}}{3 - \sqrt{2}}$
5. Express $\frac{5 - 2\sqrt{10}}{3\sqrt{5} + \sqrt{2}}$ in the form m + n√2, where, m and n are rational numbers

READING ASSIGNMENT

NGM BK 1 PG 187 – 195; Ex 17c nos 3 and 6

WEEKEND ASSIGNMENT

Given that sin ϴ = 4/5 and ϴ is acute

1. find cos ϴ (a) 5/3 (b) 3/5 (c) 4/3 (d) 4/5
2. find tan ϴ (a) 4/5 (b) 3/5 (c) 5/4 (d) ¾
3. find cosec ϴ (a) 4/5 (b) 3/5 (c) 5/4 (d) ¾

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4. find sec Θ (a) 5/3 (b) 5/4 (c) ¾ (d) 5/2

5. find cot Θ (a) 3/5 (b) 4/5 (c) 5/4 (d) 5/3

THEORY

1a.) Prove that $\frac{1}{1 + \cos\Theta} + \frac{1}{1 - \cos\Theta} = 2\,\text{cosec}^2\,\Theta$

b.) Given that sin Θ = 5/ 13 and Θ is acute, find (i) cos Θ (ii) tan Θ (iii) sec Θ (iv) cosec Θ (v) cot Θ

2a) Copy and complete the table below, giving corresponding values of Θ from 0^o to 360^o

Θ	0	30	60	90	120	150	180	210	240	270	300	330	360
Cos Θ	1	0.87	0.5	0	- 0.5								

b) Hence draw the graph of cos Θ, using 2cm to 0.5 on y-axis and 1cm to 30^o x-axis

bi) Construct a table for y = cosx – 3sinx for values of x from 0^0 to 180^0 at intervals of 20^0.

ii) Using a scale of 2cm to 20^0 on the x-axis and 2cm to 1 unit on the y-axis, draw the graph of y= cosx -3sinx.

iii) Use your graph to find the value(s) of x correct to the nearest degree for which (i) 3tanx = 1(ii) 2 + cosx = 3sinx.

WEEK NINE **DATE……………**

TOPIC: LOGIC

CONTENT

- Logical Statements
- Negations
- Conditional statements and bi-conditional statements.
- Identification of Antecedence & Consequence of Simple Statement

LOGICAL STATEMENTS

A logical statement is a declaration verbal or written that is either true or false but not both.

A true statement has a truth value T

A false statement has a truth value F

Logical statements are denoted by letters p, q, r ……

Questions, exclamations, commands and expression of feelings are not logical statements.

Example: Which of the following are logical statements?

i. Nigeria is an African country (Statement)

ii. Who is he? (Not statement)

iii. If I run I shall not be late (Statement)

iv. Japanese are hardworking people (Statement)

v. What a lovely man! (Not statement)

vi. The earth is conical in shape (Statement)

vii. If I think of my family (Not statement)

viii. Take the pencil away (Not statement)

Evaluation

State which of the statements is a logical statement

Name: ____Class: ________________

1. Caesar was great leader
2. Oh Mansa Musa, you are wonderful!
3. is he a serious teacher at all?
4. If 6 is an odd number, then 3 + 5 = 10
5. Stop talking to the boy
6. The Broking House in Ibadan is a magnificent building

NEGATION

Given a statement p, the negation of p written ~p is the statement 'it is false that p" or "not p"

If P is true (T), ~p is false (F) and if P is false (F) ~p is true (T).

The relationship between P and ~p is shown in a table called a truth table

P	~p
T	F
F	T

Example I: Let P be the statement 'Nigeria is a rich country' then ~p is the statement 'It is false that Nigeria is a rich country or 'Nigeria is not a rich country'

Example II: Let r be the statement 3 + 4 = 8 then ~p is the statement 3 + 4 ≠ 8

Example III: Let q be the statement 'isosceles triangle are equiangular' then ~q is the statement 'it is false that isosceles triangles are equiangular or 'isosceles triangle are not equiangular'.

Evaluation

1. Write the negation each of the following statements.
    1. It is very hot in the tropics.
    2. He is a handsome man.
    3. The football captain scored the first goal.
    4. Short cuts are dangerous.
2. Write the negation of each of the following avoiding the word 'not' as much as possible.
1. He was present in school yesterday.
2. His friend is younger than my brother.
3. She is the shortest girl in the class.
4. He obtained the least mark in the examination.

Reading Assignment: Further Maths projects Ex. 9a Q 3 – 7.

CONDITIONAL STATEMENTS

Let q stand for the statement 'Femi is a brilliant student' and r stand for the statement 'Femi passed the test'. One way of combing the two statement is 'If Femi is a brilliant student then Femi passed the test' or 'If q then r'

The student 'If q then r' is a combination of two simple statements q and r. It is called a compound statement.

Symbolically, the compound statement can be written as follows: 'If q then r' as q ⇒ r

The statement q ⇒ r is real as

q Implies r or

If q then r or q if r

The symbol ⇒ is an operation. In the compound statement q ⇒ r, the statement q is called the antecedent while the sub statement r is called the consequence of q ⇒ r.

The truth or falsity table for q ⇒ r is shown below.

q	r	q ⇒ r
T	TT	

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Name: ____Class: ________________

T	F	F
F	T	T

F FT

Example: If q is the statement 'I am a male' and r is the statement 'The sun will rise'
Consider the statements.

a. If I am a male then the sun will rise
b. If I am a male then the sun will not rise
c. If I am not a male then the sun will rise
d. If I am not a male then the sun will not rise

The statement (a), (c) and (d) are all true but b is not true b and c the antecedent is true and the consequent is false.

CONVERSE STATEMENT: The statement q ⇒ p is called the converse of the statement p ⇒p. e.g. Let p be the statement 'a triangle is equiangular' and q the statement 'a triangle is equilateral'.

The State p ⇒p means if a triangle is equiangular then u is equilateral.

The statement q ⇒ p means if a triangle is equilateral then u is equiangular.

INVERSE STATEMENT: This statement ~p ⇒~ q is called the inverse of the statement p ⇒ q.
If p is the statement 'a triangle is equiangular and q is the statement 'a triangle is equilateral' the statement ~p ⇒~ q is the statement 'if a triangle is not equiangular then it is not equilateral'.

CONTRAPOSITIVE STATEMENTS: The statement ~q ⇒~ p is called the contrapositive statement of p ⇒ q.
If p is the statement 'I can swim' and q is the statement 'I will win' then the statement ~q ⇒~ p is the statement 'If I cannot swim then I cannot win'.

Evaluation
If p is the statement 'it rains sufficiently' and q the statement 'the harvest will be good' write the symbol of these statements.

(i) If it rains sufficiently then the harvest will be good.
(ii) If it doesn't rain sufficiently then it doesn't
(iii) If the harvest is poor then it doesn't rain sufficiently.
(iv) It doesn't rain sufficiently.
(v) If it doesn't rain sufficiently then the harvest will be good.

IDENTIFICATION OF ANTICEDENCE AND CONSEQUENCE OF SIMPLE STATEMENTS.

1. Bi-conditional statements
2. The Chain Rule

1. **BICONDITIONAL STATEMENTS:** If p and q are statements such that p ⇒ q and q ⇒ p are valid, then p and q
Imply each other or p is equivalent to q and we write p ⇔ q. The statement p ⇔ q is called a biconditional
Statement of p and q and the statement p and q are equivalent to each other.

p ⇔ q could be read as
q Is equivalent to p or
q If and only if p or
p if and only if q or
if p then q and if q then p

The truth or falsity of p ⇔ q is shown below.

P	Q	P ⇔ q
T	T	T
T	F	F
F	T	F

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F	F	T

A bi-conditional statement is true when two sub-statements have the same truth value.
E.g. If p is the statement 'the interior angle of a polygon are equal' and q is the statement 'a polygon is regular'.

P ⇒ q is the statement 'if the interior angles of a polygon are equal then the polygon is regular'.
q ⇒ P is the statement 'if a polygon is regular then the interior angles of the polygon are equal'.
P ⇒ q and q ⇒ p
P ⇔ q
P and q are equivalent to each other.
Examples: Let p be the statement 'Mary is a model'
Let q be the statement 'Mary is beautiful'

Consider these statements.

a. Mary is a model if and only if she is beautiful.
b. Mary is a model if and only if she is ugly.
c. Mary is not a model if and only if she is beautiful.
d. Mary is not a model if and only if she is ugly.

Statements a and d are true b and c the sub-statements have the same truth value. Statements b and c are false because the sub-statements have different truth values.

2. THE CHAIN RULE: If p, q and r, are three statements such that p ⇒ q and q ⇒ r.

Example I: Consider the arguments
Premise T_1: If a student works very hard, he passes his examination
Premise T_2: If a student passes his examination he is awarded a certificate.
Conclusion T_3: If a student works very hard, he is awarded a certificate.

SOLUTION
Let p be the statement "a student works very hard"
Let q be the statement "a student passes his examination"
Let r be the statement "a student is awarded a certificate"

'The argument has the following structural form.
p ⇒ q and q ⇒ r ∴ p ⇒ r
This argument follows the chain rule link hence u is said to be valid.

Example II: Consider the arguments
T_1: Soldiers are disciplined
T_2: Good leaders are disciplined men
T_3: Soldiers are good leaders.

SOLUTION
Let p be the statement 'X is a seller'
Let q be the statement 'X is a disciplined man'
Let r be the statement 'X is a good leader'

The argument has the following structural form.
T_1 : p ⇒ q
T_2 : r ⇒ q
T_3 : p ⇒ r
The argument does not follow the format of the chain rule, hence it is not valid.

Evaluation I

Give an outline of the structural form of the following arguments and state whether or not it is valid.
T_1 : It is necessary to stay healthy in order to live long.
T_2 : It is necessary to eat balanced diet in order to stay healthy.

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T_3 : It is necessary to eat balanced diet in order to lives long.

Evaluation II

(1) Let P be the statement: "He is funny" and q be the statement: "He is serious". Write each of the following in simple English (i) p v q (ii) p ^ q (iii) p^ ~q (iv) ~pv~q

(2) If p and q represent two statements "he is good in physics" and "he is good in mathematics" respectively. Write the following in symbolic form; "he is good in physics if and only if he is good in mathematics".

General Evaluation

(1) Find the truth value of these statements.

a. If 11 > 8 then -1< -8

b. If 3 + 4 ≠ 10 then 2 + 3 ≠ 5

(2) Find the values of x satisfying $2^{3x+1} - 3(2^{2x}) + 2^{x+1} = 2^x$

(3) Solve the equation $3^{2x} - 30(3^x) + 81 = 0$

(4) Solve the simultaneous equations $2x + y = 3$; $4x^2 - y^2 + 2x + 3y = 16$.

Reading Assignment: F/Maths Project 1 pages 126 – 130 Exercise 9b Q 2, 3 and 4

WEEKEND ASSIGNMENT

P is the statement 'Ayo has determination and q is the statement 'Ayo will succeed'. Use this information to answer these questions. Which of these symbols represent these statements?

1. Ayo has no determination.

A. P ⇒ q B. ~ p ⇒ q C. ~ p

2. If Ayo has no determination then he won't succeed.

A. ~p ⇒~ q B. p ⇒~ q C. p ⇒ q D. p ⇒~ q

3. If Ayo won't succeed then he has no determination.

A. ~q ⇒ p B. ~q ⇒~q C. ~q ⇒ p D. q ⇒ p

4. If Ayo has determination then he will succeed.

A. ~p ⇒ q B. ~p ⇒~ q C. ~q ⇒~ p D. p ⇒ q

5. If Ayo has no determination then he will succeed.

A. ~p ⇒ q B. ~q ⇒~ p C. ~p D. ~p ⇒~ q

THEORY

1. Write down the inverse, converse and contrapositive of each of these statements.

(i) If the bank workers work hard they will be adequately compensated.

(ii) If he is humble and prayerful, he will meet with God's favour.

(iii) If he sets a good example, he will get a good followership.

2. Consider the following statements P: Some dogs are tame Q: All tame animals are small.

Which of the following is a valid conclusion from the above statements?

(i) All dogs are tame. (ii) No dog is small. (iii) All small animals are tame. (iv) Some dogs are small.

(v) All tame animals are dogs.

WEEK TEN **DATE……………**

TOPIC: Logical reasing continues

CONTENT:

- Connectives; (Disjunction and conjunction)
- Tautology and contradiction

Disjunction: In disjunction two statement can be combined by the use of the connective to **the truth table.** The truth table technique is used to establish whether or not two logical statement are equivalent.

Let p = He is a pastor and q = He is a singer

The above statement can be written as either he is a pastor or he is a singer.

Hence, in logical symbols; the statement can be written as p or q, where or means v i.epvq.

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NOTE: the statement Pvq is false when both p and q are the false otherwise pvq is true.
The truth table for the above statement is given or presented as:

p	q	Pvq
T	T	T
T	F	T
F	T	T
F	F	F

CONJUNCTION: When the connective and is used to combine two statement thus, we have **conjunction.**
Let p = Lagos is in Nigeria
Let q = 3 is an odd number
Thus, the above statement can be combined using the connective "and" as in : Lagos is in Nigeria and 3 is an odd number and it can be written as; p and q, where and is symbolically represented as ∧ i.e ∧ means "and". Hence, p and q = p∧q.
The above statement can be illustrated using a truth table.
NOTE: the statement p∧q is true when the sub statement p and q are both true otherwise p∧q is False.

p	q	p∧q
T	T	T
T	F	F
F	T	F
F	F	F

TAUTOLOGY: A compound statement which is always true irrespective of the truth values of the sub statement is called **TUATOLOGY**. It is represented as T.

Example: Use the truth table to show that the statement pv~p is a tautology.

p	~p	pv~p
T	F	T
T	F	T
F	T	T
F	T	T

From the above table it can be observed that the last column has the truth value T. Hence, the statement is **TAUTOLGY.**

CONTRADICTION: A compound statement which is always False irrespective of the truth value of the sub statement is called **CONTRADICTION.** It is usually denoted by F.

Example: Use the truth table to show that the statement p∧~p is a tautology.

p	~p	p∧~p
T	F	F
T	F	F
F	T	F
F	T	F

From the above table it can be observed that the last column has the truth value F. Hence, the statement is **CONTRADICTION.**

EVALUATION:

1. Copy and complete the truth table below:

P	q	r	qvr	~p∧ (qvr)

T	T	T		T
T	T	F		
T	F	T	T	
T	F	F	F	F
F	T	T		
F	T	F		
F	F	T		
F	F	F		F

2. Use the truth table technique to establish the following results:
(i) $p \wedge q = q \wedge p$
(ii.) $pv(q \wedge r) = (pvq)vr$
(iii) $\{p \wedge (\sim pvq)\}Vq$ is a tautology

GENERAL EVALUATION:

1. Draw the truth table for $\sim (p \rightarrow \sim q)$
Using the truth tables, prove that:
2. $p \wedge \{(\sim p \wedge p)V(\sim p \sim \wedge q)\}$ is a contradiction.
3. $\{(pv\sim q) \wedge (\sim pv\sim q)\}Vq$ is tautology.

Reading Assignment: F/Maths Project 2 pages 30 Exercise 3 Q 9 and 12

WEEKEND ASSIGNMENT
1. Let p = She is short and q = She is beautiful. Write each of the following in symbolic form using p and q. (i) She is short and beautiful (ii) She is short and but not beautiful (iii) It is false that she is tall and beautiful (iv) She is neither short nor beautiful.
Use the truth table technique to show that
2. $p \leftrightarrow q = (p \rightarrow q) \wedge (q \rightarrow p)$
3. $(p \wedge q) \wedge \sim (pvq)$ is a contradiction.
4. $(\sim pv\sim q)v\sim (pvr)v(qvr)$ is a tautology.

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