AP Calculus AB Summer Work

Welcome!

This packet includes a sampling of problems that students *entering* Calculus should be able to answer.

In Calculus, it's rarely the calculus that'll get you; it's the algebra. Students entering Calculus absolutely *must* have a strong foundation in algebra. Most questions in this packet were included because they concern skills and concepts that will be used extensively in Calculus. Others have been included not so much because they are skills that are used frequently, but because being able to answer them indicates a strong grasp of important mathematical concepts and—more importantly—the ability to problemsolve.

This packet will not be collected, but you must bring it to class the first day anyway. (If you're the sort of student who doesn't do homework unless forced to, Calculus might not be the best place for you…). It is extremely important for all students to review the concepts contained in this packet and to be prepared for a **test of prerequisite skills that will take place on or before our third class.** This grade goes in the gradebook as a test. Students whose scores show they were not prepared for the assessment probably either

a) don't have the mathematical prerequisite skills necessary for success in Calculus, or

b) don't have the work ethic necessary for success in Calculus. **Either way, you may be advised to leave the course.** To be clear, I won't kick you out if you really and truly want to stay and are willing to do the work to make up your deficit.

This is not a class where every problem you see on tests and quizzes is identical to problems you've done dozens of times in class. This is because the AP test itself (and, truly, all "real" mathematics) requires you to take what you know and apply it, rather than to simply regurgitate a rote process.

A word on calculators: while calculators have a place in Calculus, two-thirds of the exam is non-calculator. Therefore, nearly all the coursework will be completed without the use of a calculator. We won't even touch them until late January. If you've been using a calculator for trig, this summer is the time to memorize the unit circle.

Now that I've said all that, I encourage you to take a deep breath and start working. If you have the basics down and you put in the work needed, you'll see how amazing Calculus is!

A: SuperBasic Algebra Skills

A1. True or false. If false, change what is underlined to make the statement true.

a. $\left(x^3\right)^4 = x^{\underline{12}}$ T F

b. $x^{\frac{1}{2}}x^3 = x^{\underline{\frac{3}{2}}}$ T F

c. $(x + 3)^2 = \underline{x^2 + 9}$ T F

d. $\dfrac{x^2 - 1}{x - 1} = \underline{x}$ T F

e. $(4x + 12)^2 = \underline{16}(x + 3)^2$ T F

f. $\underline{3} + 2\sqrt{x - 3} = 5\sqrt{x - 3}$ T F

g. If $(x + 3)(x - 10) = \underline{2}$, then $x + 3 = \underline{2}$ or $x - 10 = \underline{2}$. T F

A2. More basic algebra.

a. If 6 is a zero of f, then _____ is a solution of $f(2x) = 0$.

b. Lucy has the equation $2(4x + 6)^2 - 8 = 16$. She multiplies both sides by ½. If she does this correctly, what is the resulting equation?

c. Simplify $\dfrac{2 \pm 4\sqrt{10}}{2}$

d. Rationalize the denominator of $\dfrac{12}{3 + \sqrt{x - 1}}$

e. If $f(x) = 3x^2 + x + 5$, then $f(x + h) - f(x) =$ (Give your answer in simplest form.)

f. A cone's volume is given by $V = \frac{1}{3}\pi r^2 h$. If $r = 3h$, write V in terms of h.

T: Trigonometry

You should be able to answer these <u>quickly</u>, *without* referring to a unit circle or a calculator.

T1. Find the value of each expression, in exact form.

a. $\sin \frac{2\pi}{3}$

b. $\cos \frac{11\pi}{6}$

c. $\tan \frac{3\pi}{4}$

d. $\sec \frac{5\pi}{3}$

e. $\csc \frac{7\pi}{4}$

f. $\cot \frac{5\pi}{6}$

T2. Find the value(s) of x <u>in $[0, 2\pi)$</u> which solve each equation (again, without a calculator).

a. $\sin x = \frac{\sqrt{3}}{2}$

b. $\cos x = -1$

c. $\tan x = \sqrt{3}$

d. $\sec x = -2$

e. $\csc x$ is undefined

f. $\cot x = 1$

T3. Solve the equation. Find the value(s) of x <u>in $[0, 2\pi)$</u>, if any.

a. $\sin 3x = 1$

b. $2\sqrt{3}\cos(\pi x) = 3$

c. $\tan 2x = 0$

d. $4\sec x + 1 = 9$

e. $\csc(4x + 3) = 0$

f. $3\cot 6x + \sqrt{3} = 0$

T4. Solve by factoring. Find the value(s) of x <u>in $[0, 2\pi)$</u>, if any.

a. $4\sin^2 x + 4\sin x + 1 = 0$

b. $\cos^2 x - \cos x = 0$

c. $\sin x \cos x - \sin^2 x = 0$

d. $x \tan x + 3\tan x = x + 3$

F: HigherLevel Factoring

F1. Solve by factoring.

a. $x^3 + 5x^2 - x - 5 = 0$

b. $4x^4 + 36 = 40x^2$

c. $(x^3 - 6)^2 + 3(x^3 - 6) - 10 = 0$

d. $x^5 + 8 = x^3 + 8x^2$

F2. Solve by factoring. You should be able to solve each of these *without* multiplying the whole thing out. (In fact, for goodness' sake, please *don't* multiply it all out!)

a. $(x + 2)^2 (x + 6)^3 + (x + 2)(x + 6)^4 = 0$

b. $(2x - 3)^3 (x^2 - 9)^2 + (2x - 3)^5 (x^2 - 9) = 0$

c. $(3x + 11)^5 (x + 5)^2 (2x - 1)^3 + (3x + 11)^4 (x + 5)^4 (2x - 1)^3 = 0$

d. $6x^2 - 5x - 4 = (2x + 1)^2 (3x - 4)^2$

F3. Solve. Each question *can* be solved by factoring, but there are other methods, too.

a. $a(3a + 2)^{1/2} + 2(3a + 2)^{3/2} = 0$

b. $\sqrt{2x^2 + x - 6} + \sqrt{2x - 3} = 0$

c. $2\sqrt{x + 3} = x + 3$

L: Logarithms and Exponential Functions

L1. Expand as much as possible.

a. $\ln x^2y^3$

b. $\ln \dfrac{x+3}{4y}$

c. $\ln 3\sqrt{x}$

d. $\ln 4xy$

L2. Condense into the logarithm of a single expression.

a. $4\ln x + 5\ln y$

b. $\frac{2}{3}\ln a + 5\ln 2$

c. $\ln x - \ln 2$

d. $\dfrac{\ln x}{\ln 2}$

(contrast with part **c**)

L3. Solve. Give your answer in exact form (NOT DECIMALS).

a. $\ln (x + 3) = 2$

b. $\ln x + \ln 4 = 1$

c. $\ln x + \ln (x + 2) = \ln 3$

d. $\ln (x + 1) - \ln (2x - 3) = \ln 2$

L4. Solve. Give your answer in exact form (NOT DECIMALS).

a. $e^{4x+5} = 1$

b. $2^x = 8^{4x-1}$

c. $100e^{x \ln 4} = 50$

R: Rational Expressions and Equations

R1.

	Function	Domain	Horiz. Asym., if any	Vert. Asym.(s), if any
a.	$f(x)=\dfrac{4x^2+7x-15}{8x^2-14x+5}$			
b.	$f(x)=\dfrac{3(4+x)^2-48}{x}$			
c.	$f(x)=\dfrac{6x+4}{\sqrt{3x^2-10x-8}}$		skip	

R3. Find the *x*coordinates where the function's output is zero and where it is undefined.

a. For what real value(s) of x, if any, is the output of the function $f(x)=\dfrac{x^2+4}{e^{6x}-1}$

…equal to zero? …undefined?

b. For what real value(s) of x, if any, is the output of $g(x)=\dfrac{\cos^2(\pi x)}{\sin x+2}$ …

…equal to zero? …undefined?

R4. Simplify completely.

a. $\dfrac{2}{\sqrt{x^2+4}}-\dfrac{x^2+4}{3}$ (Don't worry about rationalizing)

b. $\dfrac{3}{\left(\frac{4}{x}\right)^2+1}$ (Your final answer should have just one numerator and one denominator)

c. $\dfrac{5}{x^2+3x+2}-\dfrac{2x}{x^2+2x+1}$

d. $\dfrac{3}{(x+2)^{1/2}}+\dfrac{x}{(x+2)^{5/2}}$ (Don't worry about rationalizing)