Pythagoras
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Mini-PBL project	
Teacher data sheet: Teaching Guide	
Title	The Title declares most of the project and is probably the first spark to wake the interest of students. It must be direct, clear, motivating and descriptive of the real-life issue which it addresses.
SDG attended	Using this UN graphics, we mark such SDG which this project works.
Content units	The project may cover **2 or 3 content units**, as minimum. As the course advances, more units can be considered, but we may take care not to design a too long activity. The key concept to attend is the spiral curriculum strategy, focusing the review processes to recover students on risk to fail, and remark the connection between the content units of the course.
Sessions	Here we advance the number of sessions in the classroom we dedicate to work on this project. However, the students may know in advance that, in general, all the projects will require autonomous work, following the ECTS metric.
Hours of autonomous work	Here we may pay special attention and be careful not to generate an overtasking project. This is relevant since if you don't measure this autonomous part, the students' attitude and performance will be seriously affected. The exceed of work out of classroom affects to another subjects, impact negatively on the next mini-PBL proposed and, more disheartening for us, the goals of the mini-PBL will be displaced by the urgencies and rush (deep reflections, careful writing, checking results by different ways, discuss with classmates, etc). The worst consequence of an overtaking proposal is the cheating between students. You can introduce ways to avoid or reduce this bad practices, but regular teaching generates a high stressing workflow for both students and professors, if you have to manage additional control of the students' ethics performance.
Competences to be developed	Your subject has a list of competencies to be achieved by your students. Here is the place where those related with this project should be listed.

Recall always the sense of competences, don't mix with contents to be explained. These kinds of activities are one of the most favorable to work competencies since the combination of tasks and problems promote more the "know how to do" than the "know by repetition".

Here follows a list that can be used as example[1]. The first group of competencies are to do with the ***ability to ask and answer questions in and with mathematics:***

- **Thinking mathematically (mastering mathematical modes of thought)**
  such as
  - *posing questions* that are characteristic of mathematics, and knowing the kinds of answers (not necessarily the answers themselves or how to obtain them) that mathematics may offer;
  - understanding and handling the *scope* and *limitations* of a given *concept*.
  - *extending* the scope of a *concept* by abstracting some of its properties; *generalizing results* to larger classes of objects;
  - *distinguishing* between different *kinds of mathematical statements* (including conditioned assertions ('if-then'), quantifier laden statements, assumptions, definitions, theorems, conjectures, cases):
- **Posing and solving mathematical problems**
  such as
  - *identifying, posing,* and *specifying* different kinds of mathematical *problems* –pure or applied; open-ended or closed;
  - *solving* different kinds of mathematical problems (pure or applied, open-ended or closed), whether posed by others or by oneself, and, if appropriate, in different ways.
- **Modeling mathematically** (i.e. analyzing and building models)
  such as
  - *analyzing* foundations and properties of existing models, including assessing their range and validity
  - *decoding* existing models, i.e. translating and interpreting model elements in terms of the 'reality' modeled
  - *performing active modeling* in a given context
    - structuring the field
    - mathematising
    - working with(in) the model, including solving the problems it gives rise to
    - validating the model, internally and externally
    - analyzing and critiquing the model, in itself and vis-à-vis possible alternatives

[1] https://hqpbl.org

	- communicating about the model and its results - monitoring and controlling the entire modeling process. ● **Reasoning mathematically** such as ○ *following* and *assessing chains of arguments*, put forward by others ○ *knowing* what a mathematical *proof* is (not), ands how it differs from other kinds of mathematical reasoning, e.g. heuristics ○ *uncovering* the *basic ideas* in a given line of argument (especially a proof), including distinguishing main lines from details, ideas from technicalities; ○ *devising* formal and informal mathematical *arguments*, and *transforming* heuristic arguments to valid proofs, i.e. *proving statements.* The other group of competencies are to do with the ***ability to deal with and manage mathematical language and tools*:** ● **Representing mathematical entities (objects and situations)** such as ○ *understanding* and *utilizing* (decoding, interpreting, distinguishing between) different sorts of representations of mathematical objects, phenomena and situations; ○ understanding and utilizing the *relations between different representations* of the same entity, including knowing about their relative strengths and limitations; ○ *choosing* and *switching* between representations. ● **Handling mathematical symbols and formalisms** such as ○ *decoding* and *interpreting symbolic and formal* mathematical *language*, and understanding *its relations to natural language*; ○ understanding the *nature* and rules of *formal mathematical systems* (both syntax and semantics); ○ *translating* from *natural language* to *formal/symbolic language* ○ *handling* and manipulating statements and *expressions* containing *symbols* and *formulae*. ● **Communicating in, with, and about mathematics** such as ○ *understanding others*' written, visual or oral 'texts', in a variety of linguistic registers, about matters having a mathematical content;

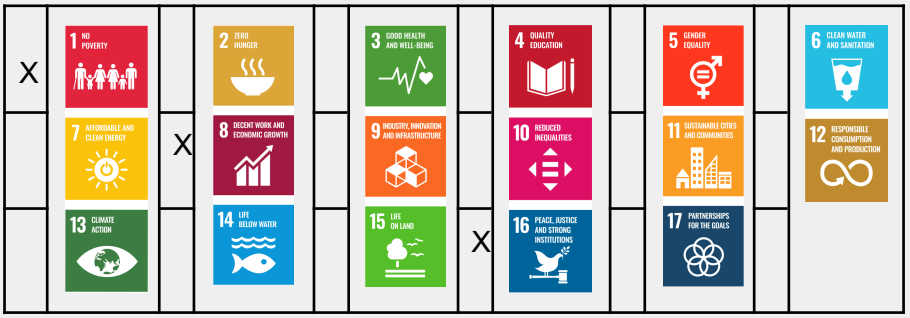
	○ *expressing oneself*, at different levels of theoretical and technical precision, in oral, visual or written form, about such matters. ● **Making use of aids and tools (IT included)** such as ○ *knowing* the *existence* and *properties* of various tools and aids for mathematical activity, and their range and limitations; ○ being able to *reflectively* use such aids and tools.
Key competencies for sustainability	● **Systems thinking competency:** the abilities to recognize and understand relationships; to analyse complex systems; to think of how systems are embedded within different domains and different scales; and to deal with uncertainty. ● **Anticipatory competency:** the abilities to understand and evaluate multiple futures – possible, probable and desirable; to create one's own visions for the future; to apply the precautionary principle; to assess the consequences of actions; and to deal with risks and changes. ● **Normative competency:** the abilities to understand and reflect on the norms and values that underlie one's actions; and to negotiate sustainability values, principles, goals, and targets, in a context of conflicts of interests and trade-offs, uncertain knowledge and contradictions. ● **Strategic competency:** the abilities to collectively develop and implement innovative actions that further sustainability at the local level and further afield. ● **Collaboration competency:** the abilities to learn from others; to understand and respect the needs, perspectives and actions of others (empathy); to understand, relate to and be sensitive to others (empathic leadership); to deal with conflicts in a group; and to facilitate collaborative and participatory problem solving. ● **Critical thinking competency:** the ability to question norms, practices and opinions; to reflect on own one's values, perceptions and actions; and to take a position in the sustainability discourse. ● **Self-awareness competency:** the ability to reflect on one's own role in the local community and (global) society; to continually evaluate and further motivate one's actions; and to deal with one's feelings and desires. ● **Integrated problem-solving competency:** the overarching ability to apply different problem-solving frameworks to complex sustainability problems and develop viable, inclusive and equitable solution options that promote sustainable development, integrating the above-mentioned competences.

ICT tools	Here is probably one of the items where you show to the students how it is going to be your support. In this section we recommend to list the features which can be used in the PBL resolution. We don't list software, this search by the students must be part of the project. As example, we should provide a list like this: - Graphics: explicit, implicit, 2D, 3D,... - Solving equations and/or systems: graphically, numerically, algebraically, … - Calculus calculator: derivatives, integrals,.. - Vector and matrix calculator: graphically, numerically,..
Context: project estatement	This section is where the project is presented to the student. The introduction, the core topic and all the information will help students to allocate the tasks and problems posted late. Here you may include the references, graphics, news from media, official reports (UN, EU, OCDE, UNICEF,...), and any source which helps to acquire a wider idea about the SDG problem we address. This effort to "put in context" the problem will connect mathematics with real-life, with Earth challenges and Humanity needs. Don't reduce this part too much thinking that students won't pay attention. When we teach mathematics by using SDG we are promoting the reflection of our students, future professionals, on the major challenges of Mankind and Earth. We are constructing Global Citizens.
Tasks and problems	Obviously, this is the section to collect all the aims of the project, by the activities and tasks to be done. In the format you prefer: classical problems, inquiries, modelization, generalization, etc. Here you have to be creative, diverse and stimulating, focus more on the competences to develop than the difficulty or the range of contents to cover. List the problems/tasks in crescent order, try to use previous results on the next ones, and drive along the content units selected (spiral curriculum) recalling the students the connection between such units.
Outcomes expected	This is a critical list: The student may know perfectly what you expect from him. PBL can't be a hidden game. This training will be useful in their professional life: the deep analysis of a problem, the design of the solution and the presentation of the results must be a well structured process, and PBL helps. However, this list can't limitate the creativity of the student, you should provide a guide of minimum output expected: - Graphics fitting the solution - Tables of data used/obtained in solutions - Numerical results explained and put in context - Capture of ICT tools solutions used - Sequence of steps followed - Remark computations done by hand and done by ICT tools - Provide complete answer to questions

	- All the results must be presented in the context of the problem
Guide for Learning	This section will evolve along you and your students would acquire experience on working in mini-PBL projects. We don't usually provide learning tips with our syllabus: Here is critical. Mainly at the beginning of the course, the students need guides on new activities, and feel your support on a well structured pack of suggestions on how to address the problems posted. Here some examples: - Read carefully the problem statement and the tasks posted. Always maintain a global view of all the projects. - Identify, or try to do a first draft match, the content units of your lecture notes involved in every task. Take your lecture notes open and review before starting to solve the problems. - Match output expected with the tasks posted, at least as first draft approach. This will give you ideas about which tools are needed (ICT tools, hand calculations, data collection, …). - Follow the order of the tasks, try to increase the knowledge of the problem while you are solving the activities. - Always think that maybe there are different ways to solve a problem. - Use ICT tools to avoid hard computations and check your solutions are correct in different ways if possible. - The solutions are always part of a context, expressing such a final solution totally integrated in the problem posted. - Be sure you answer the complete questions. Partial solving could generate missing information for the following questions. - Always try to solve the questions by yourself. If the project can be done in groups, discuss in groups to confirm and detect fails or weaknesses, confront strategies, discuss presentation format, etc. Working in groups doesn't mean work less but work better. - ……
Guide for Teaching	Here we should develop all the key facts and procedures which will guide the teachers to provide the advice, help and hints needed to present and launch the mini-PBL to students. From our point of view, this section also could be the place where we present the backoffice of the expected results and goals of the project, the desired strategies to be developed by students, the list of tools and phases in the development of the tasks. Here we can provide to teachers the mathematical insights we desire

	to be achieved by the students along the construction and solution of the mini-PBL project.
Assessment	One of the main items, which used to worry students and always affects students's perception of the activity. We recommend the following scheme: **Alternative Assessment** Assessment for learning starts with outcomes, proceeds with projects, products, and performances that map to the outcomes, and completes the loop with assessment and feedback to students. Alternative assessment provides avenues to assess projects effectively. Alternative assessment **assesses acquisition of knowledge and skills in ways other than the conventional methods such as traditional paper-and pencil tests. It actively involves students in a process that combines what is taught, how it is taught, and how it is evaluated.** *Characteristics of alternative assessment* ❖ Authentic, often in real-life environments, with real-world challenges. ❖ Interdisciplinary in nature emphasizing on specific knowledge as well as general skills such as transfer of information across settings. ❖ Involves negotiation and interpersonal skills as well as decision making skills. ❖ Involves mastery of a task before progressing to the next task. ❖ Involves mastery assessment of periodic performance ❖ Gives responsibility to learners for directing and managing their own learning. **Tools for Assessment** Checklist | Anecdotal records | Calendar records | Exhibition Oral Presentation | Performing a Skill | Conducting Experiments | Demonstration **Types of Assessment** • **Performance-based Assessment:** assess application of skills and competencies mastered in completing activities or tasks through observation. • **Authentic Assessment:** assess "real-life" and planning skills,

	creativity, knowledge integration, and collabora- tion abilities outside the school environment. This can be achieved by using a predetermined set of criteria for instance rubrics, a scoring scale incorporating a set of essential criteria for the task and appropriate levels of performance for each criterion used. • **Portfolio Assessment:** evaluates the compilation of work and processes attested in efforts and success of a particular project or area. Examinees are required to review and select items that best demonstrate their learning. Examples of portfolios can be paper-based, computer-based or a combination of both. • **Journal Assessment:** assess the continual documenta- tion of examinee's expressions, feelings, and experiences through checklists and keeping of logs. **Rubrics** Rubrics are authentic assessment tools designed to simulate real-life activity where students are engaged in solving real-life problems. It is particularly useful in assessing complex and subjective criteria. Formative assessment best describes rubrics and it becomes an ongoing part of the whole teaching and learning process. Its assessment tools comprise the rating scale, a set of evaluation criteria and descriptors. See ANNEX 1 for Rubric
Others: References	

Mini-PBL project	
Student data sheet: Learning Guide	
Title	It must be direct, clear, motivating and descriptive of the real-life issue which it addresses.
SDG attended	Using this UN graphic, we mark such SDG which this project works. 
Content units	The project may cover **2 or 3 content units**, as minimum.
Sessions	Number of sessions in the classroom we dedicate to work on this project.
Hours of autonomous work	The students may know in advance that, in general, all the projects will require autonomous work, following the ECTS metric.
ICT tools to be used	As example, we should provide a list like this: - Graphics: explicit, implicit, 2D, 3D,... - Solving equations and/or systems: graphically, numerically, algebraically, … - Calculus calculator: derivatives, integrals,.. - Vector and matrix calculator: graphically, numerically,..
Context: project estatement	This section is where the project is presented to the student. The introduction, the core topic and all the information will help students to allocate the tasks and problems posted late.
Tasks and problems	List the problems/tasks in difficulty crescent order. Activity 1: - *Problem 1.1* - *Problem 1.2* - … Activity 2: …… Activity n: - *Problem n.1* …..
Outcomes expected	Guide of minimum output expected:

	- Graphics fitting the solution - Tables of data used/obtained in solutions - Numerical results explained and put in context - Capture of ICT tools solutions used - Sequence of steps followed - Remark computations done by hand and done by ICT tools - Provide complete answer to questions - All the results must be presented in the context of the problem
Guide for learning	Here some examples: - Read carefully the problem statement and the tasks posted. Always maintain a global view of all the projects. - Identify, or try to do a first draft match, the content units of your lecture notes involved in every task. Take your lecture notes open and review before starting to solve the problems. - Match output expected with the tasks posted, at least as first draft approach. This will give you ideas about which tools are needed (ICT tools, hand calculations, data collection, …). - Follow the order of the tasks, try to increase the knowledge of the problem while you are solving the activities. - Always think that maybe there are different ways to solve a problem. - Use ICT tools to avoid hard computations and check your solutions are correct in different ways if possible. - The solutions are always part of a context, expressing such a final solution totally integrated in the problem posted. - Be sure you answer the complete questions. Partial solving could generate missing information for the following questions. - Always try to solve the questions by yourself. If the project can be done in groups, discuss in groups to confirm and detect fails or weaknesses, confront strategies, discuss presentation format, etc. Working in groups doesn't mean work less but work better. - ……
Assessment	See ANNEX 1 for Rubric
Others:	

References	

ANNEX 1: RUBRIC

Category	4=Excellent	3=Good	2=Low	1=Poor
Mathematical Concepts	Explanation shows complete understan- ding of the mathe- matical concepts used to solve the problem(s).	Explanation shows substantial understan- ding of the mathema- tical concepts used to solve the problem(s).	Explanation shows some understan- ding of the mathe- matical concepts needed to solve the problem(s).	Explanation shows very limited unders- tanding of the underlying concepts needed to solve the problem(s) OR is not written.
Mathematical Terminology and Notation	Correct terminology and notation are always used, making it easy to understand what was done.	Correct terminology and notation are usually used, making it fairly easy to understand what was done.	Correct terminolo- gy and notation are used, but it is sometimes not easy to understand what was done.	There is little use, or a lot of inappropriate use, of terminology and notation.
Strategy/Procedure	Typically, uses an efficient and effective strategy to solve the problem(s).	Typically, uses an effective strategy to solve the problem(s).	Sometimes uses an effective strategy to solve problems, but does not do it consistently.	Rarely uses an effective strategy to solve problems.
Completion	All problems are completed.	All but one of the problems are completed.	All but two of the problems are completed.	Several of the problems are not completed.
Mathematical Errors	90-100% of the steps and solutions have no mathematical errors.	Almost all (85-89%) of the steps and solu- tions have no mathematical errors.	Most (75-84%) of the steps and solu- tions have no mathematical errors.	More than 75% of the steps and solu- tions have mathe- matical errors.

Sources Checking				
Working with Others	Student was an engaged partner, listening to suggestions of others and working cooperatively throughout lesson.	Student was an engaged partner but had trouble listening to others and/or working cooperatively.	Student cooperated with others, but needed prompting to stay on-task.	Student did not work effectively with others.
Neatness and Organization	The work is presented in a neat, clear, organized fashion that is easy to read.	The work is presented in a neat and organized fashion that is usually easy to read.	The work is presented in an organized fashion but may be hard to read at times.	The work appears sloppy and unorganized. It is hard to know what information goes together.
Diagrams and Sketches	Diagrams and/or sketches are clear and greatly add to the reader's understanding of the procedure(s).	Diagrams and/or sketches are clear and easy to understand.	The work is presented in an organized fashion but may be hard to read at times.	Diagrams and/or sketches are difficult to understand or are not used.
ICT tools used				

Sources of examples

Special interest for any teacher is to have access to a high variety of sources for examples and problems to supply the mini-PBL projects. In fact, it is enough to give a basic applied problem to generate a bigger list of activities based on that.

Every course, in all universities around the world, graduate students present thousands of degree, master and PhD thesis where you can find for sure a list of examples/applications where a graphic or dataset can be the start point for constructing a mini-PBL project.

We encourage you to find your own sources, ask colleagues from your university for precise use of mathematics in any scientific area. Then connect such applications with SDG and construct your mini-PBL.

For help in such searching, in the following table we group basic scientific areas with the SDG to provide a list of suggestions for search math applications with SDG's interest label. Obviously, this is not a closed list, the items are interchangeable, and anyone can find an application of interest in the thousands of research papers and reports published every year all round the world.

SUSTAINABLE DEVELOPMENT GOALS	**Scientific areas or studies**
1 NO POVERTY	❖ Population studies and databases ❖ Incomes & Costs reduction ❖ Logistic & Transportation ❖ Equality studies
2 ZERO HUNGER	❖ Agriculture ❖ Nutrition ❖ Plagues ❖ Pollution
3 GOOD HEALTH AND WELL-BEING	❖ Medical databases ❖ Epidemiology ❖ Pharmacokinetic ❖ Microbiology ❖ Genetic ❖ Sports ❖ Aging

	❖ Education databases ❖ Psychology ❖ Neuroscience ❖ Human & childhood behavior ❖ Economy of Education ❖
	❖ Incomes inequalities ❖ Population proportion on labor sectors ❖
	❖ Water supply databases ❖ Pollution ❖ Microbiology ❖ Engineering ❖
	❖ Energy production and consume databases ❖ Engineering ❖ Consume ❖ Optimization of resources ❖ Penetration of renewable energies
	❖ Labor databases ❖ Social studies ❖ Share market ❖ Bank products ❖ Economy and enterprise ❖ Tourism ❖ Digital business

	❖ Industry and production databases ❖ Engineering ❖ Mobility ❖ Artificial Intelligence ❖ New Materials ❖ Nanotechnology ❖ Connectivity ❖ Logistic & Transportation ❖ 4th Industrial Revolution: Industry 4.0
	❖ Social lacks and gaps databases ❖ Social studies ❖ Population studies ❖ Incomes/costs reduction ❖ Digital access
	❖ Quality living databases ❖ Population studies ❖ Optimization ❖ Logistic & Transportation ❖ Civil Engineering ❖ Architecture ❖ Baggage management ❖ Energy ❖ New materials for urban furniture: maintenance, cleaning, preventing damage,... ❖ Electric and autonomous mobility ❖ Air pollution ❖ Noise reduction ❖ Mobility ❖

	❖ Consume and human behavior databases ❖ Marketing ❖ Social networks ❖ Raw and new materials ❖ Energy ❖ Engineering
	❖ Climate change databases ❖ Deforestation ❖ Desertification ❖ Atmospheric physics ❖ Overheating ❖ CO2 reduction ❖ Carbon footprint
	❖ Climate change and oceans ❖ Acidification of oceans ❖ Sustainable fishing ❖ Microbiology on oceans ❖ Microplastic and oceans ❖ Waste and biodegradable materials
	❖ Climate change and land sustainability ❖ Sustainable farming ❖ Residues and waste processing ❖ Waste and biodegradable materials
	❖ Social studies databases ❖ Economy ❖ Cooperation for development ❖

	❖ Social studies ❖ Social networks ❖ Civil movements ❖

References

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