

5.4 Optimization notes

Solve the following problems given the constraints.

1.) Production Cost Minimization

- Objective: **Minimize production cost**
- Cost Function: $C(x) = 1/3x^3 - 3x^2 - 16x$
- Where x represents the production quantity

Determine the production quantity that minimizes the total cost.

2. Fence Area Maximization

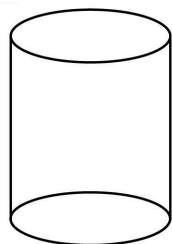
- Objective: Maximize the **area of a rectangular field**
- Given Area Function: $A(w) = w(100 - w)$
- Where w is the width of the field
- Constraint: Total available fencing is 200 feet

Find the width and length that provides the maximum area.



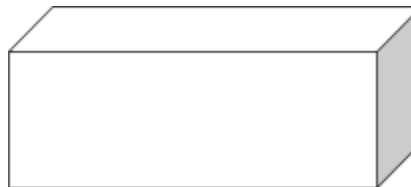
3.) Cylindrical Can

- Objective: Maximize the **volume** of a cylindrical can
- Given Volume Function: $v = \pi r^2 h$
- Constraint: Total surface area is fixed at 100 square inches
- Surface Area Constraint: $2\pi r^2 + 2\pi r h = 100$
- Solve for h in terms of r to solve:



4.) Package Design

- Objective: Find the dimensions that **minimize the surface area** of a rectangular package in which the base is a square.
- Surface Area Function: $S = 2x^2 + 4xy$
- Volume function: $v = x^2 y$
- Volume Constraint: $V = 500$ cubic inches



Set up, but do not solve the following problems.

5.) Farmer's Fence

A farmer wants to create a rectangular field using a river as one of its boundaries. The farmer has 1200 meters of fencing available and wants to maximize the area of the field. The river runs straight and will form one side of the rectangle, eliminating the need to fence that side.

Sketch:

Clearly define your variables:

Develop the objective function:

Establish the constraints:

Set up optimization model:

6.) Package Design

A shipping company needs to design a rectangular cardboard box with an open top and square base. The box must have a volume of exactly 32,000 cubic centimeters. The cost of materials is \$2 per square centimeter for the bottom, \$1 per square centimeter for the sides, and \$0 per square centimeter for the top (which is open). The company wants to minimize the total material cost.

Sketch:

Clearly define your variables:

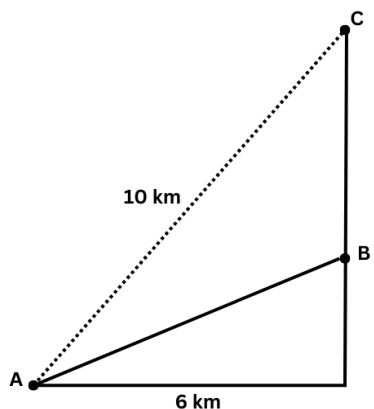
Develop the objective function:

Establish the constraints:

Set up optimization model:

7.) Optimal path: A hiker needs to travel from Point A to Point C, with a river crossing at Point B. The hiker can travel on land at 3 kilometers per hour and can use a boat to float down the river at 5 kilometers per hour. The straight-line distance from A to C is 10 kilometers and the shortest distance to the river is 6 kilometers. The hiker wants to find the point on the river (Point B) that minimizes the total travel time. (Neglect the width of the river.) ([diagram link](#))

Sketch:



Clearly define your variables:

Develop the objective function:

Establish the constraints:

Set up optimization model:

General Strategy

1. Understand
2. Sketch situation (if applicable)
3. Identify the *objective function* (a mathematical model)
4. Graph the function (if calculator is allowed)
5. Find the critical points and interpret their meaning (if calculator is not allowed)
6. Optimize the objective function
7. Interpret your solution

1.) Find two numbers whose sum is 20 and whose product is as large as possible.

3.) Maximize the volume of a box with an open top.

Suppose we have a 20 x 25 in. sheet of tin. We want to cut squares out of each corner of the sheet and fold up the sides to create a box. How large should the squares be to maximize the volume of the box?

4.) A man wants to build a rectangular enclosure for his herd. He only has \$900 to spend on the fence and wants the largest size for his money. He plans to build the pen along the river on his property, so he does not have to put a fence on that side. The side of the fence parallel to the river will cost \$5 per foot to build, whereas the sides perpendicular to the river will cost \$3 per foot. What dimensions should he choose?

5.) **Sine wave problem:** Find the dimensions and area of the largest rectangle that can be inscribed between a sine curve and the x-axis.

6.) **Can Problem:** Minimize the material used to create a cylindrical can with 1000 cm^3 .