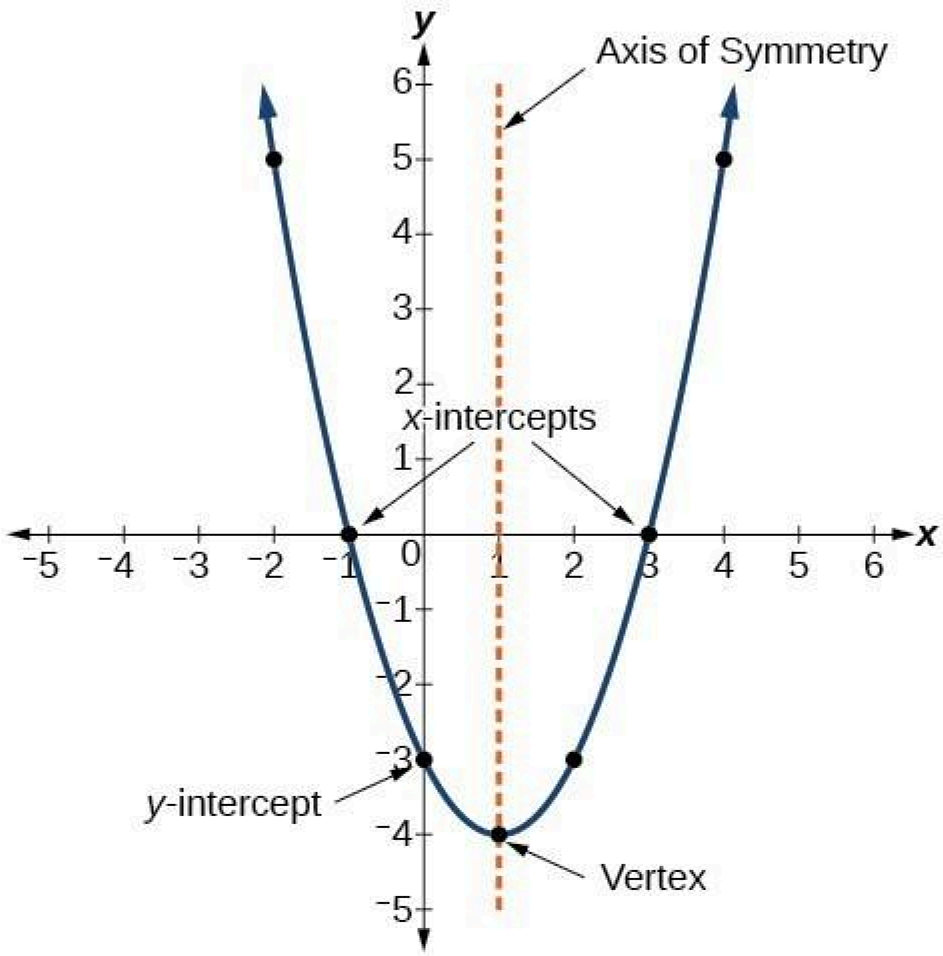
See this page in the course material.

Learning Outcomes

- Identify the vertex, axis of symmetry, [latex]y[/latex]-intercept, and minimum or maximum value of a parabola from it’s graph.
- Identify a quadratic function written in general and vertex form.
- Given a quadratic function in general form, find the vertex.
- Define the domain and range of a quadratic function by identifying the vertex as a maximum or minimum.

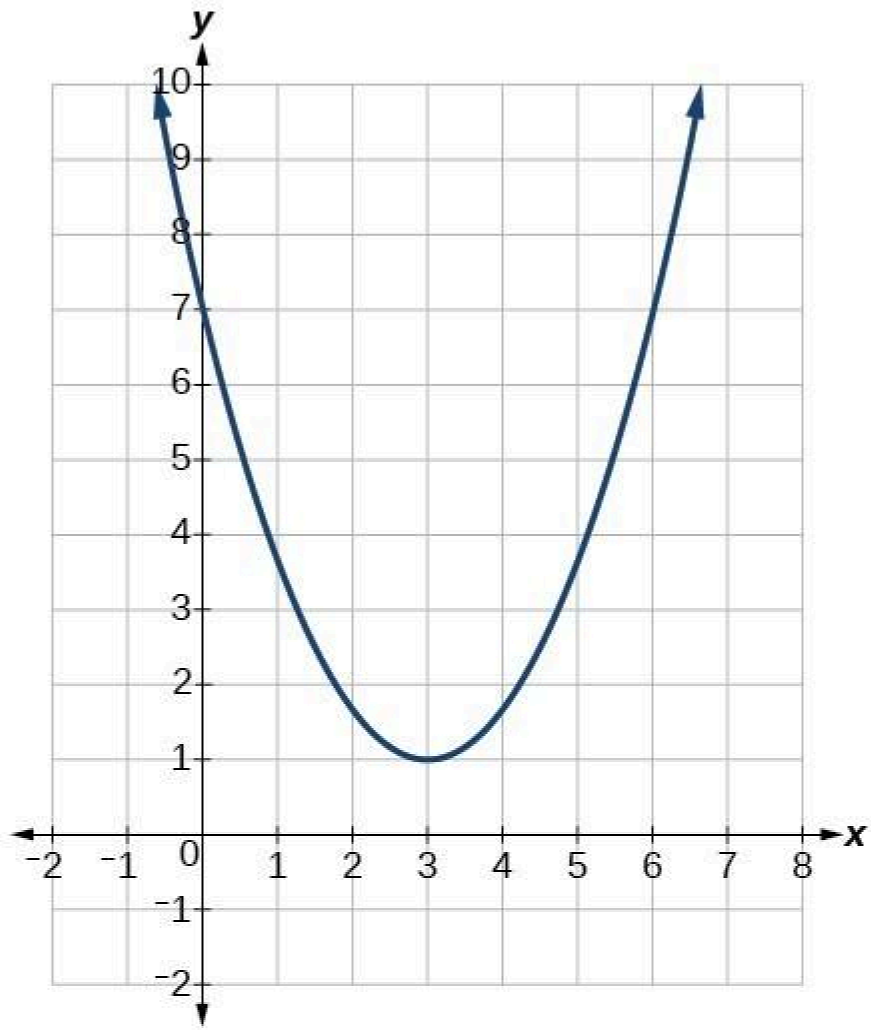
The graph of a quadratic function is a U-shaped curve called a **parabola**. One important feature of the graph is that it has an extreme point, called the **vertex**. If the parabola opens up, the vertex represents the lowest point on the graph, or the **minimum value** of the quadratic function. If the parabola opens down, the vertex represents the highest point on the graph, or the **maximum value**. In either case, the vertex is a turning point on the graph. The graph is also symmetric with a vertical line drawn through the vertex, called the **axis of symmetry**.



The [latex]y[/latex]-intercept is the point at which the parabola crosses the [latex]y[/latex]-axis. The [latex]x[/latex]-intercepts are the points at which the parabola crosses the [latex]x[/latex]-axis. If they exist, the [latex]x[/latex]-intercepts represent the **zeros**, or **roots**, of the quadratic function, the values of [latex]x[/latex] at which [latex]y=0[/latex].

Example: Identifying the Characteristics of a Parabola

Determine the vertex, axis of symmetry, zeros, and *y*-intercept of the parabola shown below.



Show Solution

The vertex is the turning point of the graph. We can see that the vertex is at [latex](3,1)[/latex]. The axis of symmetry is the vertical line that intersects the parabola at the vertex. So the axis of symmetry is [latex]*x*=3[/latex]. This parabola does not cross the [latex]x[/latex]-axis, so it has no zeros. It crosses the [latex]y[/latex]-axis at (0, 7) so this is the [latex]y[/latex]-intercept.

Equations of Quadratic Functions

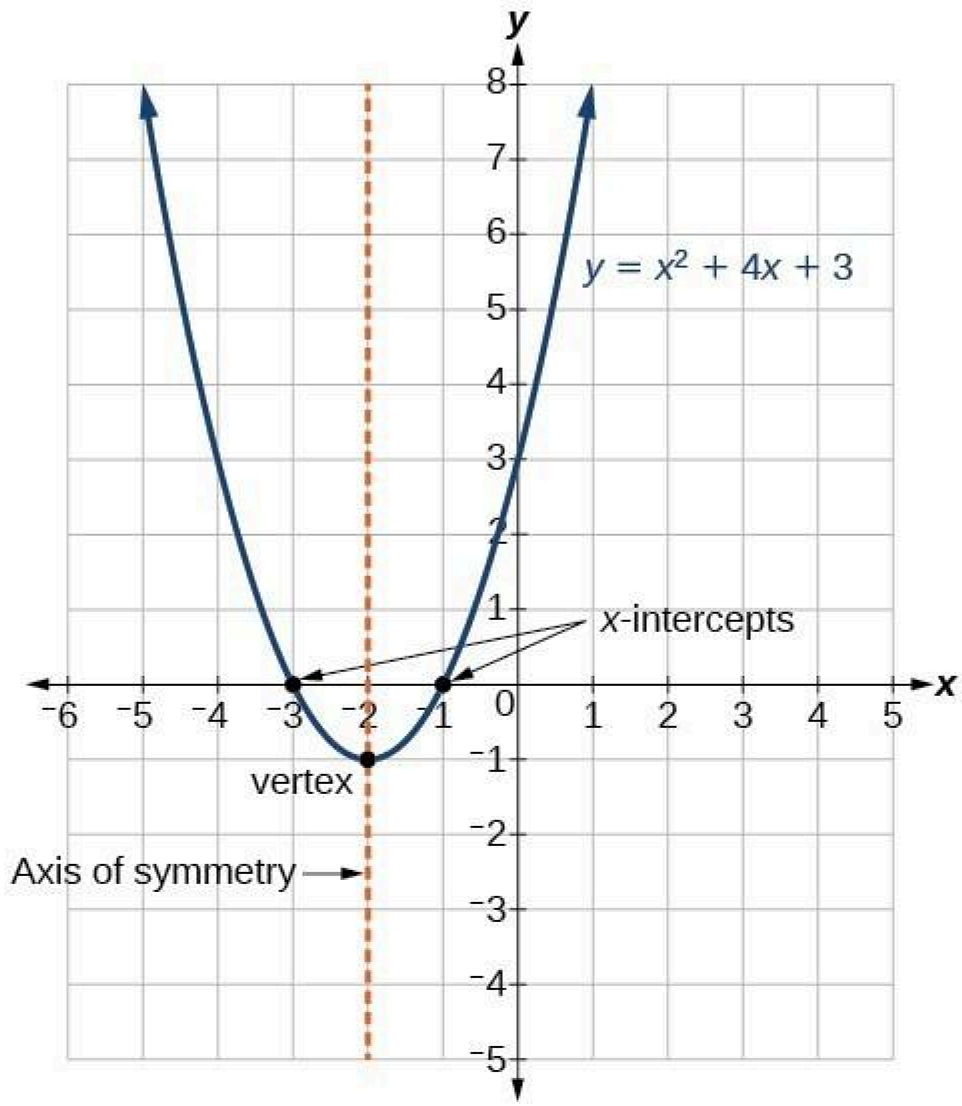
The **general form of a quadratic function** presents the function in the form

[latex]f\left(x\right)=a{x}^{2}+bx+c[/latex]

where [latex]a[/latex], [latex]b[/latex], and [latex]c[/latex] are real numbers and [latex]a\ne 0[/latex]. If [latex]a>0[/latex], the parabola opens upward. If [latex]a<0[/latex], the parabola opens downward. We can use the general form of a parabola to find the equation for the axis of symmetry.

The axis of symmetry is defined by [latex]x=-\dfrac{b}{2a}[/latex]. If we use the quadratic formula, [latex]x=\dfrac{-b\pm \sqrt{{b}^{2}-4ac}}{2a}[/latex], to solve [latex]a{x}^{2}+bx+c=0[/latex] for the [latex]x[/latex]-intercepts, or zeros, we find the value of [latex]x[/latex] halfway between them is always [latex]x=-\dfrac{b}{2a}[/latex], the equation for the axis of symmetry.

The figure below shows the graph of the quadratic function written in general form as [latex]y={x}^{2}+4x+3[/latex]. In this form, [latex]a=1,\text{ }b=4[/latex], and [latex]c=3[/latex]. Because [latex]a>0[/latex], the parabola opens upward. The axis of symmetry is [latex]x=-\dfrac{4}{2\left(1\right)}=-2[/latex]. This also makes sense because we can see from the graph that the vertical line [latex]x=-2[/latex] divides the graph in half. The vertex always occurs along the axis of symmetry. For a parabola that opens upward, the vertex occurs at the lowest point on the graph, in this instance, [latex]\left(-2,-1\right)[/latex]. The [latex]x[/latex]-intercepts, those points where the parabola crosses the [latex]x[/latex]-axis, occur at [latex]\left(-3,0\right)[/latex] and [latex]\left(-1,0\right)[/latex].



The **standard form of a quadratic function** presents the function in the form

[latex]f\left(x\right)=a{\left(x-h\right)}^{2}+k[/latex]

where [latex]\left(h,\text{ }k\right)[/latex] is the vertex. Because the vertex appears in the standard form of the quadratic function, this form is also known as the **vertex form of a quadratic function**.

Given a quadratic function in general form, find the vertex of the parabola.

One reason we may want to identify the vertex of the parabola is that this point will inform us where the maximum or minimum value of the output occurs, [latex]k[/latex], and where it occurs, [latex]h[/latex]. If we are given the general form of a quadratic function:

[latex]f(x)=ax^2+bx+c[/latex]

We can define the vertex, [latex](h,k)[/latex], by doing the following:

- Identify [latex]a[/latex], [latex]b[/latex], and [latex]c[/latex].
- Find [latex]h[/latex], the [latex]x[/latex]-coordinate of the vertex, by substituting [latex]a[/latex] and [latex]b[/latex] into [latex]h=-\dfrac{b}{2a}[/latex].
- Find [latex]k[/latex], the [latex]y[/latex]-coordinate of the vertex, by evaluating [latex]k=f\left(h\right)=f\left(-\dfrac{b}{2a}\right)[/latex]

Example: Finding the Vertex of a Quadratic Function

Find the vertex of the quadratic function [latex]f\left(x\right)=2{x}^{2}-6x+7[/latex]. Rewrite the quadratic in standard form (vertex form).

Show Solution

The horizontal coordinate of the vertex will be at

[latex]\begin{align}h&=-\dfrac{b}{2a}\ \\[2mm] &=-\dfrac{-6}{2\left(2\right)} \\[2mm]&=\dfrac{6}{4} \\[2mm]&=\dfrac{3}{2} \end{align}[/latex]

The vertical coordinate of the vertex will be at

[latex]\begin{align}k&=f\left(h\right) \\[2mm]&=f\left(\dfrac{3}{2}\right) \\[2mm]&=2{\left(\dfrac{3}{2}\right)}^{2}-6\left(\dfrac{3}{2}\right)+7 \\[2mm]&=\dfrac{5}{2}\end{align}[/latex]

So the vertex is [latex]\left(\dfrac{3}{2},\dfrac{5}{2}\right)[/latex]

Rewriting into standard form, the stretch factor will be the same as the [latex]a[/latex] in the original quadratic.

$$f\left(x\right)=2{\left(x-\frac{3}{2}\right)}^{2}+\frac{5}{2}$$

Try It

Given the equation $g\left(x\right)=13+{x}^{2}-6x$, write the equation in general form and then in standard form.

Show Solution

$g\left(x\right)={x}^{2}-6x+13$ in general form; $g\left(x\right)={\left(x - 3\right)}^{2}+4$ in standard form

Finding the Domain and Range of a Quadratic Function

Any number can be the input value of a quadratic function. Therefore the domain of any quadratic function is all real numbers. Because parabolas have a maximum or a minimum at the vertex, the range is restricted. Since the vertex of a parabola will be either a maximum or a minimum, the range will consist of all y-values greater than or equal to the y-coordinate of the vertex or less than or equal to the y-coordinate at the turning point, depending on whether the parabola opens up or down.

A General Note: Domain and Range of a Quadratic Function

The domain of any **quadratic function** is all real numbers.

The range of a quadratic function written in general form $f\left(x\right)=a{x}^{2}+bx+c$ with a positive a value is [latex]f\left(x\right)\ge f\left(-\frac{b}{2a}\right)[/latex], or [latex]\left[f\left(-\frac{b}{2a}\right),\infty \right)[/latex]; the range of a quadratic function written in general form with a negative [latex]a[/latex] value is [latex]f\left(x\right)\le f\left(-\frac{b}{2a}\right)[/latex], or [latex]\left(-\infty ,f\left(-\frac{b}{2a}\right)\right][/latex].

The range of a quadratic function written in standard form [latex]f\left(x\right)=a{\left(x-h\right)}^{2}+k[/latex] with a positive [latex]a[/latex] value is [latex]f\left(x\right)\ge k[/latex]; the range of a quadratic function written in standard form with a negative [latex]a[/latex] value is [latex]f\left(x\right)\le k[/latex].

How To: Given a quadratic function, find the domain and range.

1. The domain of any quadratic function as all real numbers.
2. Determine whether [latex]a[/latex] is positive or negative. If [latex]a[/latex] is positive, the parabola has a minimum. If [latex]a[/latex] is negative, the parabola has a maximum.
3. Determine the maximum or minimum value of the parabola, [latex]k[/latex].
4. If the parabola has a minimum, the range is given by [latex]f\left(x\right)\ge k[/latex], or [latex]\left[k,\infty \right)[/latex]. If the parabola has a maximum, the range is given by [latex]f\left(x\right)\le k[/latex], or [latex]\left(-\infty ,k\right][/latex].

Example: Finding the Domain and Range of a Quadratic Function

Find the domain and range of [latex]f\left(x\right)=-5{x}^{2}+9x - 1[/latex].

Show Solution

As with any quadratic function, the domain is all real numbers or [latex]\left(-\infty,\infty\right)[/latex].

Because [latex]a[/latex] is negative, the parabola opens downward and has a maximum value. We need to determine the maximum value. We can begin by finding the [latex]x[/latex]-value of the vertex.

[latex]h=-\dfrac{b}{2a}=-\dfrac{9}{2\left(-5\right)}=\dfrac{9}{10}[/latex]

The maximum value is given by [latex]f\left(h\right)[/latex].

[latex]f\left(\dfrac{9}{10}\right)=5{\left(\dfrac{9}{10}\right)}^{2}+9\left(\dfrac{9}{10}\right)-1=\dfrac{61}{20}[/latex]

The range is [latex]f\left(x\right)\le \dfrac{61}{20}[/latex], or [latex]\left(-\infty ,\dfrac{61}{20}\right][/latex].

Try It

Find the domain and range of [latex]f\left(x\right)=2{\left(x-\dfrac{4}{7}\right)}^{2}+\dfrac{8}{11}[/latex].

Show Solution

The domain is all real numbers. The range is [latex]f\left(x\right)\ge \dfrac{8}{11}[/latex], or [latex]\left[\dfrac{8}{11},\infty \right)[/latex].

See this interactive in the course material.

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