May Problem of the Month

First Rate

ANSWER KEY

Level A

Austin lands on every step; Dylan skips a step and lands on every other step. The picture shows an 8-step staircase (footprints on steps 1, 2, 4, 6, 8 with Dylan and Austin's prints visible).

1. Answer: Austin. Austin lands on every single step, so he needs one jump per step. Dylan covers two steps with each jump, so he needs fewer jumps to reach the same height. Taking smaller steps means more of them.

2. Answer: 4 jumps. Dylan lands on every other step: steps 2, 4, 6, 8. That is 8 ÷ 2 = 4 jumps.

3. Answer: 8 jumps. Austin lands on each step: steps 1, 2, 3, 4, 5, 6, 7, 8. That is 8 jumps.

4. Answer: Dylan. In 5 jumps Austin reaches step 5. In 5 jumps Dylan reaches step 10 (5 × 2), which is higher — in fact past the top of an 8-step staircase. Dylan is farther up.

5. Answer: Dylan. Dylan finishes in 4 jumps while Austin needs 8, so Dylan took fewer jumps.

6. Answer: Answers vary — both reasonable conclusions should be accepted with justification. The problem does not state how fast each boy jumps, so the "winner" depends on what students assume. Common, acceptable answers:

- If each jump takes the same amount of time, Dylan wins because he needs only 4 jumps versus Austin's 8.
- If Dylan's bigger jumps take longer (he has to jump twice as far each time), it could be a tie or Austin could win. The key idea is that "fewer jumps" does not automatically mean "faster" unless we know the time per jump.

Level B

Tom: 6 yards in 4 seconds. Diane: 5 yards in 3 seconds.

Who reaches 30 yards first?

1. Find each runner's time for 30 yards using unit rates.

Tom's speed = 6 ÷ 4 = 1.5 yards per second. Time for 30 yards = 30 ÷ 1.5 = 20 seconds.

Diane's speed = 5 ÷ 3 ≈ 1.67 yards per second. Time for 30 yards = 30 ÷ 1.67 = 18 seconds.

Answer: Diane gets to 30 yards first (18 seconds vs. 20 seconds).

Who runs faster, and how can you compare their speeds?

Answer: Diane runs faster.

Compare using equivalent rates. The cleanest comparison is yards per second (unit rate):

Tom = 1.5 yd/s, Diane ≈ 1.67 yd/s. Since 1.67 > 1.5, Diane is faster.

You can also scale both to a common time. In 12 seconds (a common multiple of 4 and 3): Tom runs 18 yards, Diane runs 20 yards. Same time, more distance for Diane.

Level C

Tammy cleans 1/2 of the area per hour. Melissa cleans 1/3 of the area per hour.

1. Add their rates to get their combined rate (fraction of the area per hour).

1/2 + 1/3 = 3/6 + 2/6 = 5/6 of the area per hour.

2. Time = total work ÷ rate. They need to clean 1 whole area.

Time = 1 ÷ (5/6) = 6/5 hours.

Answer: 6/5 hours = 1 1/5 hours = 1 hour 12 minutes.

Level D

1,500 m race = 3¾ laps of a 400 m track. Jaali's best time = 3 min 29.4 s = 209.4 s, run at a steady pace.

Set up Jaali's pace (the target to beat):

Jaali's average speed = 1500 ÷ 209.4 ≈ 7.16 m/s. At a steady pace each 400 m lap takes about 209.4 ÷ 3.75 ≈ 55.84 s.

First part of the race (the first three laps, 1,200 m):

When Jaali has 300 m to go, Jaali has run 1,200 m. The time for that = (1200/1500) × 209.4 ≈ 167.5 s.

To be no more than 50 m behind at that moment, you must have run at least 1,200 − 50 = 1,150 m in those 167.5 s.

Answer: Average speed for the first part ≈ 1150 ÷ 167.5 ≈ 6.87 m/s (a touch slower than Jaali, as planned).

The kick (your finishing sprint):

You start your kick at 1,150 m, so you still have 1500 − 1150 = 350 m to run. Jaali's final 300 m takes (300/1500) × 209.4 ≈ 41.9 s. To win, you must cover your 350 m in that same time or less.

Answer: Kick speed needed ≈ 350 ÷ 41.9 ≈ 8.36 m/s to tie; run a bit faster than this to win.

If Jaali runs two seconds faster (207.4 s):

Jaali's average speed rises to 1500 ÷ 207.4 ≈ 7.23 m/s, and the final 300 m now takes ≈ 41.5 s instead of 41.9 s.

Answer: You have slightly less time for your 350 m kick, so the required kick speed rises to about 350 ÷ 41.5 ≈ 8.44 m/s. You would need to start your kick a little earlier, lag a little less, or sprint a little faster to still win.

Level E

The receiver runs 10 yd straight, turns right, then runs toward the sideline at 8 yd/s. Based on the figure, the fixed leg of the right triangle (the vertical distance, y) is 16 yards; x is how far the receiver has run since turning; z is the straight-line distance between quarterback and receiver.

Part 1 — receiver catches the ball 12 yards after turning (x = 12):

Distance: $z = \sqrt{12^2 + 16^2} = \sqrt{144 + 256} = \sqrt{400} = 20$ yards. (A 12-16-20 right triangle.)

Rate: $dz/dt = x\cdot(dx/dt) / z = (12 \times 8) / 20 = 96 / 20 = 4.8$ yd/s.

Answer: The quarterback and receiver are 20 yards apart, and that distance is increasing at 4.8 yards per second.

Part 2 — the distance at the catch is z = 17.3 yards:

Find x: $x = \sqrt{z^2 - y^2} = \sqrt{17.3^2 - 16^2} = \sqrt{299.29 - 256} = \sqrt{43.29} \approx 6.58$ yards.

Rate: $dz/dt = x\cdot(dx/dt) / z = (6.58 \times 8) / 17.3 \approx 52.6 / 17.3 \approx 3.04$ yd/s.

Answer: The receiver had run about 6.6 yards after turning, and the distance between them was increasing at about 3.0 yards per second.

Part 3 — explain the relationship:

As the receiver runs farther past the turn (larger x), three things happen together: the straight-line distance z gets larger, and the rate dz/dt that the distance is changing also gets larger — but dz/dt never reaches the receiver's full 8 yd/s.

The reason is the geometry. Only the part of the receiver's motion that points directly away from the quarterback adds to the distance. Early on (small x), much of the receiver's run is "sideways" relative to the quarterback, so the distance changes slowly. As x grows, the receiver's path lines up more and more with the line connecting the two players, so a bigger share of the 8 yd/s goes into separating them. In the limit, when x is huge compared to the fixed 16 yards, dz/dt approaches 8 yd/s but never quite equals it.

In short: farther catch ⇒ greater distance z ⇒ faster rate of separation dz/dt, with $dz/dt = 8x/z$ always less than 8.