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B.Sc. (Hons.) Chemistry (Semester – 2nd)
MATHEMATICS II
Subject Code: BMATH5201
Paper ID: [19131614]

Time: 03 Hours **Maximum Marks: 60**

Instruction for candidates:

1. Section A is compulsory. It consists of 10 parts of two marks each.
2. Section B consist of 5 questions of 5 marks each. The student has to attempt any 4 questions out of it.
3. Section C consist of 3 questions of 10 marks each. The student has to attempt any 2 questions.

Section – A **(2 marks each)**

Q1. Attempt the following:

a) If A, B, C are mutually exclusive and exhaustive events associated with a random experiment and $P(B) = 0.6 \times P(A)$ and $P(C) = 0.2 \times P(A)$, then find $P(A)$.

b) For two events A and B, $P(A) = 0.5$, $P(B) = 0.6$ and $P(A \cup B) = 0.8$. Find the conditional probability $P(A/B)$ and $P(B/A)$.

c) A problem in a question paper is given to 3 students in a class to be solved. The probabilities of their solving the problem are 0.5, 0.7 and 0.8 respectively. Find the probability that the problem will be solved.

d) For a moderately skewed data, the arithmetic mean is 130, the coefficient of variation is 6 and Karl Pearson's coefficient of skewness is 0.4. Find the mode.

e) A die is rolled until a 6 appears. What is the expectation of the number of rollings required?

f) Discuss the convergence of the sequence $\left\{\frac{1}{n}\right\}$, $n \in N$.

g) Write Cauchy root test.

h) Find the radius of convergence of the power series $\sum_{n=1}^{\infty} n^n x^n$.

i) State Euler's theorem of homogeneous functions of two variables.

j) Find $\frac{df}{dt}$ at $t = 0$, where $f(x, y) = x \cos cos\, y + e^x \sin sin\, y$, $x = t^2 + 1$, $y = t^3 + t$.

Section – B **(5 marks each)**

Q2. Show that Poisson distribution is a limiting case of Binomial distribution.

Q3. Find the coefficient of correlation between the values of X and Y given below:

X	2	3	4	5	6	7	8
Y	7	6	5	4	3	2	1

Q4. Find the Taylor series expansion of $f(x) = \tan tan\, x$ at $x = \frac{\pi}{4}$.

Q5. Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at the point $(x, y) = (1, 1)$, for $e^y - e^x + xy = 1$.

Q6. Prove that $\sum_{n=1}^{\infty} \left(\frac{n}{n+1}\right)^n$ is divergent.

Section – C **(10 marks each)**

Q7. a) The joint probability distributions of two random variables X and Y is given by $P(x, y) = \frac{2}{n(n+1)}$, $y = 1, 2, 3. \cdots, n\,; x = 1, 2, 3, \cdots, y$. Are the random variables X and Y independent? (4)

b) Discuss the convergence of the series: **(6)**

$$\sum_{n=1}^{\infty} \frac{1}{n^p}, \qquad p > 0.$$

Q8. a) Consider the data: 3, 5, 6, 9, 11. Find the second and third moments about the mean. **(4)**

b) If $tan\, u = \frac{x^3+y^3}{x-y}$, then show that: **(6)**

$$x^2 \frac{\partial^2 f}{\partial x^2} + 2xy \frac{\partial^2 f}{\partial x \partial y} + y^2 \frac{\partial^2 f}{\partial y^2} = (1 - 4\, sin^2 u)\, sin2u.$$

Q9. a) The random variable X is normally distributed with mean 9 and standard deviation 3. Find the probability:

i) $X \geq 15$

ii) $X \leq 15$

iii) $0 \leq X \leq 9$

It is given that $P(0 \leq Z \leq 3) = 0.4987$ and $P(0 \leq Z \leq 2) = 0.4772$. **(6)**

b) If $z = f(x, y)$, $x = r \cos cos\, \theta$, $y = r \sin sin\, \theta$, then using Jacobian, show that **(4)**

$$\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2 = \left(\frac{\partial f}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial f}{\partial \theta}\right)^2$$