NAME:	TEACHER: Js

Tamaki College

AS 91256 (V3)

Mathematics and Statistics 2.1

Apply Co-ordinate Geometry Methods in Solving Problems

Quadrilateral Task

Credits: 2

You should answer ALL questions.

Show ALL working for ALL questions.

YOU MUST SUBMIT THIS TO THE SUPERVISOR AT THE END OF THE ASSESSMENT.

Achievement	Achievement with Merit	Achievement with Excellence
Apply co-ordinate geometry methods in solving problems.	Apply co-ordinate geometry methods, using relational thinking, in solving problems	Apply co-ordinate geometry methods, using extended abstract thinking, in solving problems
Overall level of performance		

Marker:	Date:
Comment:	

Useful Formulae:

Midpoint of a line segment	$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$
Distance between two points	$\sqrt{((x_2 - x_1)^2 + (y_2 - y_1)^2)}$
Gradient	$\frac{y_2 - y_1}{x_2 - x_1}$
Equation of a line	$y = mx + c$
Parallel lines	$m_1 = m_2$
Perpendicular lines	$m_1 \times m_2 = {}^{-}1$

Useful Tool:

Link to Geogebra

Introduction

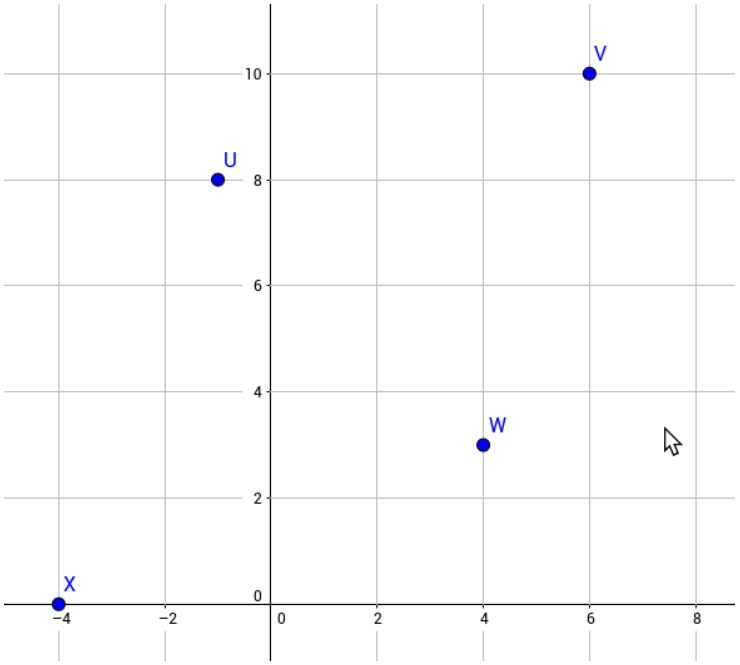
This assessment requires you to apply co-ordinate geometry methods using a specific shape on a co-ordinate plane.

Quadrilateral Task

The quadrilateral **UVWX** has vertices **U** (-1, 8), **V** (6, 10), **W** (4, 3) and **X** (-4, 0)

- Use co-ordinate geometry methods to investigate the properties of quadrilateral UVWX. Explain your reasoning.
- Use co-ordinate geometry methods to investigate the properties of the diagonals (UW and VX) of quadrilateral UVWX. Explain your reasoning.

The quality of your discussion and reasoning will determine your overall grade. Show your calculations. Use appropriate mathematical statements. Clearly communicate your strategy and method at each stage of your solution.



Line UV
U (-1, 8) and V (6, 10)
x_1 = -1, y_1 = 8 and x_2 = 6, y_2 = 10

Gradient of UV

$$\frac{10-8}{6--1} = 0.29\ (2\ d.p.)$$

Distance of UV

$$\sqrt{((6\ -\ -\ 1)^2 + (10\ -\ 8)^2)} = 7.28\ (2\ d.p.\ )$$

Line VW
V (6, 10) and W (4, 3)
x_1 = 6, y_1 = 10 and x_2 = 4, y_2 = 3

Gradient of VW

$$\frac{3-10}{4-6} = 3.5$$

Distance of VW

$$\sqrt{((4\ -\ 6)^2 + (3\ -\ 10)^2)} = 7.28\ (2\ d.p.)$$

Line WX
W (4, 3) and X (-4, 0)
x_1 = 4, y_1 = 3 and x_2 = -4, y_2 = 0

$$\frac{0-3}{-4-4} = 0.38\ (2\ d.p.)$$

$$\sqrt{((-\ 4\ -\ 4)^2 + (0\ -\ 3)^2)} = 8.54\ (2\ d.p.)$$

Line UX
U (-1, 8) and X (-4, 0)
x_1 = -1, y_1 = 8 and x_2 = -4, y_2 = 0

$$\frac{0-8}{-4--1} = 2.67\ (2\ d.p.)$$

$$\sqrt{((-\ 4\ -\ -\ 1)^2 + (0\ -\ 8)^2)} = 8.54\ (2\ d.p.)$$

Line	Distance	Gradient	Parallel	Perpendicular
UV	7.28	0.29	-	-

VW	7.28	3.5	-	-
WX	8.54	0.38	-	-
UX	8.54	2.67	-	-

The shape has two pairs of sides that are the same (UV, VW and WX, UX). The shape has no parallel lines, because no gradients are the same. There are no perpendicular lines because all the gradients are positive and two positive numbers multiplied together give a postivie number, so $m_1 \times m_2 \neq -1$. Therefore, the shape must be a kite.

Line of UW **U (-1, 8) and W (4, 3)** **$x_1 = -1$, $y_1 = 8$ and $x_2 = 4$, $y_2 = 3$** **Gradient of UW** $\frac{3-8}{4--1} = -1$	**Line of VX** **V (6, 10) and X (-4, 0)** **$x_1 = 6$, $y_1 = 10$ and $x_2 = -4$, $y_2 = 0$** **Gradient of VX** $\frac{0-10}{-4-6} = 1$
Equation of UW $y = mx + c$ $y = (-1)x + c$ $(3) = (-1)(4) + c$ $3 = -4 + c$ $7 = c$ $y = -x + 7$	**Equation of VX** $y = mx + c$ $y = (1)x + c$ $(10) = (1)(6) + c$ $10 = 6 + c$ $4 = c$ $y = x + 4$
The diagonal lines are perpendicular because $m_{UW} \times m_{VX} = -1$	
Point of intersection $y = -x + 7$ $y = x + 4$ $-x + 7 = x + 4$	

$7 = 2x + 4$

$3 = 2x$

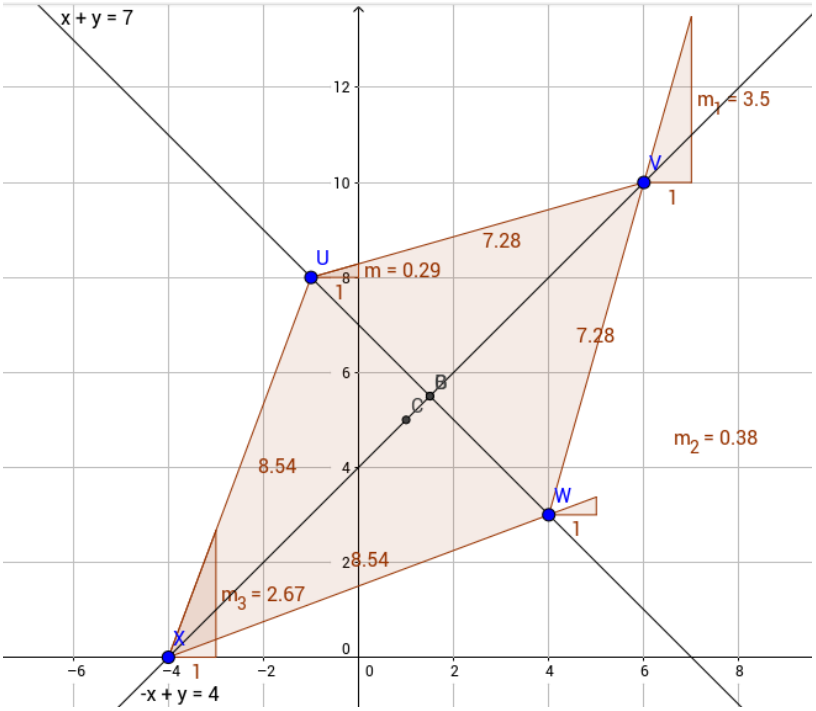
$1.5 = x$

$y = x + 4$

$y = (1.5) + 4$

$y = 5.5$

Lines intersect at (1.5, 5.5)



Assessment Schedule: Mathematics and Statistics AS 2.1 *Quadrilateral Task*

Evidence/Judgements for Achievement	Evidence/Judgements for Achievement with Merit	Evidence/Judgements for Achievement with Excellence
Apply co-ordinate geometry methods in solving problems. The student; ● Selects and uses methods ● Demonstrates knowledge of co-ordinate geometry concepts and terms ● Communicates using appropriate representations	Apply co-ordinate geometry methods, using relational thinking, in solving problems. The student; ● Selects and carries out a logical sequence of steps ● Connects different concepts or representations ● Demonstrates understanding of concepts And relates findings to a context or communicates their thinking using appropriate mathematical statements.	Apply co-ordinate geometry methods, using extended abstract thinking, in solving problems. The student ● Devises a strategy to investigate or solve a problem ● Identifies relevant concepts in context ● Develops a chain of logical reasoning ● Forms a generalisation And uses correct mathematical statements or communicates mathematical insight.
Students are expected to use at least two different co-ordinate geometry methods. Working **must** be shown and they need to indicate what the calculated answer represents. The student finds the gradients and lengths of the sides of quadrilateral UVWX. For example the student might; ● *Find the lengths of line segments UV, WV, XW and UX.* *Potential Solutions - Lengths* $\|UV\| = \sqrt{53} = 7.28$ $\|WV\| = \sqrt{53} = 7.28$ $\|XW\| = \sqrt{73} = 8.54$ $\|UX\| = \sqrt{73} = 8.54$ ● *Find the gradients of line segments UV, WV, XW and UX.* *Potential Solutions - Gradients* m_{UV} = (= 0.286) m_{WX} = (= 0.375) m_{VW} = (= 3.5) m_{UX} = (= 2.667)	Relational thinking could be shown by the student making conclusions about the properties of quadrilateral UVWX based on their gradient and length calculations Working **must** be shown in context and a conclusion clearly stated. For example the student might do at least two of the following (at least one conclusion based on lengths and one on gradients); ● *Works out and states that the quadrilateral has two pairs of equal sides (joined, not opposite pairs)* $\|UV\| = \sqrt{53} = 7.28$ $\|WV\| = \sqrt{53} = 7.28$ $\|XW\| = \sqrt{73} = 8.54$ $\|UX\| = \sqrt{73} = 8.54$ ● *Calculate all gradients correctly and conclude that the quadrilateral has no parallel sides* m_{UV} = (= 0.286) m_{WX} = (= 0.375)	Extended abstract thinking could be shown by the student making further observations about the diagonals of the quadrilateral. *Proves that VX is the perpendicular bisector of UW, but not vice versa, as it is perpendicular and runs through its midpoint. (or equivalent)* The proof would most likely include the following steps: ● *Works out gradients of the diagonals and concludes (with justification)that the diagonals are perpendicular* $m_{VX} = 1$ $m_{UW} = -1$ $m_{VX} \times m_{UW} = -1$ ● *Works out the equations of the diagonals and solves them simultaneously to find the point of intersection and investigates to see if this point is the same as the mid-points of the diagonals.* *Equation VX: y = x + 4* *Equation UX: y = –x + 7* *Lines meet at (1.5, 5.5)*

● *Other Calculations from diagonals investigation:* *Gradients:* $m_{VX} = 1$ $m_{UW} = -1$ *Equations.* *Equation VX:* $y = x + 4$ *Equation UX :* $y = -x + 7$ ***Mid-points*** Mid-Point VX = (1, 5) Mid-point UW = (1.5, 5.5) ***Note 1:*** *Skills must be directly related to the solution of the problem.*	m_{VW} = (= 3.5) m_{UX} = (= 2.667) ● *Calculate all gradients correctly and conclude with justification (e.g* $m_{WX} \times m_{UX} = 1$ *and it needs to be −1) that no sides are perpendicular (or no right angled corners)* ● *.From diagonals investigation* *Works out gradients of the diagonals and concludes (with justification)that the diagonals are perpendicular* $m_{VX} = 1$ $m_{UW} = -1$ $m_{VX} \times m_{UW} = -1$	*Mid-Point VX =* $(\frac{6+-4}{2}, \frac{10+0}{2})$ = *(1, 5) Mid-point UW =* $(\frac{-1+4}{2}, \frac{8+3}{2})$ = *(1.5, 5.5)* *Midpoint UW is the intersection of the diagonals but mid-point VX is not.* *N.B. Students may investigate the same property as above by calculating the lengths of the line segments between the vertices and the point of intersection.*

Final grades will be decided using professional judgement based on a holistic examination of the evidence provided against the criteria in the Achievement Standard.