

MaT313 Spring2024
Sample Quiz 3

Work must be done on paper.

Review your notes and homework for Lesson 6.
Only students who have completed Lesson 6 may take Quiz 3.
The Quiz 3 HW may only be completed after completing Lesson 7.

You will have only 20 minutes to complete this quiz.

Set a timer for 20 minutes, photo what is done in time, copy and paste photos into your quiz doc using the googledoc app on your phone or tablet. Google will record the time that this is done. If you do not post photos before 25 minutes, you will have to retake the quiz with a new question next week.

Finish the rest of the work for homework and add later.

This is the quiz template:

$$1pt \textcircled{1} \begin{bmatrix} \\ \end{bmatrix} \begin{pmatrix} \\ \end{pmatrix} =$$

Template!

$$1pt \textcircled{2} \begin{bmatrix} \\ \end{bmatrix} \begin{pmatrix} \\ \end{pmatrix} =$$

$$1pt \textcircled{3} \begin{bmatrix} \\ \end{bmatrix} \begin{pmatrix} \\ \end{pmatrix} =$$

$$1pt \textcircled{4} \begin{bmatrix} \\ \end{bmatrix} \begin{pmatrix} \\ \end{pmatrix} =$$

$$1pt \textcircled{5} \begin{bmatrix} \\ \end{bmatrix} \begin{pmatrix} \\ \end{pmatrix} =$$

Quiz 3 HW on Null Spaces is worth 5 pts.

must complete lesson 7 to request the Quiz 3 HW.

Quiz 3 HW (5pts) Find the nullspace of $\begin{bmatrix} & \\ & \end{bmatrix}$

- using our algorithm to complete the row reduction
- writing the null space in position/direction form.
- checking the solution using matrix multiplication

We have only one sample quiz so review Lesson 7 and remember to show all work to get credit:

$$1pt \text{ (1)} \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{pmatrix} 2 \\ 2 \end{pmatrix} =$$

$$1pt \text{ (2)} \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \begin{pmatrix} 4 \\ 4 \end{pmatrix} =$$

$$1pt \text{ (3)} \begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} =$$

$$1pt \text{ (4)} \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} =$$

$$1pt \text{ (5)} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix} \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} =$$

Quiz 3 HW on Null Spaces is worth 5 pts. (Do when ready)

Quiz 3 HW (5pts) Find the nullspace of $\begin{bmatrix} 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 2 \end{bmatrix}$

SAMPLE

Here are the solutions:

MAT313

Quiz 3

Prof Sonmani

$$1pt \quad ① \quad \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{pmatrix} 2 \\ 2 \end{pmatrix} = \begin{pmatrix} 0 \cdot 2 + 1 \cdot 2 \\ -1 \cdot 2 + 0 \cdot 2 \end{pmatrix} = \begin{pmatrix} 0+2 \\ -2+0 \end{pmatrix} = \begin{pmatrix} 2 \\ -2 \end{pmatrix}$$

$$1pt \quad ② \quad \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \begin{pmatrix} 4 \\ 4 \end{pmatrix} = \begin{pmatrix} 1 \cdot 4 + 1 \cdot 4 \\ 0 \cdot 4 + 0 \cdot 4 \end{pmatrix} = \begin{pmatrix} 4+4 \\ 0+0 \end{pmatrix} = \begin{pmatrix} 8 \\ 0 \end{pmatrix}$$

$$1pt \quad ③ \quad \begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \cdot 1 + 2 \cdot 0 \\ 1 \cdot 1 + 2 \cdot 0 \end{pmatrix} = \begin{pmatrix} 1+0 \\ 1+0 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$1pt \quad ④ \quad \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \cdot 1 + 0 \cdot 1 + 1 \cdot 1 \\ 0 \cdot 1 + 1 \cdot 1 + 0 \cdot 1 \\ 1 \cdot 1 + 0 \cdot 1 + 0 \cdot 1 \end{pmatrix} = \begin{pmatrix} 0+0+1 \\ 0+1+0 \\ 1+0+0 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

$$1pt \quad ⑤ \quad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix} \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 1 \cdot 1 + 0 \cdot 2 + 0 \cdot 3 \\ 0 \cdot 1 + 2 \cdot 2 + 0 \cdot 3 \\ 0 \cdot 1 + 0 \cdot 2 + 3 \cdot 3 \end{pmatrix} = \begin{pmatrix} 1+0+0 \\ 0+4+0 \\ 0+0+9 \end{pmatrix} = \begin{pmatrix} 1 \\ 4 \\ 9 \end{pmatrix}$$

Must
show
all this
work!

Spts

Quiz 3 HW on Null Spaces is worth 5 pts. (downen ready)

Quiz 3 HW (5pts) Find the nullspace of $\begin{bmatrix} 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 2 \end{bmatrix}$

The solution for HW is in null space lesson.

The solutions are explained in [Video313S23-Q3-sample](#).

The Quiz 3 HW may only be completed after completing Lesson 7. Once you complete that lesson you will be sent the Quiz 3 HW.

Quiz3HW is based on Lesson 7.

To find the null space of a matrix you will solve a homogeneous system using an Augmented Matrix. You will do row reduction to Echelon form and then more row reduction to reduced echelon form as taught in Lesson 5. Then you will write the solution set in position direction form as taught in Lesson 5 and Lesson 7. Finally you will check your answer using matrix multiplication.

First Sample Quiz3HW with solution:

(much easier than actual Quiz3HW)

Null space of $\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$

$$\left(\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \end{array} \right) \xrightarrow{P_3 \rightarrow P_3 - P_1} \left(\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right)$$

Reduced Echelon

leaders
are x and y
 $z = z$ (free)

$$\begin{aligned} x + z &= 0 \\ y &= 0 \\ \hookrightarrow x &= -z \end{aligned}$$

$$\left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -z \\ 0 \\ z \end{pmatrix} : z \in \mathbb{R} \right\}$$

$$= \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} = z \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} : z \in \mathbb{R} \right\}$$

To check that

$$\left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -z \\ 0 \\ z \end{pmatrix} : z \in \mathbb{R} \right\}$$

is the null space of $\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$

In fact we only need
to check the direction
vector

$$\begin{pmatrix} -2 \\ 0 \\ 2 \end{pmatrix} = z \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$$

$$A \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1(-1) + 0 \cdot 0 + 1 \cdot 1 \\ 0 \cdot (-1) + 1 \cdot 0 + 0 \cdot 1 \\ 1 \cdot (-1) + 0 \cdot 0 + 1 \cdot 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \checkmark$$

If there are two
direction vectors
check both

Scoring:

1 point for correct first row action

1 point for rest of the row actions to Reduced Echelon Form correct

1 point solving for leaders and finding free variables

1 point for solution set with correct directions

1 point for checking

Second Sample Quiz3HW with solution:

Example

Find the Null Space for $\begin{bmatrix} 2 & 2 \\ 1 & 3 \\ 3 & 5 \end{bmatrix}$

We are

Solving:

Solve it using Augmented Matrices and Reduced Echelon Form

$$\begin{bmatrix} 2 & 2 \\ 1 & 3 \\ 3 & 5 \end{bmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \longrightarrow \begin{bmatrix} 2 & 2 & | & 0 \\ 1 & 3 & | & 0 \\ 3 & 5 & | & 0 \end{bmatrix}$$

next do row reduction to Echelon Form using only this

change 1st leader into a 1 using scale (or switch if zero)

$$P_1 \rightarrow \frac{1}{2}P_1 \longrightarrow \begin{bmatrix} 1 & 1 & | & 0 \\ 1 & 3 & | & 0 \\ 3 & 5 & | & 0 \end{bmatrix}$$

all entries in row 1 are divided by 2
restore copies

box the leader

make all entries below the leader into zeroes

using skew $P_i \rightarrow P_i - kP_1$
because leader is in row 1

$$P_2 \rightarrow P_2 - P_1 \\ P_3 \rightarrow P_3 - 3P_1 \longrightarrow \begin{bmatrix} 1 & 1 & | & 0 \\ 0 & 2 & | & 0 \\ 0 & 2 & | & 0 \end{bmatrix}$$

copy row 1
 $3-3(1)$ $5-3(1)$

change the second leader into a 1 by scaling (or switch if needed)

$$P_2 \rightarrow \frac{1}{2}P_2 \longrightarrow \begin{bmatrix} 1 & 1 & | & 0 \\ 0 & 1 & | & 0 \\ 0 & 2 & | & 0 \end{bmatrix}$$

box the next leader

Make all entries below the leader into zeroes

using skew $P_i \rightarrow P_i - kP_2$
because leader is in row 2

$$P_3 \rightarrow P_3 - 2P_2 \longrightarrow \begin{bmatrix} 1 & 1 & | & 0 \\ 0 & 1 & | & 0 \\ 0 & 0 & | & 0 \end{bmatrix}$$

Echelon Form

Continue to Reduced Echelon Form

Make all entries above the last leader into zeroes

using skew $P_i \rightarrow P_i - kP_2$

because last leader is in row 2

$$P_1 \rightarrow P_1 - P_2 \longrightarrow \begin{bmatrix} 1 & 0 & | & 0 \\ 0 & 1 & | & 0 \\ 0 & 0 & | & 0 \end{bmatrix}$$

Reduced Echelon Form

Rewrite as a linear system

$$\begin{cases} 1x + 0y = 0 \\ 0x + 1y = 0 \\ 0x + 0y = 0 \end{cases}$$

$$\begin{cases} x = 0 \\ y = 0 \end{cases}$$

(no free variables)

Solve for leaders

$$x = 0$$

$$y = 0$$

The Null Space is $\left\{ \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \right\}$

To check this second one multiply the matrix by the 0 vector. [Here is an extra video](#) reminding you how to do row reduction first to Echelon form and then reduced Echelon form as in the solutions reviewed above.