Seminario di Algebra & Geometria

Dipartimento di Matematica “Guido Castelnuovo”
SAPIENZA Università di Roma
A.A. 2023/2024

Mercoledì 27 Settembre 2023 - ore 14:00

Sebastian Goette (Universität Freiburg)

Homotopy Associative Submanifolds in G2-manifolds

Associative submanifolds are certain calibrated submanifolds in G2-manifolds. There is the hope that counting them will reveal subtle information about the underlying G2-structure. On the other hand, certain singular associatives can be resolved in exactly three different ways, so a naive count will be meaningless.

In this talk, we will define homotopy associatives as cobordism classes of threedimensional submanifolds that are adapted to the G2-structure in a rather weak sense. We will see that a given cobordism class can be interpreted as a homotopy associative in exactly three different ways. This might help us to define a consistent counting scheme even when the naive number of associatives in a given cobordism class changes due to singularities.

Mercoledì 4 Ottobre 2023 - ore 14:00

Silvia Sabatini (Universität zu Köln)

Sulla classificazione di varietà simplettiche monotone di complessità uno

Spesso molti esempi di varietà simplettiche con azioni Hamiltoniane di gruppi vengono dalla geometria algebrica; questo è il caso, per esempio, per le varietà simplettiche toriche, nelle quali la dimensione del toro che agisce è metà della dimensione della varietà.
In questo talk consideriamo il caso in cui la varietà simplettica presenta una struttura monotona, ovvero, quando la prima classe di Chern del fibrato tangente è esattamente la forma simplettica (in coomologia).

In questo caso, la congettura di Fine e Panov asserisce che tutte le varietà monotone di dimensione 6 con un'azione Hamiltoniana di un toro di dimensione 1 sono diffeomorfe a varietà di Fano.
Più in generale, è interessante chiedersi sotto quali ipotesi una varietà simplettica monotona con l'azione Hamiltoniana di un toro è diffeomorfa o simplettomorfa ad una varietà di Fano.
In questo talk si discuterà il caso in cui l'azione sia di complessità uno (la dimensione del toro è metà della dimensione della varietà meno uno).

Mercoledì 11 Ottobre 2023 - ore 14:00

Giuseppe Pipoli (Università dell'Aquila)

Constant mean curvature hypersurfaces in H^n x R with small planar boundary

Denoting with H^n the n-dimensional hyperbolic space, we show that constant mean curvature hypersurfaces in H^n x R with small boundary contained in a horizontal slice P are topological disks, provided they are contained in one of the two half-spaces determined by P.

This is the analogous in H^n x R of a result in R^3 by A. Ros and H. Rosenberg.

The proof is based on geometric and analytic methods : from one side the constant mean curvature equation is a quasilinear elliptic PDE on manifolds, to the other the specific geometry of the ambient space produces some peculiar phenomena.

This talk is based on a joint work with Barbara Nelli.

Mercoledì 18 Ottobre 2023 - ore 14:00

Misha Feigin (University of Glasgow)

Subalgebras of Cherednik algebras

A rational Cherednik algebra is a flat deformation of a skew product of the Weyl algebra and a Coxeter group W. I am going to discuss two interesting subalgebras of Cherednik algebras going back to the work of Hakobyan and the speaker from 2015. They are flat deformations of skew products of quotients of the universal enveloping algebras of gl(n) and so(n), respectively, with W. I would also like to explain their relations with generalised Howe duality and geometry of particular nilpotent orbits.

Mercoledì 8 Novembre 2023 - ore 14:00

Lorenzo Foscolo (Sapienza Università di Roma)

Hypertoric varieties, W-Hilbert schemes and Coulomb branches

Motivated by physics, in the late 1990s Sen discussed a construction of complete hyperkähler metrics in (real) dimension 4 and so-called ALF (asymptotically locally flat) asymptotics as a "superposition" of simpler explicit building blocks, namely Z_2-invariant multi-Taub-NUT metrics and the Atiyah-Hitchin metric. These metrics were then produced by Cherkis–Kapustin and Cherkis–Hitchin, amongst others, via twistor theory and Nahm's equations. In this talk I will discuss joint work with R. Bielawski about a higher dimensional version of this story. We study transverse equivariant Hilbert schemes of hypertoric varieties invariant under the action of a Weyl group W. We investigate the conjectural hyperkähler metric on these spaces in terms of twistor theory and Nahm's equations and discuss the relation of (symplectic quotients of) such Hilbert schemes with the Coulomb branches of 3-dimensional N=4 supersymmetric gauge theories in theoretical physics, recently defined as holomorphic symplectic varieties by Braverman–Finkelberg–Nakajima.

Mercoledì 15 Novembre 2023 - ore 14:00

Alessio Corti (Imperial College London)

How to make log structures

I discuss how to practically put a log structure on a toroidal crossing space, and hopefully sketch applications to smoothing toric Fano varieties and log birational geometry. This is work in progress with Helge Ruddat.

Mercoledì 22 Novembre 2023 - ore 14:00

Cédric Lecouvey (Université de Tours)

Positively multiplicative graphs, affine Grassmannians and finite automatons

Positively multiplicative graphs are graphs whose adjacency matrix can be embedded in a matrix algebra admitting a distinguished basis labelled by its vertices with nonnegative structure constants . It is easy to get such graphs from the group algebra of the character algebra of a finite group. Other simple examples are obtained from classical bases of symmetric functions. More subtlety, it is also possible to define numerous multiplicative graphs from the affine Grassmannian associated to an affine Weyl group. These graphs are then related to interesting

probabilistic models (random walks in alcoves, TASEP etc.) and the related positively multiplicative graphs give finite automatons recognizing reduced expressions of affine Grassmannian elements. The talk will consist in an introduction to these notions and problems. This is a work in collaboration with J. Guilhot (IDP Tours) and P. Tarrago (LPSM Paris).

Mercoledì 29 Novembre 2023 - ore 14:00

Patrick Graf (Universität Bayreuth)

Equality in the Miyaoka--Yau inequality and uniformization of klt pairs of general type

Let (X, D) be a projective klt pair, where K_X + D is ample and D has standard coefficients. Guenancia and Taji have shown that a suitable version of the famous Miyaoka--Yau inequality holds in this setting. I will show that in the equality case (X, D) is a quotient of the unit ball, in the orbifold sense. Joint with Benoît Claudon (Rennes) and Henri Guenancia (Toulouse).

Mercoledì 6 Dicembre 2023 - ore 14:00

Bruno Klingler (Humboldt Universität zu Berlin)

Abelian differentials and their periods

An abelian differential is a smooth projective curve endowed with an algebraic one-form. I will discuss the uniformization of the moduli of abelian differentials provided by their periods, its arithmetic and functional transcendence properties. Joint work with Leonardo Lerer.

Mercoledì 13 Dicembre 2023 - ore 14:00

Bruno Anglès (Université de Caen)

On the j-invariants of CM rank two Drinfeld modules

We will investigate the arithmetic properties of the j-invariant of a rank two Drinfeld module having CM by an order of an « imaginary » quadratic function field. This talk is based on a joint work with Cécile Armana (Besançon, France), Vincent Bosser (Caen, France) and Fabien Pazuki (Copenhagen, Denmark).

Mercoledì 10 Gennaio 2024 - ore 14:00

Riccardo Zuffetti (Technische Universität Darmstadt)

The Lefschetz decomposition of the Kudla-Millson theta function

In the 80's Kudla and Millson introduced a theta function in two variables, nowadays known as the Kudla--Millson theta function. This behaves as a Siegel modular form with respect to one variable, and as a closed differential form on an orthogonal Shimura variety with respect to the other variable. In this talk we show that the Lefschetz decomposition of (the cohomology class of) this theta function provides simultaneously the modular decomposition in Eisenstein, Klingen and cuspidal parts. Time permitting, we will report on geometric applications.
This is joint work with J. Bruinier.

Mercoledì 17 Gennaio 2024 - ore 14:00

Salvatore Floccari (Universität Bielefeld)

The Hodge conjecture for sixfolds of generalized Kummer type

The Hodge conjecture is a central problem in modern algebraic geometry. It is notoriously difficult to attack, and we still lack general evidence towards its validity. In my talk I will present a proof of the Hodge conjecture for all six-dimensional hyper-Kähler varieties of generalized Kummer type, i.e. those arising via deformation of Beauville's generalized Kummer varieties built from abelian surfaces. The result presented yields the first complete families of projective hyper-Kähler varieties of dimension larger than two for which the Hodge conjecture is verified. As I will explain, a key ingredient for the proof is the construction of a K3 surface naturally associated to a sixfold of generalized Kummer type.

Mercoledì 24 Gennaio 2024 - ore 14:00

Charles Young (University of Hertfordshire)

Raviolo vertex algebras, higher current algebras, and raviolo conformal blocks

Vertex algebras formalise the properties of what physicists would call operator product expansions (OPEs) in chiral conformal field theories (CFTs). One way to motivate the axioms of vertex algebras is by first defining conformal blocks in genus zero, and then studying their limits in which marked points collide.

One would like to generalise this story to higher settings: "higher" in the sense of higher dimensions, but also in the sense of higher/homotopy/differential graded (dg) algebras. In recent months, an elegant and comparatively accessible instance of such higher vertex algebras has been introduced by Garner and Williams. They are called raviolo vertex algebras, and are associated to manifolds of real dimension three admitting a transverse holomorphic foliation; that is, roughly, manifolds having one complex-holomorphic and one topological direction.

I will describe these raviolo vertex algebras, and go on to show that they, too, arise from the limiting behaviour of certain raviolo conformal blocks, which I will introduce in the talk. In particular I will describe a certain configuration space of ravioli, and a model in dg commutative algebras of the derived sections of its structure sheaf.

This talk is based on work in preparation, joint with Luigi Alfonsi and Hyungrok Kim.

Mercoledì 31 Gennaio 2024 - ore 14:00

Giulio Bresciani (SNS Pisa)

Real versus complex plane curves

We prove that a smooth, complex plane curve of odd degree can be defined by a polynomial with real coefficients if and only if it is isomorphic to its complex conjugate; there are counterexamples in even degree. Even though the statement is completely elementary, it was not known until now. Our proof is stack-theoretic. More generally, we prove a result about fields of moduli of plane curves.

Mercoledì 14 Febbraio 2024 - ore 14:00

Christian Bär (Universität Potsdam)

A holographic index theorem and applications to scalar curvature geometry

In the first part of the talk I will discuss a "holographic" index theorem for compact manifolds with boundary. It relates the index of a boundary value problem to the index of an operator on the boundary. It will then be applied to scalar curvature geometry. We will show a rigidity theorem for scalar curvature on certain warped product spaces. This implies, in particular, Llarull's theorem for the punctured sphere. A version where a lower bound on scalar curvature is replaced by the dominant energy condition for initial data sets is also discussed. This is based on joint work with Simon Brendle, Aaron Chow, Bernhard Hanke, and Yipeng Wang.

Mercoledì 21 Febbraio 2024 - ore 14:00

Carlo Collari (Università di Pisa)

Groebner methods and magnitude homology

In this talk we show how to apply the framework developed by Sam and Snowden to study structural properties (eg. bound on rank and order of torsion) of graph homologies, in the spirit of Ramos, Miyata and Proudfoot. In particular, we focus on magnitude homology for graphs, which was introduced by Hepworth and Willerton.

The talk is organised as follows; we start with a short introduction to modules over categories and to the theory of Groebner categories. Then, we introduce magnitude homology and see some examples. Finally, we will see how to use the theory of Groebner categories to obtain information on magnitude (co)homology.

Mercoledì 28 Febbraio 2024 - ore 14:00

Brandon Williams (RWTH Aachen University)

Modular forms with poles on hyperplane arrangements

I will describe a family of hyperplane arrangements in lattices of signature (n, 2) for which the graded rings of modular forms with poles on those hyperplanes are freely generated. The largest example is a ring of modular forms for the lattice $2U \oplus D_{11}$, which is a polynomial algebra on 14 generators.
This is joint work with Haowu Wang.

Mercoledì 6 Marzo 2024 - ore 14:00

Thomas Krämer (Humboldt Universität zu Berlin)

Arithmetic finiteness of very irregular varieties

I will discuss joint work with Marco Maculan in which we prove the Shafarevich conjecture for a large class of irregular varieties over number fields. Our proof combines the method of Lawrence-Sawin with a big monodromy theorem from previous work with Javanpeykar, Lehn and Maculan. If time permits, I will briefly sketch at the end some recent progress which uses Hodge modules to rule out exceptional groups in the monodromy theorem.

Mercoledì 13 Marzo 2024 - ore 14:00

Dror Varolin (Stony Brook)

Metric Positivity for Holomorphic Vector Bundles

Holomorphic line bundles play many important roles in complex analytic geometry. In the higher rank case, much less is known, but there have been important advances in the last 15 years. After a review of notions of positivity and their consequences, I will discuss some of the more recent results in the subject, often comparing with the rank-1 case.

Mercoledì 20 Marzo 2024 - ore 14:00

Claudio Procesi (Sapienza Università di Roma)

Algebre di Cayley-Hamilton e pseudocaratteri

Il titolo si riferisce al teorema di Cayley-Hamilton che esprime il fatto che una matrice n x n su un anello commutativo A soddisfa il suo polinomio caratteristico.

La Teoria delle algebre di Cayley-Hamilton è una assiomatizzazione di questo fatto. Gli pseudocaratteri sono funzioni su un gruppo a valori in un anello commutativo che generalizzano i caratteri, appaiono in Teoria dei Numeri per studiare deformazioni di rappresentazioni. Un risultato recente su tali deformazioni potrebbe dare un esempio mancante nella teoria delle algebre di Cayley-Hamilton in caratteristica positiva.

Mercoledì 3 Aprile 2024 - ore 14:00

Leonardo Patimo (Università di Pisa)

Singular Light Leaves

In this talk, we will describe how to construct a basis of Bott-Samelson bimodules, called singular light leaves. Bott-Samelson bimodules are algebraic objects that correspond geometrically to resolutions of singularities of Schubert varieties. In the nonsingular case, that is for Schubert varieties in full flag varieties, Libedinsky introduced a basis (called light leaves) between Bott-Samelson bimodules which has been extensively used to compute the decomposition behavior in the Hecke category, for example enabling the discovery of counterexamples to Lusztig's conjecture on representations in characteristic p. We will thoroughly extend Libedinsky's result to the singular setting. To do this, we will employ the language of

diagrammatic calculus associated with Frobenius extensions. Our construction has concrete applications in computing intersection forms, as well as more theoretical implications pointing towards a diagrammatic definition of the Hecke category. This is a joint project with B. Elias, H. Ko, and N. Libedinsky.

Mercoledì 10 Aprile 2024 - ore 14:00

Erwan Rousseau (Université de Bretagne Occidentale)

A generalization of the Bloch-Ochiai theorem

The classical Bloch-Ochiai theorem states that a complex projective manifold with irregularity larger than its dimension has no Zariski dense entire curve. I will present a generalization of this theorem in the setting of pairs. (Joint work with S. Kebekus).

Mercoledì 17 Aprile 2024 - ore 14:00

Alessio Cela (ETH Zürich)

Fixed-curve counts in algebraic varieties

The counts of algebraic curves in an algebraic variety satisfying specific geometric conditions are referred to as Gromov-Witten invariants of the variety. In my talk, I will focus on these invariants when, in addition, the complex structure of the curve is fixed, explaining the advantages that such a choice affords.

Mercoledì 24 Aprile 2024 - ore 14:00

Marc Troyanov (EPFL Lausanne)

Asymptotic Geometry in SOL

SOL is one of Thurston's eight classical homogeneous Riemannian geometries, possibly the most exotic one. To get some insight of this geometry, it might be helpful to visualize the shape of a large spheres in SOL. Clearly, the first challenge is to compute, or at least estimate, the Riemannian distance between two points. In this talk, I will propose a way to circumvent this difficulty by replacing the Riemannian metric with an asymptotically equivalent Finsler metric, inspired by architectural cardboard models. This alternative Finsler metric offers the double advantage of explicit computability of distances, coupled with a rapid convergence that closely aligns with the Riemannian metric, thus simplifying our understanding and representation of SOL geometry. As concrete applications, I will show how to represent the shape of large spheres in SOL and I will compute the volume entropy of this manifold.

Mercoledì 8 Maggio 2024 - ore 14:00

Gabriele Viaggi (Sapienza Università di Roma)

Hausdorff dimension of hyperconvex representations of surface groups

A discrete and faithful representation of a surface group in PSL(2,C) is said to be quasi-Fuchsian when it preserves a Jordan curve on the Riemann sphere. Classically these objects lie at the intersection of several areas of mathematics and have been studied (for example) using complex dynamics, Teichmüller theory, and 3-dimensional hyperbolic geometry. From a dynamical perspective, an important invariant of such representations is the Hausdorff dimension of the invariant Jordan curves (typically a very fractal object). It is elementary to see that this number is always at least 1. A celebrated result of Bowen establishes it is equal to 1 if and only if the quasi-Fuchsian representation is Fuchsian, that is, it is conjugate in PSL(2,R). I will first describe this classical picture and then report on recent joint work with James Farre and Beatrice Pozzetti where we prove a generalization of Bowen's result for the much larger class of hyperconvex representations of surface groups in PSL(d,C) (where d is arbitrary).

Mercoledì 15 Maggio 2024 - ore 14:00

Roberto Pirisi (Università di Napoli Federico II)

Brauer groups of moduli problems and enumerative geometry

The Brauer group, classifying Azumaya algebras up to Morita equivalence, is a fundamental invariant in number theory and algebraic geometry. Given a moduli problem M (e.g. smooth curves of a given genus, K3 surfaces, abelian varieties of a given dimension...) one can consider an element of the Brauer group of M as a way to functorially assign to any family X -> S in M(S) an element in the Brauer group of S.

If we consider the moduli problem M_g of smooth curves of a given genus, the Brauer groups of M_{1,1} (the moduli problem of elliptic curves) and M_2 are known over a vast generality of bases, for example Br(M_{1,1}) is known when the base is any field or the integers; the Brauer group of M_g for g at least 4 is known to be trivial over the complex numbers through topological methods. The case g=3 is open over any base.

In a recent paper with Andrea di Lorenzo (Università di Pisa) we show that over any field k of characteristic zero the Brauer group of M_3 is equal to a direct sum of Br(k)

and a copy of Z/2Z. To our surprise, the proof of this result goes through one of the most well-known theorems in classical enumerative geometry: there are exactly 27 lines lying on a cubic surface in P^3.

Mercoledì 22 Maggio 2024 - ore 14:00

George Lusztig (Massachusetts Institute of Technology)

Decomposing a reductive group into strata

Let G be a reductive connected group over an algebraically closed field of characteristic p . Of particular importance in the study of G is the set u(G) of unipotent conjugacy classes. It is known that the cardinal of u(G) depends on p. We propose to define another set str(G) which is closely related to u(G) but is independent of p. But while u(G) is a subset of the set of conjugacy classes, str(G) is a quotient of that set. Its elements can be viewed as parts of a partition of G.
We also show that the set of conjugacy classes in the Weyl group is partitioned according to str(G).

Mercoledì 29 Maggio 2024 - ore 14:00

Ralf Schiffler (University of Connecticut)

Cluster algebras and knot theory

Cluster algebras are commutative algebras with a special combinatorial structure. A cluster algebra is a subalgebra of a field of rational functions in several variables that is generated by a distinguished set of generators called cluster variables. These cluster variables are constructed recursively from an initial seed by a process called mutation. The algebra depends on the choice of an initial quiver (=oriented graph) which governs the mutation process.

Cluster algebras are related to a number of research areas including representation theory of algebras and Lie algebras, combinatorics, algebraic and hyperbolic geometry, dynamical systems, and string theory. In this talk, we will present our recent work with Véronique Bazier-Matte establishing a connection between cluster algebras and knot theory. To every knot (or link) diagram, we associate a cluster algebra in which we identify a cluster whose cluster variables realize the Alexander polynomial of the knot.

Mercoledì 05 Giugno 2024 - ore 14:00

Claudio Onorati (Università di Bologna)

Modular vector bundles on the Fano variety of a cubic fourfold

In this talk I will report on a joint work in progress with E. Fatighenti, in which we study some special vector bundles on the Fano variety of lines of a cubic fourfold. We will see that these bundles are modular, but not atomic, and we will make some remarks on their infinitesimal deformations. In the first part of the talk I will introduce the main definitions, providing background and motivations. In the second part I will state the main results and we will look at what comes next; time permitting, we will comment on some parts of the proofs.

Mercoledì 12 Giugno 2024 - ore 14:00

Yukako Kezuka (Institut de Mathématiques de Jussieu-PRG)

Non-commutative Iwasawa theory of abelian varieties

Non-commutative Iwasawa theory has emerged as a powerful framework for understanding deep arithmetic properties over number fields contained in a p-adic Lie extension and their precise relationship to special values of complex L-functions. This talk aims to explore non-commutative Iwasawa theory over global function fields. We consider an abelian variety A defined over various base fields F, and discuss its arithmetic over the cyclotomic Z_p-extension and more general p-adic Lie extensions. After reviewing some known results over number fields, we shift our focus to the case of global function fields. In this context, we compare the arithmetic of A over different p-adic Lie extensions without assuming the finiteness of the Selmer group of A over the base field F.

Mercoledì 19 Giugno 2024 - ore 14:00

Andrea Petracci (Università di Bologna)

On moduli spaces of Fano varieties and their singularities

Fano varieties are projective varieties with “positive curvature”. Examples of Fano varieties are projective spaces, products of projective spaces, Grassmannians and hypersurfaces in projective spaces of low degree. Recently, K-stability (which is the algebro-geometric version of Kähler-Einstein metrics in complex-differential geometry) has been used to construct compact (actually projective) moduli spaces of Fano varieties. In this talk I will explain how to use combinatorial methods (e.g.

Minkowski sums of polytopes and toric geometry) to produce singular points on moduli spaces of Fano varieties. This is based on work with Anne-Sophie Kaloghiros.

Mercoledì 26 Giugno 2024 - ore 14:00

Johannes Nordström (University of Bath)

Complete cohomogeneity one solitons for G_2 Laplacian flow

Bryant's Laplacian flow is an analogue of Ricci flow that seeks to flow an arbitrary initial closed G_2-structure on a 7-manifold toward a torsion-free one, to obtain a Ricci-flat metric with holonomy G_2. This talk will give an overview of joint work with Mark Haskins and Rowan Juneman about complete self-similar solutions on the anti-self-dual bundles of CP^2 and S^4, with cohomogeneity one actions by SU(3) and Sp(2) respectively. We exhibit examples of all three classes of soliton (steady, expander and shrinker) that are asymptotically conical. In the steady case these form a 1-parameter family, with a complete soliton with exponential volume growth at the boundary of the family. All complete Sp(2)-invariant expanders are asymptotically conical, but in the SU(3)-invariant case there appears to be a boundary of complete expanders with doubly exponential volume growth.