Unit 2
DC Circuits

A. Current, Resistance, and Resistivity

Key terms

Term	Meaning
Current (I)	Measure of how much charge passes through a given area over time. SI units of $Ampere$ (A).
$Ampere$ (A).	Current equivalent to transferring 1 coulomb of charge per second. SI units of $\frac{C}{s}$.
Direct Current (DC)	Constant flow of charge in one direction.
Resistor	Device used to reduce current flow.
Resistance (R)	Measure of how much an object resists current flow. Depends on material, length, and cross sectional area. SI units of Ohms (Ω).
Resistivity (ρ)	Measure of how much a specific material resists current flow. SI units of $\Omega \cdot m$.
Ohm (Ω)	The unit of electrical resistance. SI units of $\frac{kg^2 \cdot m^2}{s^3 \cdot A^2}$.

Equations

Equation	Symbols	Meaning words
$I = \frac{\Delta q}{\Delta t}$	I is current, Δq is net charge, and Δt is change in time.	Current is the change in charge over the change in time.
$R = \frac{\rho l}{A}$	R is resistance, ρ is resistivity, l is length, and A is cross sectional area.	Resistance is proportional to resistivity and length, and inversely proportional to cross sectional area.

How to visualize the current

Current measures the flow of charges through an area over time. Figure 2.1 shows a wire with charges q moving to the left through the wire, which has a cross sectional area A. Imagine we counted how many charges passed through the cross sectional area in one second. This rate is the current.

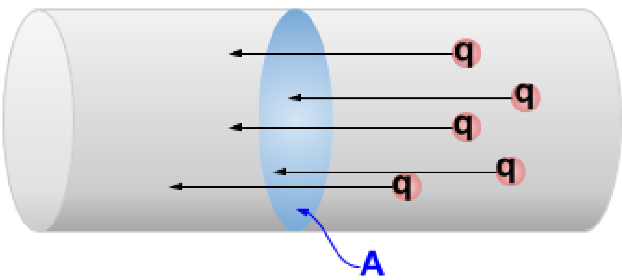


Figure 2.1. Charges q flowing through a cross sectional area A in a wire.

Finding the current direction

Current direction is designated by the symbol I along with an arrow and always refers to the flow of positive charge as shown in Figure 2.2.A. This is sometimes called **conventional current**.

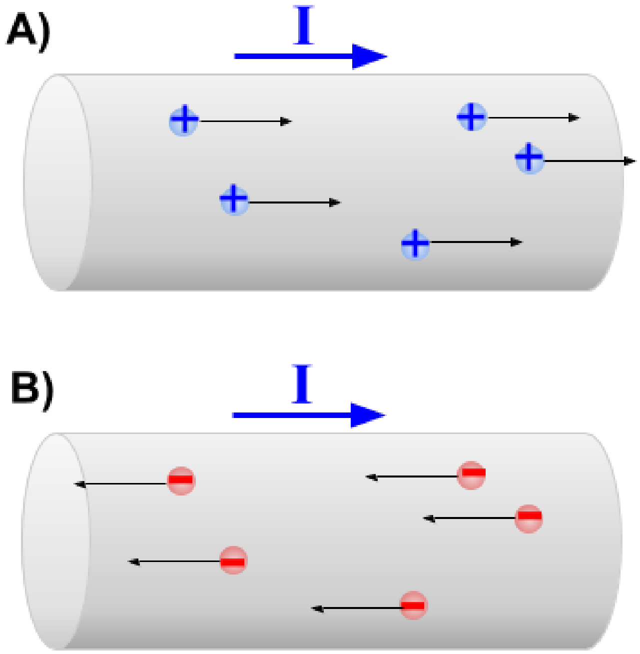


Figure 2.2. A) Conventional current describes the flow of positive charges. B) Electron current in a wire is the opposite direction of the conventional current.

In conductors such as wires, the electrons are the only charge that move. The electrons flow opposite to I (see Figure 2.2.A). The direction of the flow of electrons is called **electron current**, and its direction is opposite to I (see Figure 2B). The convention of I representing the flow of positive charge is a historical convention that is equivalent to negative charge flowing in the opposite direction.

[Do positive charges ever actually flow in the direction of the current?]
Yes. In liquids and gases, positive ions can flow in the direction of the current.

What does resistance depend on?

Resistance depends on an object's size, shape, and material. In Figure 2.3 below, the cylinder's resistance is directly proportional to its length l. The longer the cylinder, the higher the resistance.

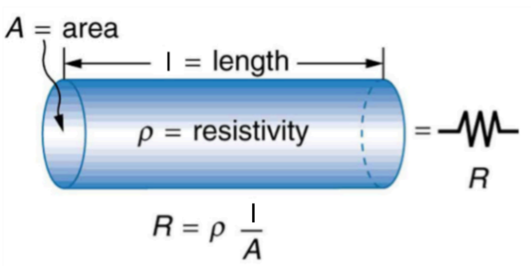


Figure 2.3. A uniform cylinder of length l and cross sectional area A. The longer the cylinder, the greater its resistance. The larger its cross-sectional area A, the smaller its resistance. Image credit: Adapted from OpenStax College Physics. Original image from OpenStax, CC BY 4.0

Additionally, the resistance is inversely proportional to the cross sectional area A. If the diameter of the cylinder is doubled, the cross-sectional area increases by a factor of 4. Therefore, resistance decreases by a factor of 4.

The resistivity ρ of a material depends on the molecular and atomic structure, and is temperature-dependent. For most conductors, resistivity increases with increasing temperature.

B. Electric Potential Difference and Ohm's Law

Key terms

Term	Meaning
Battery	Device that transforms chemical energy into electrical energy. An ideal battery has no internal resistance.
Electric potential difference (ΔV)	Energy change per unit charge between two points. Also called voltage or electric potential. Has SI units of Volts $V = \frac{J}{C}$.
Electromotive force (EMF, ϵ)	EMF is the potential difference produced by a source such as an ideal battery. Has SI units of V.

Equations

Equation	Symbols	Meaning words
$I = \frac{\Delta V}{R}$	I is current, ΔV is electric potential difference, and R is resistance.	Current is directly proportional to electric potential difference and inversely proportional to resistance.

Ohm's Law

Ohm's law states that for some devices there is a relationship between electric potential difference, current, and resistance.

The equation is: $I = \frac{\Delta V}{R}$

Where I is current, ΔV is electric potential difference, and R is resistance.

How are electric potential difference and current related?

For a given resistance R, increasing the electric potential difference ΔV increases the current I and vice versa.

How are current and resistance related?

For a given electric potential difference ΔV, if the resistance R increases, then the current I decreases and vice versa.

How are resistance and electric potential difference related?

For a given current I, if the electric potential difference ΔV increases, then the resistance R also increases and vice versa.

Analyzing electric potential difference across a resistor using Ohm's law

If the current encounters resistance, the electric potential difference decreases according to Ohm's law. We sometimes call this a voltage drop.

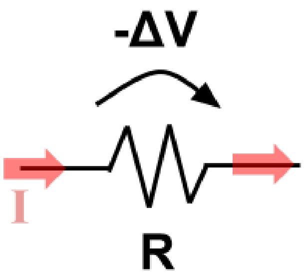


Figure 2.4. Electric potential drop across a resistor

Analyzing electric potential difference and current across a battery

A common source of electric potential is a battery, which is represented in diagrams by the symbol below (Figure 2.5). The short side is the negative end, with a lower electric potential, and the long side is the positive end, with a higher electric potential.

Electrons flow from the negative terminal to the positive terminal. Conventional current III travels from the positive terminal (higher electric potential), through the circuit, and finally to the negative end (lower electric potential).

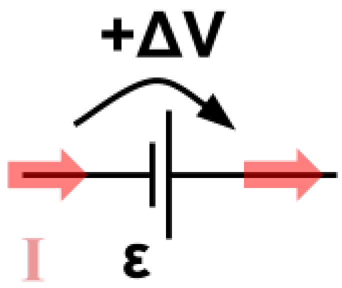


Figure 2.5. The symbol for a battery. The long side is the positive terminal, and the short side is the negative terminal.

Current flow and electric potential difference can be better understood by using the analogy of a boulder rolling down a hill. At the top of the hill, the boulder has a lot of gravitational potential energy. Similarly, an electron has a lot of stored energy in the form of electric potential energy when it is at the negative terminal of a battery. The boulder will naturally fall toward the ground where potential energy is lower. The electron at the negative terminal of a battery will naturally flow toward the positive terminal, where the electric potential is lower.

As the boulder falls downward, the stored energy is converted to kinetic energy. As the electron flows across electrical components, the stored energy is converted into various forms of energy such as heat and light.

Common mistakes and misconceptions

Sometimes people think all devices follow Ohm's law. However, a device is only ohmic when the current is directly proportional to the electric potential difference, and inversely proportional to the resistance. If we plotted an electric potential vs. current graph for an ohmic device, the relationship would be linear (see Figure 2.6).

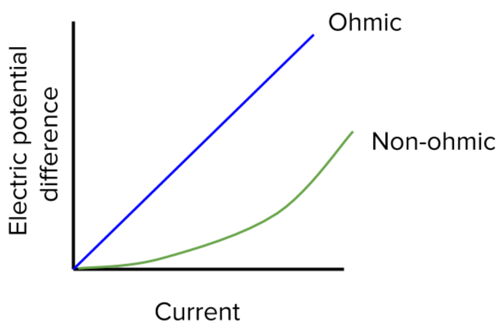


Figure 2.6. Relationship between electric potential difference and current for ohmic and non-ohmic devices.

Some devices such as light bulbs are non-ohmic. This means that their electric potential difference-current graphs are non-linear, as in Figure 2.6. For non-ohmic devices, we can't use $I = \frac{\Delta V}{R}$ to solve for an unknown.

C. DC Circuit and Electrical Power

Equations

Equation	Symbols	Meaning words
$P = I\,\Delta V$	P is power, I is current, and ΔV is electric potential difference.	The rate at which energy is transferred from a resistor is equal to the product of the electric potential difference across the resistor and the current through the resistor. Scalar quantity with units of Watts (W).

[Are there other ways to write an equation for power?]

Yes! We can make substitutions using Ohm's law, $I = \frac{\Delta V}{R}$

We can substitute for I to get:

$P = I\,\Delta V$

$= \frac{\Delta V}{R}\,\Delta V$

$= \frac{\Delta V^2}{R}$

We can substitute for ΔV to get:

$P = I\,\Delta V$

$= I\,I\,R$

$= I^2\,R$

Circuit component definitions and symbols

Resistor (R)

Resistors are electrical components that resist current and expends voltage within a circuit.

Figure 2.7. Symbol for resistor.

Battery (ϵ)

Batteries are electrical components that provide electrical energy.

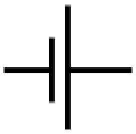


Figure 2.8. Symbol for battery. The short end is the negative terminal and the long end is the positive terminal.

Batteries have positive and negative terminals. The negative terminal is drawn with a short line, and the positive terminal is shown as a long line.

Switch (S)

Switches turn the flow of current through a circuit pathway on and off. When the switch is open, no current flows because there is a gap in the circuit (Figure 2.9).

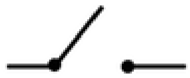


Figure 2.9. Symbol for open switch. No current flows through this location because the conductive pathway has a gap.

When the switch is closed, current can flow because the circuit is continuous (Figure 2.10).

Figure 2.10. Symbol for closed switch. Current can pass through this location because the circuit pathway is continuous.

Node

A node (or junction) is a place where two or more circuit elements join together. Figure 2.11 below shows a single node (the black dot) formed by the junction of five electrical components (abstractly represented by orange rectangles).

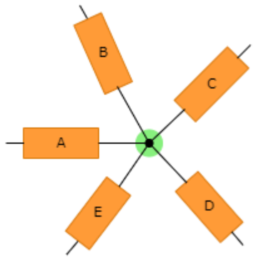


Figure 2.11. A junction (highlighted in green) between 5 different electrical components.

DC circuit types

Simple circuit

A simple circuit contains the minimum amount of components that allow it to be a functional electric circuit: a voltage source ϵ (battery), a resistor R, and a loop of wires for current I to flow around (see Figure 2.12 below). We usually ignore any resistance from the wires.

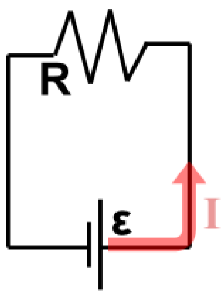


Figure 2.12. Simple circuit diagram.

In a simple circuit, the voltage supplied by the battery ϵ is the voltage expended by the resistor R, and there is only one current I in the circuit.

Closed circuit

A closed circuit has a continuous pathway for current to flow through. In other words, there are no gaps in the circuit.

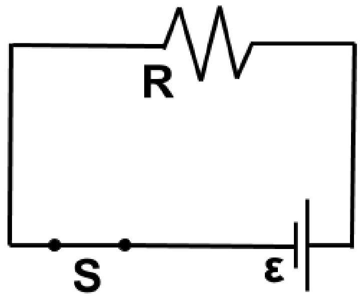


Figure 2.13. Diagram of a closed circuit.

Open circuit

An open circuit has a gap in the circuit that does not allow current to flow through. The gap can be caused by an open switch, a broken component, or broken wire.

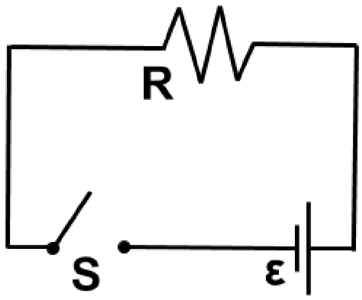


Figure 2.14. Diagram of a open circuit.

Short circuit

A short is a pathway of zero resistance within a circuit (see the blue wire in Figure 2.15). When there is a short circuit, all the current flows across the short because the current prefers the path of least resistance.

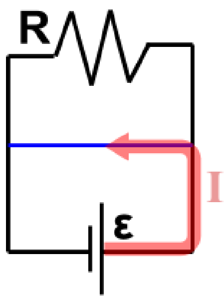


Figure 2.15. The blue wire has no resistance and is a short in this circuit. Since there is no resistance, all the current I flows across the blue wire instead of going through the resistor R.

Figure 2.16 below shows how closing a switch S can divert all the current from resistor R_2. When switch S is open (see Figure 2.16.A), the current I flows out of the positive terminal of the battery towards node N. Since the switch is open, no current flows through the switch and all the current flows through resistor R_2. When the switch is closed (see Figure 2.16.B), it forms a short around resistor R_2. Now, once the current I reaches N, the current bypasses R_2 and flows through the switch.

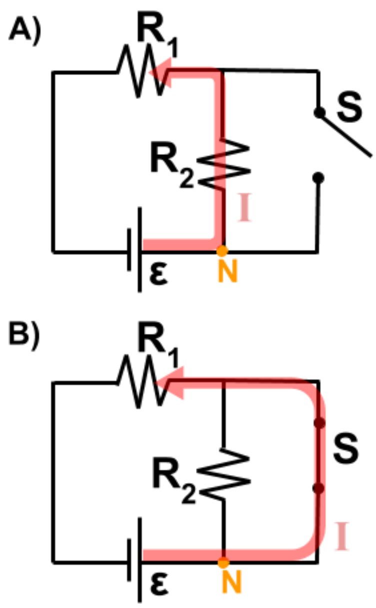


Figure 2.16. When the switch S changes from open (diagram A) to closed (diagram B), resistor R_2 is shorted out and the current I bypasses it to go through the switch.

D. Kirchhoff's Junction Rule

Key terms

Junction

Intersection of three or more pathways in a circuit. Typically represented by a dot on a circuit diagram. Also called a node.

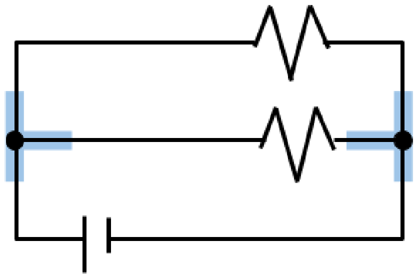


Figure 2.17. Two junctions represented by dots and the current pathways highlighted.

Branch

A path connecting two junctions.

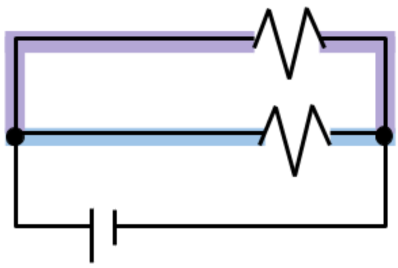


Figure 2.18. Two branches in a circuit, each highlighted in a different color.

Kirchhoff's junction rule

Kirchhoff's junction rule says that the total current into a junction equals the total current out of the junction. This is a statement of conservation of charge. It is also sometimes called Kirchhoff's first law, Kirchhoff's current law, the junction rule, or the node rule. Mathematically, we can write it as:

$$I_{in} = I_{out}$$

Junctions can't store current, and current can't just disappear into thin air because charge is conserved. Therefore, the total amount of current flowing through the circuit must be constant.

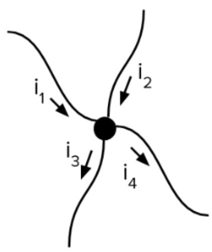


Figure 2.19 Kirchhoff's junction rule says that the current flowing into the node, i_1 and i_2, must be equal to the current flowing out of the node, i_3 and i_4.

For the total current in Figure 2.19, we can write the relationship between the current going into and out of the node as:

$$I_{in} = I_{out}$$

$$i_1 + i_2 = i_3 + i_4$$

For example, in Figure 2.20, the current into the node equals the current out of the node.

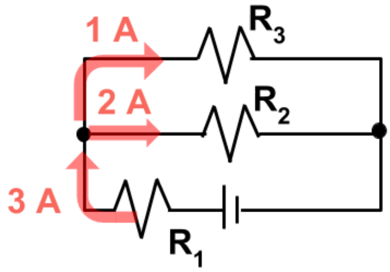


Figure 2.20 The current into the node equals the current out of the node.

The current into the node is 3 A. There are two branches out of the node. The current across resistor R_2 is 2 A and the current across resistor R_3 is 1 A, so we can write:

$$i_{in} = i_{out}$$

$$3\,A = 1\,A + 2\,A$$

$$3\,A = 3\,A$$

Yes!

E. Kirchhoff's Loop Rule

Key terms

Term	Meaning
Loop	Closed circuit that starts and ends at the same point.

Kirchhoff's loop rule

Kirchhoff's loop rule states that the sum of all the electric potential differences around a loop is zero. It is also sometimes called Kirchhoff's voltage law or Kirchhoff's second law. This means that the energy supplied by the battery is used up by all the other components in a loop, since energy can't enter or leave a closed circuit. The rule is an application of the conservation of energy in terms of electric potential difference, ΔV.

Mathematically, this can be written as:

$$\Sigma\Delta V = 0$$

How to determine the electric potential difference across a circuit component

For example, we can use Kirchhoff's loop rule to find the unknown electric potential difference across a resistor (Figure 2.21).

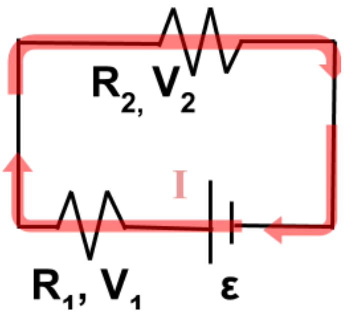


Figure 2.21 A circuit with two resistors, R_1 and R_2. V_2 is unknown.

Let's pick our starting point at the battery and go around the loop until we are back to the same point.

The electric potential increase over the battery is ϵ. Over R_1 there is an electric potential decrease of V_1. We do not know the electric potential decrease V_2 over R_2.

Now we can use the loop rule to solve for V_2 in terms of V_1 and ϵ:

$$\Sigma\Delta V = 0$$

$$-V_1 - V_2 + \epsilon = 0$$

$$V_2 = \epsilon - V_1$$

F. Resistors In Series And Parallel

Key terms

Term	Meaning
Equivalent resistance (R_{eq})	The total resistance of a configuration of resistors.

Equations

Equation	Symbols	Meaning words
$R_s = \sum_i R_i$ or $R_s = R_1 + R_2 + \ldots$	R_s is equivalent series resistance and $\sum_i R_i$ is the sum of all individual resistances R_i.	Equivalent series resistance is the sum of all the individual resistances.
$\frac{1}{R_p} = \sum_i \frac{1}{R_i}$ or $\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots$	R_p is equivalent parallel resistance and $\sum_i \frac{1}{R_i}$ is the sum of all individual resistances R_i reciprocals.	The reciprocal of the equivalent parallel resistance is the sum of all the individual resistance reciprocals.

Resistors in series and parallel

Series resistor properties

Any time we have more than one resistor in a row, the configuration is described as having the resistors in series or series resistors (Figure 2.22).

Figure 2.22. Resistors in series.

Resistors in series have some special characteristics worth remembering. Any configuration of resistors in a series will have the following properties.

- The same current flows through each resistor: $I_1 = I_2 = \ldots = I_n$
- Potential difference is distributed among series resistors: $\Delta V_s = \Delta V_1 + \Delta V_2 + \ldots + \Delta V_n$
- The resistor with the biggest resistance has the greatest voltage.
- The equivalent resistance R_s is always more than any resistor in the series configuration.

Parallel resistor properties

Another possible way to arrange resistors in a circuit is to have multiple resistors branch off from a single junction in the circuit (Figure 2.23).

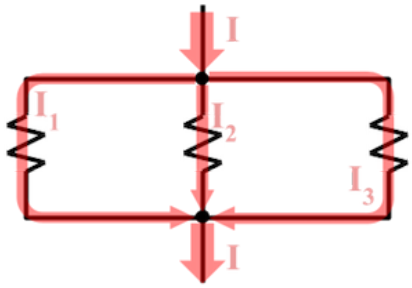


Figure 2.23. Resistors in parallel.

Resistors in parallel also have some special characteristics:

- The current is distributed across resistors: $I = I_1 + I_2 + ... + I_n$
- Potential difference is the same across all resistors in parallel: $\Delta V_1 = \Delta V_2 = ... = \Delta V_n$
- The smallest resistance gets the most current.
- The equivalent resistance R_p is always less than any resistor in the parallel configuration.
- Keep in mind that not all circuits are strictly series or parallel. Sometimes they can be a combination of both.

How to calculate equivalent resistance

Resistors in series or parallel can be replaced by a single resistor of equivalent resistance. This strategy is helpful for solving complex circuit problems because it let's us simplify the circuit.

Equivalent series resistance

We can redraw the circuit with the resistors in series replaced by a single equivalent resistor (Figure 2.24).

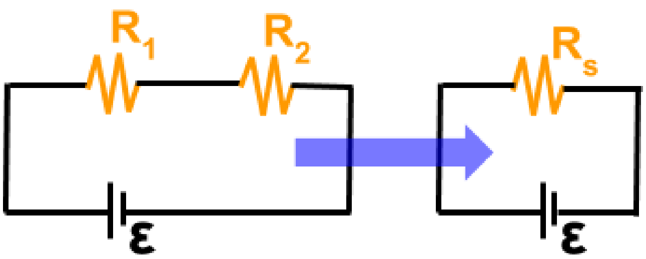


Figure 2.24. Replacing resistors in a series with an equivalent resistor R_s

We can calculate R_s from the resistances of the individual resistors in series. If $R_1 = 4\ \Omega$ and $R_2 = 8\ \Omega$, then the equivalent resistance is the sum of R_1 and R_2:

$R_s = R_1 + R_2$

$= 4\ \Omega + 8\ \Omega$

$= 12\ \Omega$

Equivalent parallel resistance

We can redraw a circuit with all resistors in parallel replaced by a single equivalent resistor (Figure 2.25).

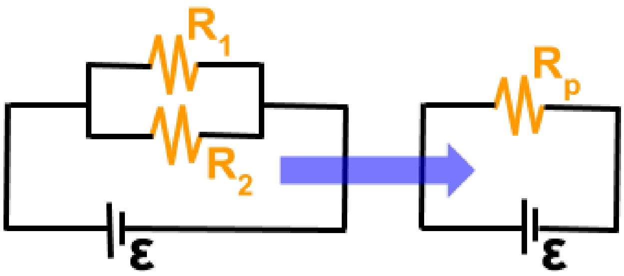


Figure 2.25. Replacing a resistors in parallel with an equivalent parallel resistor.

We can calculate R_p from the resistances of the individual resistors in parallel. If $R_1 = 4\ \Omega$ and $R_2 = 8\ \Omega$, then the equivalent resistance R_p is:

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$= \frac{1}{4\ \Omega} + \frac{1}{8\ \Omega}$$

$$= \frac{2}{8\ \Omega} + \frac{1}{8\ \Omega}$$

$$\frac{1}{R_p} = \frac{3}{8\ \Omega}$$

Now, let's be careful here. Lots of people make a mistake here: $\frac{3}{8}$ is not the equivalent parallel resistance R_p yet, it is the **reciprocal**. To solve for R_p, we need to take the reciprocal of both sides:

$$\left(\frac{1}{R_p}\right)^{-1} = \left(\frac{3}{8\ \Omega}\right)^{-1}$$

$$R_p = \frac{8\ \Omega}{3}$$

$$\approx 2.7\ \Omega$$

G. DC Ammeters And Voltmeters

Key terms

Term	Meaning
Ammeter	Instrument used to measure current.
Voltmeter	Instrument used to measure electric potential difference.

How are ammeters and voltmeters represented in a circuit?

Ammeters are typically represented by a circle with a letter A inside (Figure 2.26).

Figure 2.26. Symbol for ammeter.

Voltmeters are typically represented by a circle with a letter V inside (Figure 2.27).

Figure 2.27. Symbol for voltmeter.

How to use ammeters and voltmeters

Ammeters measure the current through components. To measure the current going through a component, the ammeter is connected in series with the components we want to investigate. Resistors in series experience the same current (Figure 2.28). Typically, ammeters have negligible resistance, so they do not affect the circuit.

For example, if we want to find the current through R_1 in the circuit below, the ammeter is placed in series with R_1.

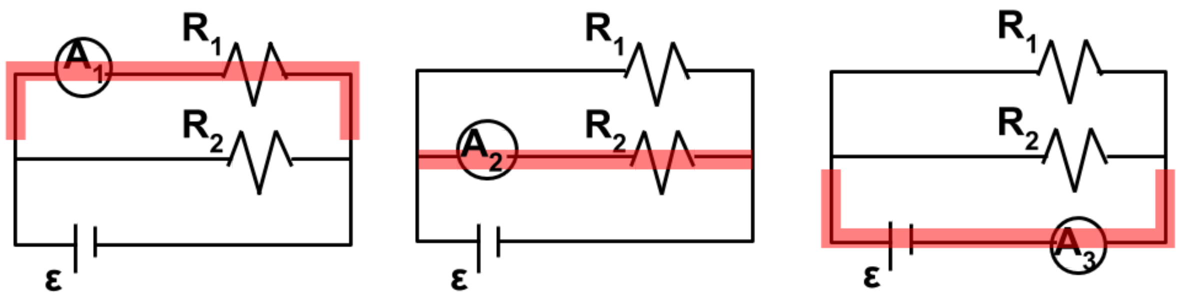


Figure 2.28. Ammeter A_1 in series with R_1 so it measures the current through R_1. Ammeter A_2 in series with R_2 so it measures the current through R_2. Ammeter A_3 in series with the battery so it measures the current through it.

Voltmeters measure the electric potential drop across components. The voltmeter is placed in parallel with the component of interest because components in parallel experience the same potential difference.

For example, if we want to measure the electric potential drop across resistor R_2, the voltmeter should be arranged in parallel with the resistor (R_2).

[Does the voltmeter affect the circuit?]
If the voltmeter's resistance is much greater than the device, then very little current flows through it and there is negligible effect on the circuit. Typically we can assume that the voltmeter's resistance is high.

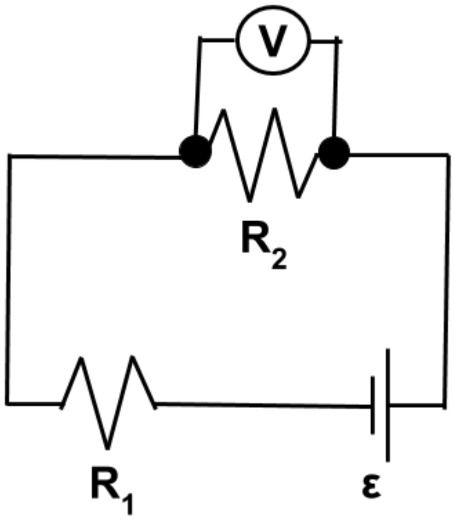


Figure 2.29. Voltmeter V in parallel measures potential drop across resistor R_2 in the circuit.

Exercises of Unit 2

Calculating resistance, voltage, and current using Ohm's law

1. A student builds a simple circuit with a single resistor of resistance R and current I, and measures an electric potential difference of ΔV_1 across the resistor. Then, the student decreases the current through the circuit to $\frac{I}{2}$.

   How does the electric potential difference across the resistor ΔV_2 of the new circuit compare with ΔV_1?

   A. $\Delta V_2 = 4\Delta V_1$ C. $\Delta V_2 = \frac{\Delta V_1}{4}$

   B. $\Delta V_2 = \frac{\Delta V_1}{2}$ D. $\Delta V_2 = 2\Delta V_1$

2. A student builds a simple circuit with a single resistor with resistance R and measures an electric ΔV across the resistor. Then, the student replaces the resistor with a new one of resistance R_2 and keeps the electric potential difference the same.

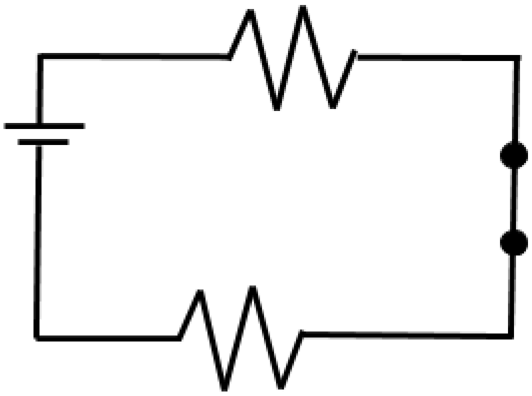
   What is the current I_2 through the new circuit in terms of the original current, I_1?

   A. $I_2 = 2I_1$ C. $I_2 = \frac{I_1}{4}$

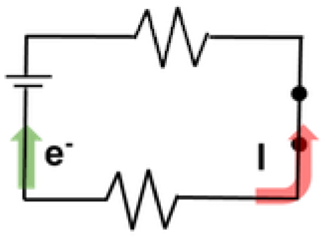
   B. $I_2 = \frac{I_1}{2}$ D. $I_2 = 4I_1$

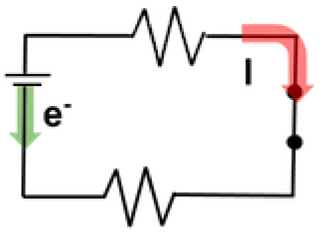
Characteristics of circuit diagrams

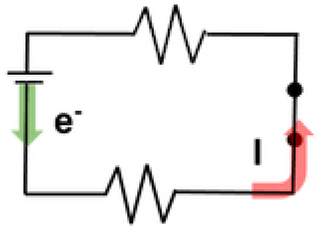
3. Below is a diagram of a circuit with a switch, two resistors, and a battery.

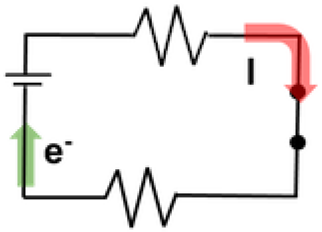


Which of the following shows the correct pathways for the conventional current I and electron current e^-?

A. 

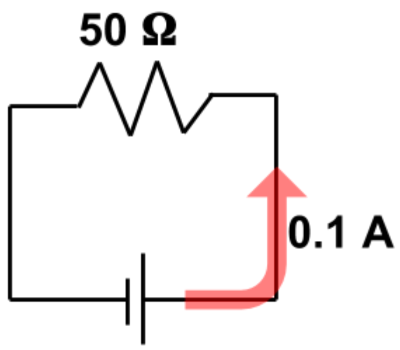
B. 

C. 

D. 

Calculations from circuit diagrams

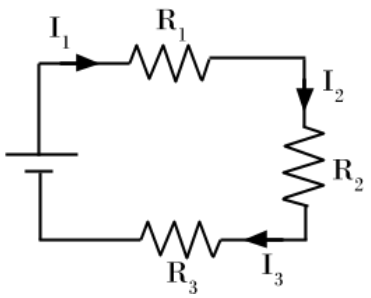
4. A student builds the circuit below.



The magnitude of the voltage across the resistor is ... V.

Kirchhoff's junction rule

5. For the circuit shown, the resistances are $R_1 > R_2 > R_3$.

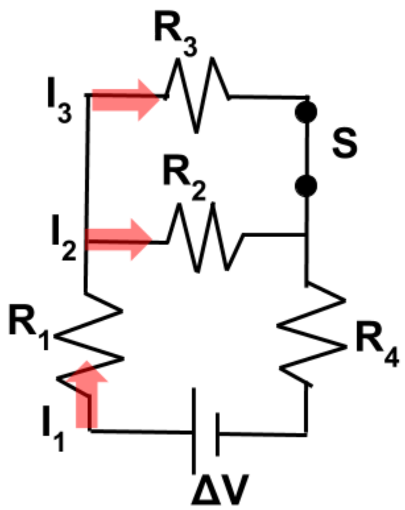


What is true about the current through each resistor?

A. $I_1 < I_3 < I_2$
B. $I_1 > I_2 > I_3$
C. $I_1 > I_3 > I_2$
D. $I_1 < I_2 < I_3$

Kirchhoff's loop rule: Symbolic problems

6. Four resistors with different resistances are connected to a battery with voltage ΔV as seen below.

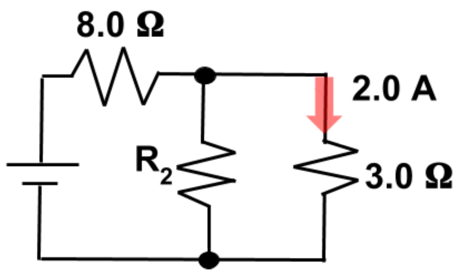


Which of the following equations correctly represent the total potential difference for a complete loop in the circuit diagram above?

A. $\Delta V - I_1(R_1 + R_4) - I_2R_2 = 0$
B. $\Delta V - I_1(R_1 + R_4) - I_2R_2 - I_3R_3 = 0$
C. $- I_2R_2 + I_3R_3 = 0$
D. $- I_2R_2 - I_3R_3 = 0$

Kirchhoff's loop rule calculations

7. For the circuit below, the current that flows through the $3.0\ \Omega$ resistor is $2.0\ A$.

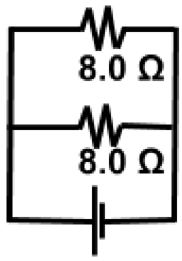


What is the magnitude of the potential difference across R_2 ?

A. $3.0\ V$
B. $2.0\ V$
C. $9.0\ V$
D. $6.0\ V$

Calculating equivalent resistance for series and parallel resistors

8. A circuit with two resistors is shown below.

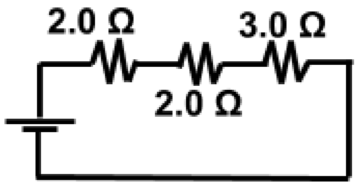


What is the equivalent resistance of the resistors?

A. $16\ \Omega$ C. $4.0\ \Omega$

B. $8.0\ \Omega$ D. $64\ \Omega$

9. A circuit with three resistors is shown below.



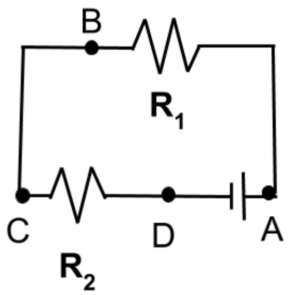
What is the equivalent resistance of the resistors?

A. $0.75\ \Omega$ C. $12\ \Omega$

B. $2.3\ \Omega$ D. $7.0\ \Omega$

Using voltmeters and ammeters to measure potential difference and current

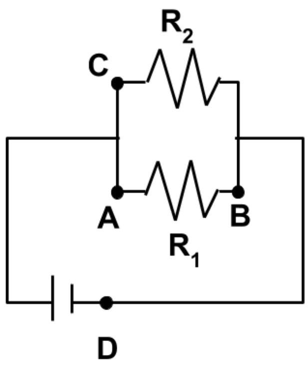
10. A student wants to measure the electric potential difference across the resistor R_2 for the circuit below.



Where should a student hook up a voltmeter to measure the electric potential difference across resistor R_2 ? (Choose 2 answers)

A. From point A to point C
B. From point A to point D
C. From point B to point D
D. From point C to point D

11. A student wants to measure the current through the resistor R_1 for the circuit below.



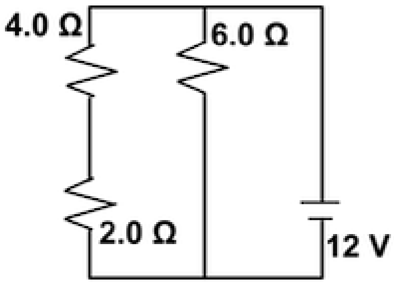
Where should a student hook up an ammeter to measure the current through the R_1 branch?

A. Point A C. Point C

B. Point B D. Point D

Advanced circuit analysis

12. A student builds the circuit below.



What is the current through the battery?

A. $2.0\,A$ C. $6.0\,A$

B. $3.0\,A$ D. $4.0\,A$