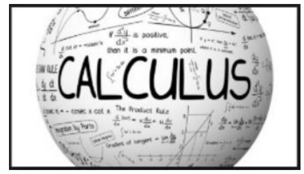
AP CALCULUS AB



Dear Prospective AP Calculus AB Student,

AP Calculus AB is a college level course covering material traditionally taught in the first semester of college calculus. To be successful in AP Calculus AB there are certain skills that have been taught to you over the previous years that you are expected to have mastered. If you do not have these skills you will find that you will consistently get problems incorrect next year, even though you understand the calculus concepts. It is frustrating for students when they are tripped up by the algebra and not the calculus. This AP Calculus AB Summer Readiness Packet is intended to help you brush up and possibly relearn these topics. It is recommended that you visit https://www.coolmath.com/precalculus-review-calculus-intro to refresh your memory on any topics covered in this packet that you need to review.

Students need a strong mathematical foundation to be ready for the rigorous work required throughout AP Calculus AB. Completing this AP Calculus AB Summer Readiness Packet should prepare you for the material. Show all work in a neat and organized manner on your own paper. Refer to your precalculus notes for explanations and examples. If you need more assistance search the web. I particularly recommend www.khanacademy.org. A graphing calculator may be used when appropriate.

If you choose not to work on this packet over the summer, you will have the first week of school to complete and turn in the packet. Because you will have other assignments to complete in AP Calculus AB and your other courses, I encourage you to do what you can over the summer to make your transition back to school easier.

Next year will be an exciting learning experience. I look forward to working with you!

Sincerely,

Mrs. Anderson

AP Calculus AB Summer Readiness Assignment

- Copy each problem on your own paper and show ALL work. Clearly label each section.
- For those problems that can be done intuitively or by using a calculator, your written justification must be part of your solution.
- It is permissible to use a math textbook, internet resources, notes from previous math courses or a study partner in an effort to understand how to work each problem, but it is a violation of the OLHS Honor Code to copy answers or another student's work.

Part I (Not Calculator Active):

1. Simplify: $2(x - 3y) - 4(3x - y)$

2. Evaluate: $\frac{x+3y}{x^2-6y^2-z^2}$ if x = -4, y = -3 and z = -1

3. Solve: $7 - y = -2$

4. Simplify: $(x - 5)(2x + 3)$

5. Simplify: $(x - 8)^2$

6. Factor: $16x^2 - 1$

7. Factor: $2a^2 + 13a - 7$

8. Solve: $2x^2 + 4x = 30$

9. Simplify: $(-5x^4)^3$

10. Simplify: $\frac{12a^5b^6}{(2a^2b)^2}$

11. Simplify: $\frac{3x^2+2x}{2x}$

12. Simplify: $\frac{x^2-4}{2-x}$

13. Solve: $\frac{12}{z-3} = \frac{15}{z+1}$

14. Solve: $\frac{y-1}{2} - \frac{y+1}{3} = 1$

15. Simplify: $\frac{\frac{7}{y+7}}{\frac{1}{y+7}-\frac{1}{y}}$

16. Express $(-3y^{-3})^3$ in terms of positive exponents.

17. Solve the following system of equations: $5x - 4y = 9$
$3x + 2y = 1$

18. A box is to be constructed from a square piece of material of side length *x* by cutting out a 2-inch square from each corner and turning up the sides. Express the volume of the box as a polynomial in the variable *x*.

19. If a football is thrown upward, its height above the ground is given by the equation $s = -16t^2 + v_0 t + s_0$, where v_0 and s_0 are the initial velocity and initial height of the football, respectively, and t is the time in seconds. Suppose the football is thrown from the top of a building that is 192 feet tall, with an initial speed of 96 feet per second.

a) Write the polynomial that gives the height of the football in terms of the variable t (time).

b) What is the height of the football after 2 seconds have elapsed? Will the football hit the ground after 2 seconds?

Part II (Not Calculator Active):

Write an equation for the specified line.

1. The line through (-3,6) and (1,-2)

2. The horizontal line through (0,2)

3. The line through (4, -12) and parallel to $4x + 3y = 12$

4. The line through (-1,2) and perpendicular to $\frac{1}{2}x + \frac{1}{3}y = 1$

5. The line through (4, -2) with x-intercept of -3

For #6-9 a) find the domain, b) find the range and c) graph the function.

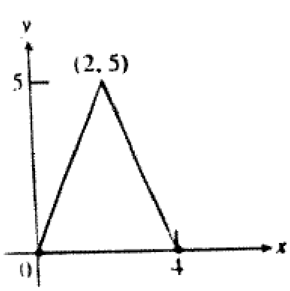
6. $y = \sqrt{16 - x^2}$

7. $y = 2e^{-x} - 3$

8. $y = \ln ln\,(x - 3) + 1$

9. $y = \{- x - 2, \quad - 2 \leq x \leq - 1\ x, \qquad - 1 < x \leq 1\ - x + 2, \quad 1 < x \leq 2$

10. Write a piecewise equation for the function:



11. Given $f(x) = \frac{1}{x}$ and $g(x) = \frac{1}{\sqrt{x+2}}$, find the following:

a) $f(g(-1))$ b) $g(f(2))$ c) $f(f(x))$ d) $g(g(x))$

Part III (Calculator Active):

1. On the closed interval [0, 10], how many times do the graphs of $y = \cos cos\, x$ and $y =- 2e^{-2x}$ intersect? Sketch the graph and explain your reasoning.

2. Given $f(x) = x^{\frac{4}{5}} - 1$, find $f(x)$. Sketch the graph and show the limit.

3. Find the roots of $y = x^3 + 1.1x^2 - 1.6x + 0.4$.

4. Find the coordinates of the intersection of $y = 3\sin sin\, x$ and $y = 0.5x + 1$.

5. Given $f(x) = \frac{x^2+3}{x} + 1$, find the relative maxima and relative minima.

6. Given the function $f(x) = x^2 - 4x$ on [0,2]:

a) Sketch the graph of the function on the interval. Scale the graph so that it is large enough to include detail.

b) Find the slope of the secant line connecting the endpoints.

7. Given $f(x) = x^2 - 5x + 2$ and $g(x) = 3 - 2x$, find the coordinates of any points of intersection and indicate them on a sketch of the graph.

8. Given $f(x) = x^4 - 7x^3 + 6x^2 + 8x + 9$

a) Sketch the graph.

b) Determine the *x* and *y* coordinates of the lowest point on the graph.

c) On the closed interval [-10,10] determine the highest and lowest values of f(*x*).

9. Given $f(x) = 3\sin sin\, 2x - 4x + 1$ on the interval [-2π,2π]

a) Find all roots to the nearest 0.001 (All trig functions are done in radian mode on the calculator).

b) Sketch the function with its roots.

Part IV (Not Calculator Active):

You should be very familiar with the following functions and be able to envision them in your mind without the advantage of a graphing calculator.

1. constant function: $f(x) = k,\ k$ is a constant
2. linear function: $f(x) = mx + b$
3. identity function: $f(x) = x$
4. absolute value function: $f(x) = |x|$
5. piece-wise (or partitioned domain) function: $f(x) = \{g(x),\ x \le n\ h(x),\ x > n$

6. greatest integer function: $f(x) = [x]$
7. quadratic function: $f(x) = ax^2 + bx + c$
8. polynomial function: $f(x) = ax^n + bx^{n-1} + \cdots + mx + n$
9. radical function: $f(x) = \sqrt{x}$
10. rational function: $f(x) = \frac{ax+b}{cx+d}$
11. exponential function: $f(x) = b^x$ where b>0 and b≠1
12. logarithmic function: $f(x) = x$

Part V (Not Calculator Active):

You should be very familiar with the six trig functions including their graphs, domain and range.

You must know the following trig identities:

$x = 1$ $\cos cos\, 2\theta = \theta - \theta$ $\sin sin\, 2\theta = 2 \sin sin\, \theta \cos cos\, \theta$
$x + 1 = x$ $\cos cos\, 2\theta = 1 - 2\theta$
$1 + x = x$ $\cos cos\, 2\theta = \theta - 1$

You must know the special angle trig values from the unit circle. Evaluate the following without using a calculator:

a) $\sin sin\, 0$ b) $\cos cos\, \pi$ c) $\tan tan\, \frac{5\pi}{6}$ d) $\sin sin\, \frac{5\pi}{3}$

e) $\cos cos\, \frac{7\pi}{4}$ f) $\sec sec\, \frac{11\pi}{6}$ g) $\cot cot\, \frac{7\pi}{4}$ h) $\csc csc\, \frac{7\pi}{6}$

i) $\frac{9\pi}{2}$) j) $\sin sin\, \frac{17\pi}{3}$ k) $\sin sin\, \frac{100\pi}{8}$ l) $\frac{13\pi}{6}$)

Part VI (Calculator Active):

Finding limits numerically-

Complete the table and use the result to estimate the limit.

1. $\frac{x-4}{x^2-3x-4}$

x	3.9	3.99	3.999	4.001	4.01	4.1
f(x)						

2. $\frac{\sqrt{4-x}-3}{x+5}$

x	-5.1	-5.01	-5.001	-4.999	-4.99	-4.9
f(x)						

Finding limits graphically-

Find each limit graphically. Use your calculator to assist in graphing.

3. $\cos \cos x$ 4. $\frac{2}{x-5}$ 5. $\{x^2 + 3,\ x \neq 1\ 2, \quad x = 1$

Evaluating limits analytically-

Solve by direct substitution when possible. Remember, if you get $\frac{0}{0}$ from direct substitution you must find the limit another way such as factoring and simplifying or by the rationalization technique.

6. $(4x^2 + 3)$ 7. $\frac{x^2+x+2}{x+1}$ 8. $\sqrt{x^2 + 4}$

9. $\cos \cos x$ 10. $\frac{x^2-1}{x-1}$ 11. $\frac{x^2+x-6}{x+3}$

12. $\frac{\sqrt{x+1}-1}{x}$ 13. $\frac{3-x}{x^2-9}$ 14. $\frac{2(x+h)-2x}{h}$

Part VII (Not Calculator Active):

Vertical Asymptotes-

Determine the vertical asymptote(s) for each function. Recall: To find the vertical asymptote(s) factor the denominator and set any factors, not common to a factor in the numerator, equal to zero.

15. $f(x) = \frac{1}{x^2}$ 16. $f(x) = \frac{x^2}{x^2-4}$ 17. $f(x) = \frac{2+x}{x^2(1-x)}$

Horizontal Asymptotes-

Case I: Degree of the numerator is less than the degree of the denominator. The asymptote is y=0.
Case II: Degree of the numerator is the same as the degree of the denominator. The asymptote is y=ratio of the leading coefficients.
Case III: Degree of the numerator is greater than the degree of the denominator. There is no horizontal asymptote. The function increases without bound. There is a slant asymptote if the degree of the numerator is one more than the degree of the denominator which can be found by synthetic or long division.

Determine all horizontal asymptotes:

18. $f(x) = \frac{x^2-2x+1}{x^3+x-7}$ 19. $f(x) = \frac{5x^3-2x^2+8}{4x-3x^3+5}$ 20. $f(x) = \frac{4x^5}{x^2-7}$

Limits as *x* approaches infinity-

This is the same process you used to find horizontal asymptotes for a rational function.

21. $\frac{2x-5+4x^2}{3-5x+x^2}$ 22. $\frac{2x-5}{3-5x+3x^2}$ 23. $\frac{7x+6-2x^2}{3+14x+x^2}$

Limits to infinity-

A rational function does not have a limit if it goes to $\pm\infty$, however you can state the direction the limit is headed if both the left and right hand side go in the same direction.

Determine each limit if it exists and state whether the limit approaches ∞ or $-\infty$.

24. $\frac{1}{x+1}$

25. $\frac{2+x}{1-x}$

26. . $\frac{2}{\sin sin\, x}$