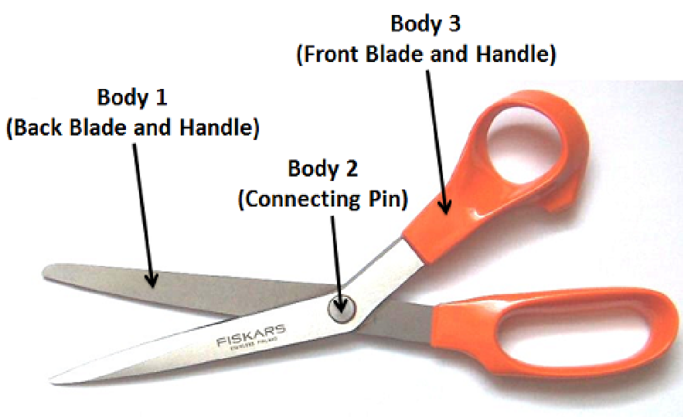
Ch 6: Equilibrium of Structures

Introduction:

Broadly defined, a structure is any set of interconnected rigid bodies. The different bodies in the structure can move relative to one another (such as the blades in a pair of scissors in the figure below) or they can be fixed relative to one another (such as the structural members within a bridge). Static analysis of structures involves the application of the equilibrium equations to solve for the forces acting on and between individual members of the structure.



[caption] An example of an engineering structure. The scissors as a whole consists of three interconnected rigid bodies.

Learning Outcomes: Upon completion of this section you should be able to:

1. Identify a structure as a truss, frame, or machine and explain why.
2. Explain why a two-force member is associated with only a single unknown value.
3. Identify two-force members in a structure.
4. Explain how Newton's Third Law applies to interaction forces between connected members in structure
5. Observe a structure and conceptualize the load path from any applied load to the structure's reactions.

Looking across the various types of equilibrium

You might be wondering how structures are different than the particle and rigid body equilibrium that you have learned so far in Statics. You can find a summary of these differences in the table below.

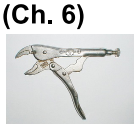
[caption] **Table 6-1:** This table compares the various types of equilibrium topics, plus solving techniques and number of equations available for each.

<table>
<tr><th>System Type</th><th>Definition</th><th>Technique to Solve</th><th>Equations and Unknowns</th></tr>
<tr><td>Single Particle (Ch. 3)</td><td>A single particle where the external forces, body forces, and reactions are all collinear and in equilibrium</td><td>Collinear forces allow us to isolate particle as FBD (cutting axial forces in 2-force supports) and $\Sigma\mathbf{F}=0$ (Note: $\Sigma\mathbf{M}=0$ not helpful as forces do not create moment about particle)</td><td>2D$\rightarrow$2 $\Sigma\mathbf{F}=0$ equations = 2 unknowns

3D$\rightarrow$3 $\Sigma\mathbf{F}=0$ equations = 3 unknowns</td></tr>
<tr><td>Single Rigid Body (Ch. 5)</td><td>A multi-force member where external loading and reactions are in equilibrium</td><td>Variety of forces and couples (from loading and reactions) require FBD of rigid body and apply $\Sigma\mathbf{F}=0$ and $\Sigma\mathbf{M}=0$ equations.</td><td>2D$\rightarrow$2 $\Sigma\mathbf{F}=0$ + 1 $\Sigma\mathbf{M}=0$ equations** = 3 unknowns

3D$\rightarrow$3 $\Sigma\mathbf{F}=0$ + 3 $\Sigma\mathbf{M}=0$ equations = 6 unknowns</td></tr>
<tr><td>Trusses (Ch. 6)
</td><td>An engineering structure that is rigid as a whole, is assumed to consist entirely of two-force members, and only carries forces at joints between members</td><td>Two-force members and loading at joints allow FBD of joints to expose axial loads in members</td><td>2D$\rightarrow$(2 $\Sigma\mathbf{F}=0$ equations)(# joints) = (# reactions) + (# members)

3D$\rightarrow$(3 $\Sigma\mathbf{F}=0$ equations)(# joints) = (# reactions) + (# members)</td></tr>
<tr><td>Frames (Ch. 6)
</td><td>An engineering structure that is rigid as a whole, but contains at least one member that is not a two-force member.</td><td rowspan="2">Mix of two-force and multiforce bodies require require FBD's of rigid bodies where interactions between bodies are equal and opposite (as per Newton's 3rd Law). Can apply $\Sigma\mathbf{F}=0$ and $\Sigma\mathbf{M}=0$ equations for each body to solve for total unknowns.</td><td rowspan="2">2D$\rightarrow$(2 $\Sigma\mathbf{F}=0$ + 1 $\Sigma\mathbf{M}=0$ equations per multiforce body) + (1 $\Sigma\mathbf{F}=0$ equation per two-force body)

3D$\rightarrow$(3 $\Sigma\mathbf{F}=0$ + 3 $\Sigma\mathbf{M}=0$ equations per multiforce body) + (1 $\Sigma\mathbf{F}=0$ equation per two-force body)</td></tr>
<tr><td>Machines (Ch. 6)
</td><td>An engineering structure that is not rigid as a whole, and contains at least one member that is not a two-force member.</td></tr>
</table>

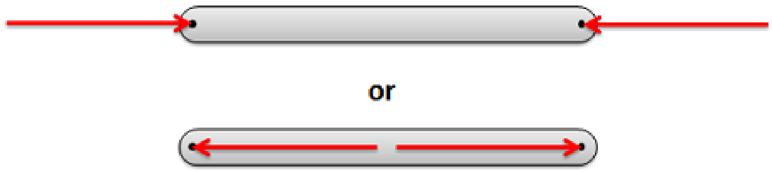
** Other combinations of equilibrium equations are possible see Ch. 5 Rigid Body Equilibrium Section XX for details

As the approach and equations vary between topics, it is important to first determine the type equilibrium problem you are solving, as you can see that there are different strategies for analyzing each type. While the details related to equilibrium of a particle or single rigid body should be reviewed; do not worry if the techniques and equations for trusses, frames, and machines are not clear at this point, we will revisit this table again at the end of the chapter as a review. One specific skill that is important to determine if a structure is a truss (made of all two-force members), rather than a frame or machine (which have at least one multi-force body).

As we can see from the above definitions, the key to determining is a structure is a truss, is being able to identify two-force members.

Two-force Members

A two-force member is any rigid body which only is subject to a pair of compression or tension forces with and no couples exerted anywhere on the body.



[caption] The two possible types of two-force members. (top) Compression pushes and (bottom) tension pulls on the ends of a two-force member.

For a two-force member to be in **equilibrium**, certain conditions need to be true about those forces. First, the two-force vectors need to be **equal and opposite** to ensure that the sum of the forces is equal to zero. Second, the force vectors need to be **collinear** to ensure that the sum of the moments are equal to zero. Specifically, the forces will share a single line of action which acts along the line connecting the two points where the forces are applied (often pins). By this definition it holds that two-force members are not required to be slender, straight members.

[insert figure of curved and bent 2 force members, one tension and another compression).

[caption] The forces in a two-force member are always share the same line of action which connects the pins on the body. Note that if two-force members are not straight, then they technically they would be required to support forces beyond compression and tension which include, internal shear forces and bending moments which are introduced in the Ch 8 Internal Loading within Rigid Bodies.

[Key point box] Equilibrium of Multi-Body Systems

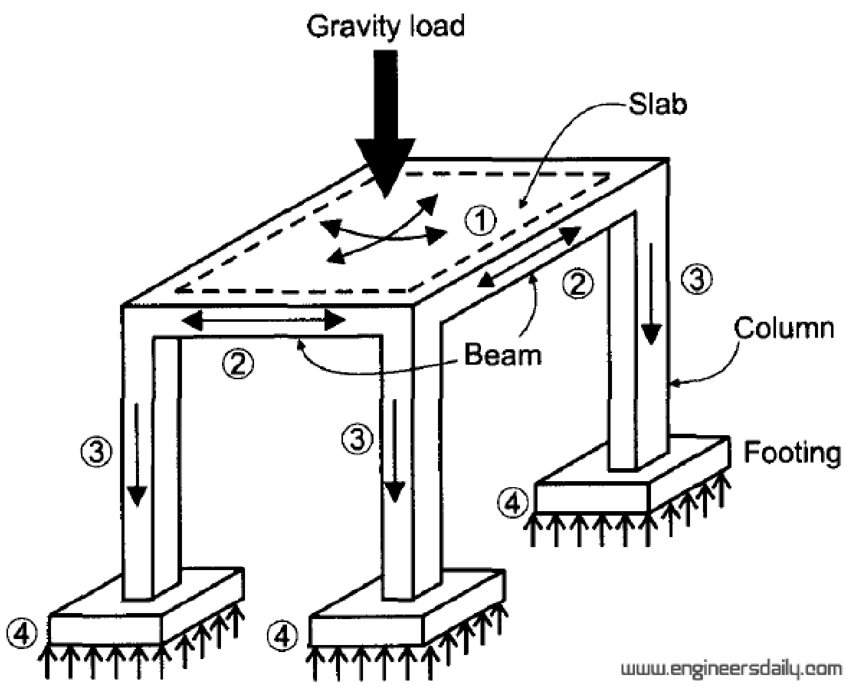
One of the governing principles which allows us to break multi-body systems in equilibrium down into smaller solvable parts is the fact that if the entire system is in equilibrium, it follows that each connection, each member, and even sections cut away from the rest of the overall system are all in equilibrium as well. This leaves us great freedom in our FBD’s to isolate the portions of the system which will expose the unknown values we wish to solve.

Interactions between members

As we are now dealing with multi-body systems we need to revisit Newton's 3rd Law (from Chapter 1), “For every action, there is an equal and opposite reaction" as we are creating multiple free body diagrams of interacting bodies. We apply law to multi-body systems as the reactions applied to one body must be transferred equal and opposite to the interacting bodies. So practically we could restate Newton's 3rd law for interaction forces, “For every force on a body, there is and equal, opposite, and collinear force acting on the connecting body." or for interaction couples "For every couple reaction on a body, there is and equal opposite couple reaction acting on the connecting body."

Thinking about Load Paths

A basic concept which can help you think about structural systems is load paths. Load paths are conceptually how an applied load (like the gravity load in the image below) are passed through the interconnected members of a structural system and end up being supported by the reactions (which in the image below are the earth beneath the footings). All structural systems, whether non-moving frames or moving machines have some sort of load path. This is also a good concept to think about for your problem solving, where we need to computationally move from our known values (often applied loads) down through the interconnected bodies of the system, solving for unknowns as you go.



Overview of Simple Trusses

Simple trusses are made purely of two-force members pinned together and organized into triangles. The benefit of simple truss design is that it is consistent with our Statics use of rigid bodies (which don't flex, stretch, or compress) and thus is independent of computations related to the bending or change in length of loaded members.

Learning outcomes: Upon completion of this section you should be able to:

1. Differentiate simple trusses from other structural systems and also identify the benefits and dangers of building with simple trusses
2. Differentiate the various types of forces acting within simple truss systems

3. For a truss in equilibrium, explain why every individual member, joint, and section cut from the truss is also equilibrium.
4. Describe the difference between tension and compression.
5. Identify zero-force members in a truss and use their presence to simplify the analysis.

Simplicity and Danger of Building Simple Trusses

Simple trusses do not rely on computation of the bending, compression, or extension of any of their members. Hence, we can assume each is a rigid body and thus design them in Statics. This simplicity also has a dark side, as simple trusses are also called fracture critical trusses as they are designed with no redundancy. Hence, if a single member failures, there is no load path for the transfer of the forces being supported by that member to hold up the bridge and it will likely collapse. You can visualize the fracture critical nature of simple trusses by thinking about a triangle with pinned corners. If you break one of the sides of a triangle, the other two sides will collapse. In a full truss made of only triangles, the collapse of one triangle starts a chain reaction as others collapse as well. While fracture critical bridges are being replaced by more robust designs, there are still thousands in service across the United States. To read more about two specific fracture critical collapses search the internet for:

- Silver Bridge Collapse Ohio River
- I-5 Bridge Collapse Skagit River

In our discussion of trusses it is worth defining the various types of forces which you will be using on truss FBD's.

[Table caption] Table XX - The table below is adapted from Table XX in Chapter 1 and highlights only the subset of forces you need to consider in truss systems. Note that due to the assumption of all two-force members in a truss system, trusses cannot support any couples and therefore the table below only includes forces.

Type of force	**Source**
Applied	Applied forces are the external forces which a rigid body or system of rigid bodies is being designed to support. They are often given in a problem and you will be asked to find the reaction and interaction forces developed by them.
Body	Body forces are gravitational weight forces of rigid bodies within a system. To maintain the assumption that trusses are formed of two-force members, all included weight forces must be split in two and distributed to the pins on either end of a truss member.
Reaction	Reaction forces come from support pins, support rollers, or support two-force members interacting with the outside world.
Interaction	Interaction forces are caused by the connection of one body with all the others it touches. Hence, when looking at the force on a pin joint, we would include all the forces coming from all members and reactions connected to that pin joint.

Internal	Internal forces are the forces developed within a rigid body. For trusses the internal forces are the compression and tension forces internal to each two-force member.

Identifying Simple Trusses

As discussed in Table 6-1 there are some rules which separate simple trusses from other systems of rigid bodies.

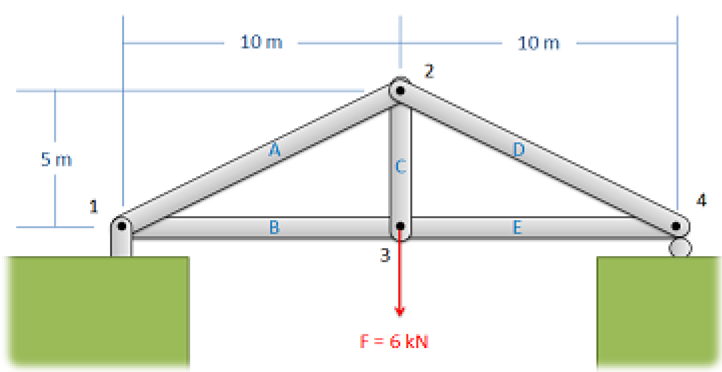
1. Simple trusses are made of all two-force members and therefore all joints are modeled as frictionless pins and all applied and reaction forces are applied to these joints.
2. Simple trusses are made of triangles, which makes them rigid when removed from supports, but also have no redundant members. The balance of equations and unknowns in a simple truss can be computed in the equation

    2\times (\text{\# of Joints}) = (\text{\# of reaction forces}) + (\text{\# of members})

$$2 \times (\text{\# of Joints}) = (\text{\# of reaction forces}) + (\text{\# of members})$$

Common Labeling Practices

After you have validated that a system can be modeled as a simple truss using the rules above, make sure that all joints and members are labeled. This will help you keep everything organized and consistent in later analysis. In this book, the joints will be labeled with letters and the members will be labeled with the joints they are connected to.



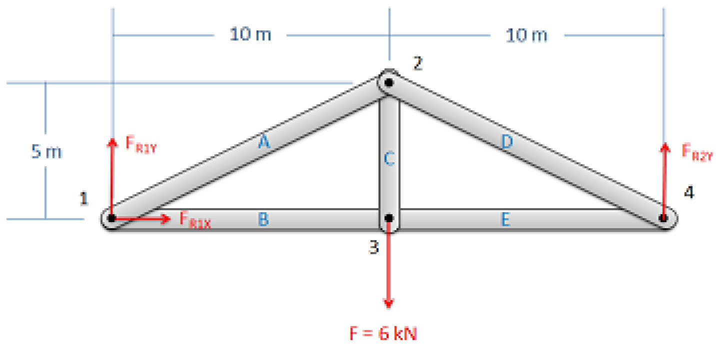
[caption] The first step in the method of joints is to label each joint and each member.

[key point box] Equilibrium of All The Parts

Trusses are in equilibrium at all spatial levels. This means that not only is the full assembly of two-force members in equilibrium, but also every individual member, joint, and even a section cut from the truss is in equilibrium as long as you have replaced all supports with reactions and added the exposed internal forces in cut members. This is because the forces inside of a truss are all in equal and opposite pairs (think about the tension forces in a 2-force member being equal and opposite pulling on each end). Hence, all interaction and internal forces which are not exposed by cutting or isolating members have an opposing force which effectively cancels them out. Only when we isolate member or joint, do we expose all the forces which come from the interaction of the surrounding bodies.

Solving for Reactions

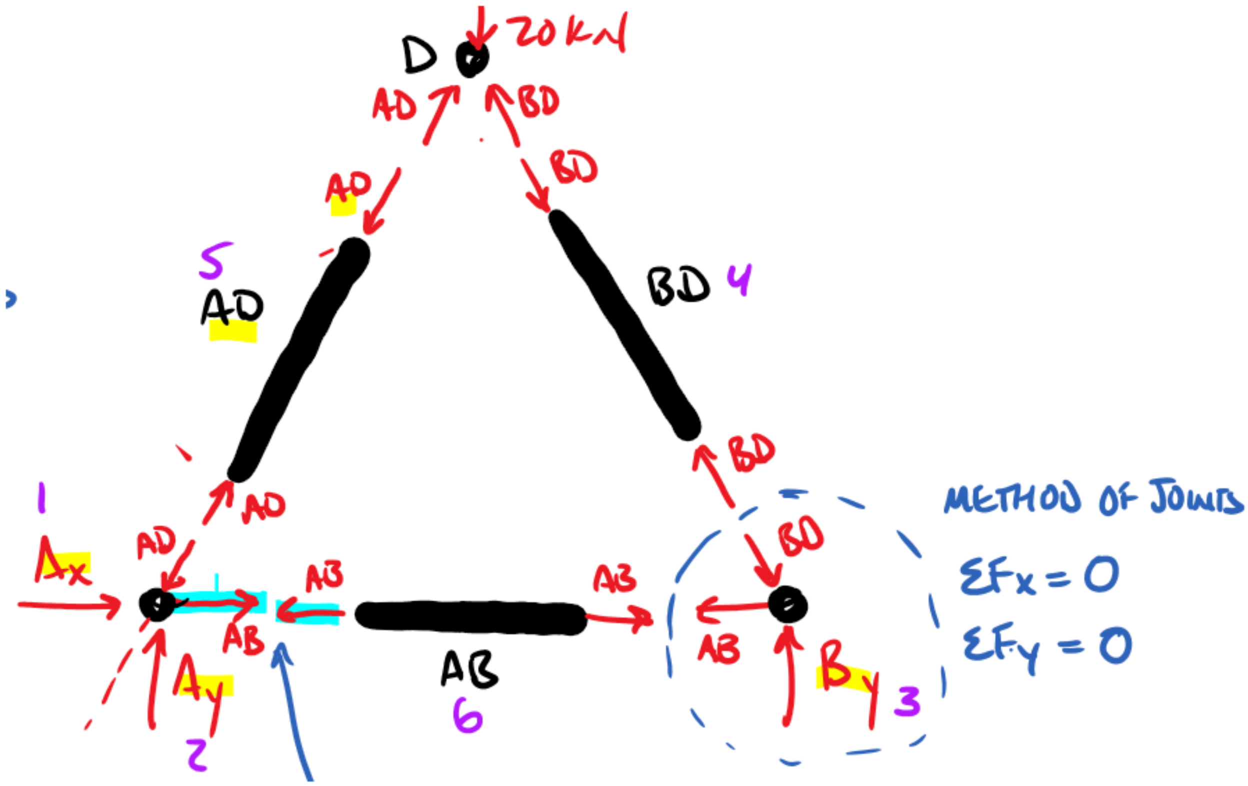
As trusses are themselves rigid bodies, you can solve for their reaction forces using a FBD of the full truss (before starting Method of Sections or Method of Joints) just like you did in Ch 5 Rigid Body Equilibrium. Hence to solve for the reactions, draw a free body diagram, write out the equilibrium equations, and solve for the external reaction forces acting on the truss structure.



[caption] Treat the entire truss as a rigid body and solve for the reaction forces supporting the truss structure.

Internal Tension and Compression in Two-Force Members

The common way to express the unknown axial force in each two-force member is to solve for the magnitude of the force as well as whether that force is putting the member in tension (T) or compression (C). To further investigate the nature of the pure axial forces of tension (T) and compression (C) let us look at the effect of tension pulling and compression pushing on each joint and member.



[caption] Notice how the tension (in AB) pulls on every member and joint, while the compression (in AC and BC) pushes on each member and joint. This diagram also helps us see that using the FBD's of two-force members we can only verify that the forces are equal and opposite, but by using the FBD's of the joints (which is detailed further in the Method of Joints below) we are able to include all reaction and applied forces as well.

When analyzing trusses, we are looking to identify the axial forces acting within each member for a given loading. In classes you take after Statics, you will be able to use these axial forces to then design the material and cross sectional properties which which will support the required loading. Later in Ch 8: Internal Loading within Rigid Bodies chapter of this book, you will learn to understand and compute the full suite of internal loadings which in addition to axial force also include shear force and bending moment.

Load Paths through Trusses

Zero-Force Members

As their name implies, zero-force members are not engaged by the loads applied to a truss. While you can find zero-force members using either Method of Joints or Sections, you can save a lot of computational time by identifying them with two simple rules

- **Rule 1:** If two members meet at an unloaded joint, then both members are zero-force members.
- **Rule 2:** If three forces (interaction, reaction, or applied forces) meet at a joint and two are collinear, then the third is a zero-force member.

- While it is probably easiest to conceptualize this rule when the third member is perpendicular to the collinear pair, it does not have to be. If you sum forces perpendicular to the collinear pair if even a component of the third member is zero, the whole member must be a zero-force member.

[insert example of truss joint + FBD of truss joint for each of Rule 1 and 2.

Key points to keep in mind regarding zero-force members include:

1. **Finding zero-force members is iterative**. Once you find some, eliminate them and go look for others.
2. **The zero-force rules above only have to apply to one end of a member** (not both) to determine that the whole member is a zero-force member. Keep in mind that all zero-force members started as two-force members which have exactly the same axial load on either end, hence if one end has zero force, the other does as well.
3. When you get rid of zero-force members, you are left with the simplest truss which will still connect the reaction and applied forces with triangles. If you misinterpret the rules you may over-eliminate members and be left with missing legs of triangles or 'floating' forces which have no load path from the remaining members.

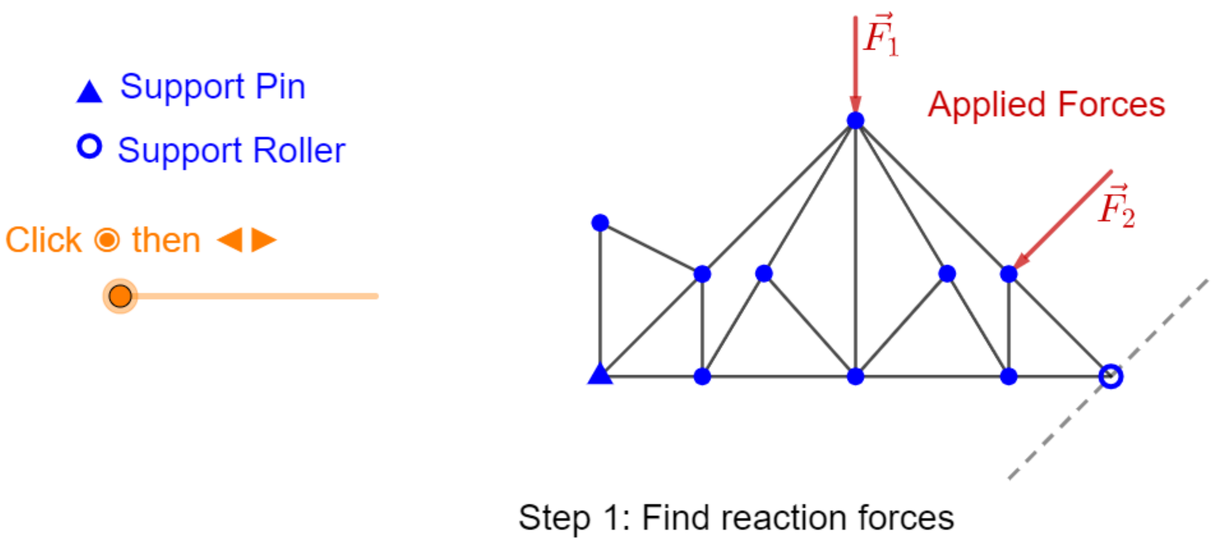


Figure FF: Interactive showing the process of finding and eliminating zero-force members. Click on the orange dot and either slide it slowly to the right or use your keyboard arrows to see each step. Source: https://www.geogebra.org/m/mjy82dtm

While finding zero-force members is a handy tool to minimize computations in Statics. In the real world, zero-force members may have minimal axial forces, but not truly 'zero force' for multiple reasons:

- The weight of members is typically ignored in Statics truss computations. Even if we distributed the member weight to the pins as required to keep the members two-force, these additional pin forces would tend to negate either rule.
- Another function of truss members is to support the members they interact with as members deform under load. When we call a member a zero force, we essentially

remove it from our analysis. One issue with this is if the member it is attached to begins buckling in compression, the zero-force member will be engaged to prevent this buckling.
- Trusses are often used over a wide array of loading conditions - think about how the position of the weight of a truck changes relative to the bridge it is driving over. Thus, while a member may have zero force for one load condition, it will likely be engaged given other loading conditions.

There are two major strategies used to identify the unknown forces in a truss, these are the Methods of Joints and the Method of Sections. The Method of Joints is usually easier if you wish to find the forces in all members, while the Method of Sections makes it easier to find the forces in a specific or specific set of members. It's also possible to mix and match methods when necessary.

The Method of Joints:

The **method of joints** is a process used to solve for the unknown forces acting on members of a **truss**. The method centers on the joints or connection points between the members, and it is useful if you need to solve for all the unknown forces in a truss structure.

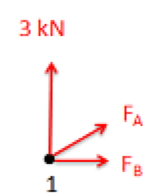
Learning outcomes: Upon completion of this section you should be able to:

1. Draw a free-body diagram of a joint in a truss.
2. Accurately pass known values of tension and compression from one joint to the next in your analysis.
3. Apply the Method of Joints to a truss to solve for unknown forces in 2D planar trusses.

Using the Method of Joints

The process used in the method of joints follows the sequence of steps below:

1. **First, determine if a truss can be modeled as a simple truss** (see Identifying Simple Trusses above)
2. **Identify and eliminate all zero-force members.** This is not required, but can minimize your computations, see Zero-Force Members above.
3. Next, determine if you need to solve for the reactions first, or if you can solve for them as part of the Method of Joints. Joints are treated as a series of equilibrium particles (which you learned about in Ch 3). As all forces on joints are concurrent, these forces can be solved with equilibrium force equations, but moment equations do not add any information. Thus each 2D joint yields two equations (<m>\sum F_x=0<\m> and <m>\sum F_y=0<\m>). To be able to solve for the unknowns at a joint in a step-by-step fashion, you'll need ≤ 2 unknowns at each joint and ≥ 1 known force at each joint. If there are no joints that satisfy this condition and you would like to solve for the unknown forces

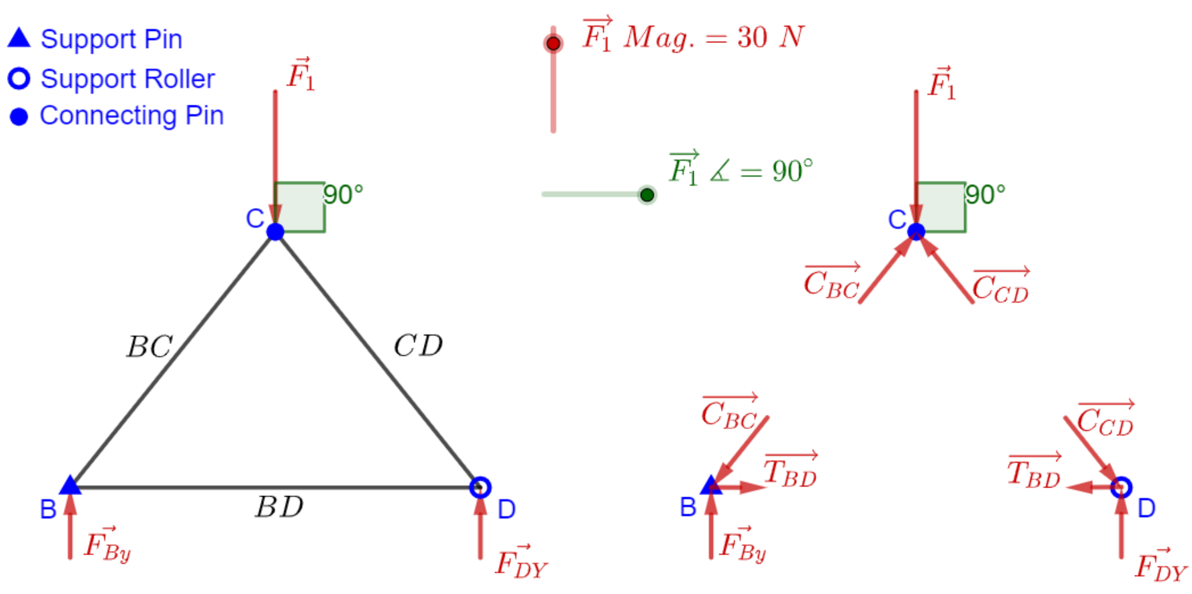


in a step-by-step manner, then you will need to solve for the truss reactions first before starting the Method of Joints.

   - Note that you could write out all the equations for each FBD and solve them all simultaneously with a linear algebra matrix solution, but only if you have a computer available as large matrices are not typically solvable with a calculator.

4. **Next, represent the forces at each joint with a free body diagram.** It is simplest to:
   - DRaw one joint FBD at a time
   - Always draw known forces in their known direction and value (whether external, reaction, or interaction),
   - Draw unknown forces in assumed directions and uniquely label them. A common practice for trusses is to assume that all unknown forces are in tension (or pulling away from the FBD of the pin) and label them based on the member they represent (like F_{AB}).
5. **Finally, write out and solve the force equilibrium equations for each of the joint FBD.** If you assumed that all forces were tensile earlier, remember that negative answers indicate compressive forces in the members.

See figure GG below to investigate how the internal tension and compression forces propagate through a simple triangular truss with a single external force <m>\\overrightarrow{F_1}.



Applied & reaction force FBD

Method of joints FBD's

Figure GG - Interactive showing how the method of joints creates FBD’s of each joint for you to solve for the internal tension and compression forces. Change the magnitude and angle ∡ of <m>\vec{F_1}<\m> to see how the reaction <m>\vec{F}<\m>, internal tension forces <m>\vec{T}<\m>, and internal compression forces <m>\vec{C}<\<m> change accordingly. If you wanted to sequentially solve this problem joint by joint you would need to meet the ≤ 2 unknowns at each joint and ≥ 1 known force at each joint criteria. Hence, you could either solve for the reactions at B and D first OR go straight into method of joints, starting at C, then moving to D, and finally to B. Source: https://www.geogebra.org/m/fvuugvb4 examples?

The Method of Sections

The Method of Sections is a more targeted process than the Method of Joints, which is used to solve for the unknown forces within a limited number of members in a truss . The method involves cutting the truss into individual sections and analyzing each section as a separate rigid body. The advantage of the Method of Sections is that the only internal member forces exposed are those which you have cut through, the remaining internal forces are not exposed and thus ignored.

Learning outcomes: Upon completion of this section you should be able to:

1. Evaluate multiple section cuts to determine which would provide the simplest solution to find the desired member forces.
2. Apply the Method of Sections to a truss to solve for unknown forces in 2D planar trusses.

Using the Method of Sections:

1. **First, determine if a truss can be modeled as a simple truss** (see Identifying Simple Trusses above).
2. **Identify and eliminate all zero-force members.** This is not required, but can minimize your computations, see Zero-Force Members above.
3. **Next, take out your imaginary chain saw and evaluate various section cuts through the entire truss.** These cuts should include some (or all) of the members you are asked to solve for. You can see various cuts in Figure XX below.
    - Note that every cut member exposes the internal axial force which exists in that member. Hence if you cut three members, you'll have three new unknowns to solve for. Exposing four (or more) members is not advised as you only have three equilibrium equations to solve for the exposed axial force unknown values.
    - Technically, the cut does not need to be a straight line nor vertical (but they often are).

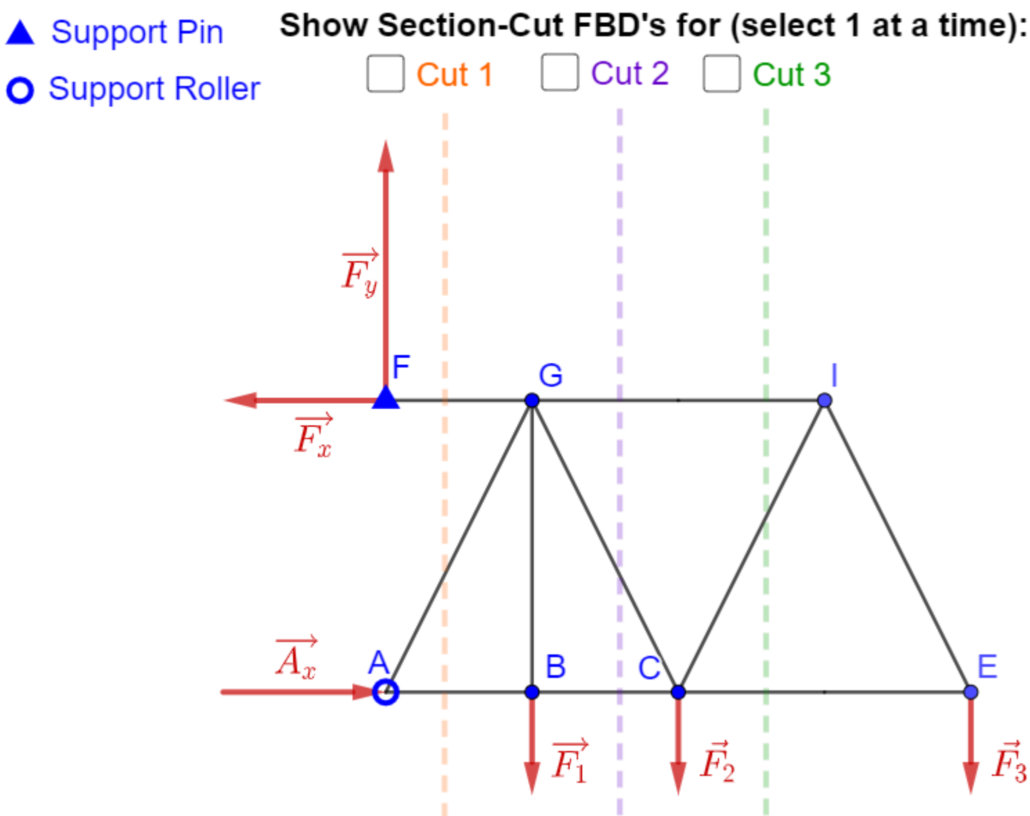


Figure XX: Interactive showing how the cantilever truss (a truss hanging off of a wall) can be cut at multiple points (1-3) With each cut new internal member forces are exposed and you could select either the FBD right or left of the cut to solve for your unknown member forces. Note tha the right section would need the reactions solved first (using the top FBD before the cut) to get down to only three unknowns, whereas the left section only has three unknowns, which could be solved for directly. Source: https://www.geogebra.org/m/s3vvnbsq

4. **Solve for reactions (if needed).** If your section FBD will include reaction forces (like the left section in Figure XX above), go back to your full section FBD and solve for the reactions you will need on your section FBD.
5. **Draw your section FBD:**
    - Always draw known forces in their known direction and value (whether external, reaction, or interaction) and make sure to include any applied and reaction force which are also applied to the portion of your truss remaining.
    - Draw unknown forces in assumed directions and uniquely label them. A common practice for trusses is to assume that all unknown forces are in tension (or pulling away from the FBD of the pin) and label them based on the member they represent (like F_{AB}).
6. **Write out and solve the force equilibrium equations for your section FBD.** If you assumed that all forces were tensile earlier, remember that negative answers indicate compressive forces in the members.
7. **If you have not solved for the required members with one section cut, then add a Method of Joints step or another Method of Sections step**. Recall that is is possible to mix and match methods when necessary.

[insert Method of Sections example(s) OR link to Jacob's two examples at
http://mechanicsmap.psu.edu/websites/5_structures/method_of_sections/methodofsections.html

Frames and Machines

Frame and **machines** are engineering structures which contains at least one multi-force member. As their name implies, multi-force members have multiple forces and/or couples applied to them and therefore are not two-force members. Note that all bodies we investigated in Ch 5 Equilibrium of Rigid Bodies were all multi-force bodies. Refer to Table 6-1 for the details of how frames and machines compare to other equilibrium systems.

Learning outcomes: Upon completion of this section you should be able to:

1. Explain the key difference between trusses and frames/machines, including why you cannot use the method of joints for frames and machines
2. Explain how a frame is different than a machine

Defining Frames vs Machines

By definition:

- **Frames** are structures designed to stay in one place and not move.
- **Machines** are non-rigid structures where the parts can move relative to one another. Note that all machines in this course will still be held in static equilibrium by interacting and applied forces.

Though there is a difference in vocabulary in describing frames and machines, they are grouped together here because we use the same process to analyze both of these structural systems.

This stool contains non-two force members (the legs) and no part can move relative to the other parts (it is rigid). Therefore this is a frame.

This pair of locking pliers contains non-two force members and has parts that can move relative to one another (it is not rigid). Therefore this is a machine

Analyzing Frames and Machines

When we talk about analyzing frames or machines, we are usually looking to identify both the external applied and reaction forces/couples acting on the structure and the interaction forces/couples acting between members within the structure.

The method we use to analyze frames and machines centers around the process of breaking the structure down into individual rigid bodies. Where the components are connected, Newton's Third Law states that each body will exert an equal and opposite force on the other body. Thus, each component will be analyzed as an independent rigid body leading to equilibrium equations for each component, and due to Newton's Third Law, some interaction forces/couples may show up acting on two bodies. All of these interaction forces/couples need to be represented as being equal and opposite. Hence the force of body A on B is equal and opposite to the force of body B on body A.

Equations and Unknowns for Frames and Machines

Remember from Chapter 5 Rigid Body Equilibrium how you were limited to three independent equations per rigid body FBD? It turns out that relationship still holds here, but now we have more FBD’s to use. Here’s a few more details on the number of equations that come from each type of two-dimensional FBD:

- **Two-force members = 1 equation:** Recall that two-force members can be recognized as either a cable or a weightless link with all forces coming from two frictionless pins.

The available equation simply tells you that the force at one pin is equal and opposite to the force on the other placing the body in tension or compression.

- **Multi-force rigid body with concurrent forces = 2 equations:** These are the same type of problems you solved in Ch. 3: Equilibrium of Particles with only two equations available $\sum Fx=0$ and $\sum Fy=0$.
- **Multi-force rigid body with offset forces and/or couples = 3 equations:** These are the most general body types where you can use $\sum Fx=0$, $\sum Fy=0$, and $\sum Mpt=0$ to solve for three unknowns.

[key point box] Why Method Joints works on Trusses, but Does Not Work on Frames and Machines

When using the method of joints, we had to make the assumption that all members in the truss were two-force members. Now dealing with the multi-force members in a frame and machine by cutting through a multi-force member, you would be exposing

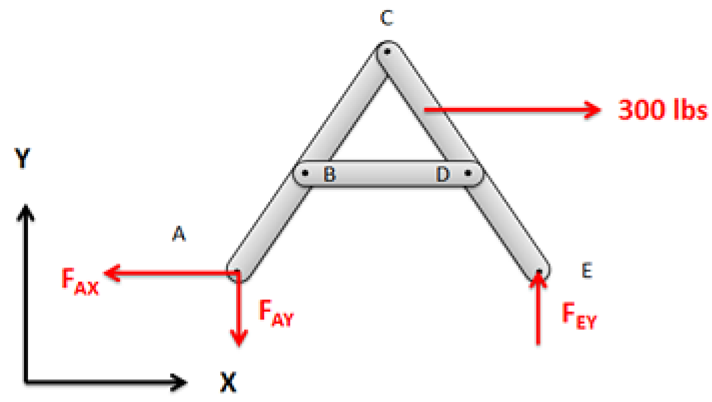
- an axial force perpendicular to your cut which is not constant throughout a multi-force rigid body (whereas the axial force in a two-force member was exactly the same from pin to pin)
- a shear force which would lie parallel to your cut. We will learn about shear forces later in Ch 8: Internal Loading within Rigid Bodies, but you do not have the tools to consider them now.
- a bending moment across your cut. We will also learn about bending moments later in Ch 8: Internal Loading within Rigid Bodies, but you do not have the tools to consider them now.

Hence, you would be exposing more three external unknown terms in every body that you cut through and your number of unknowns would quickly eclipse your number of equations rendering the problem impossible to solve. Bottom line: use method of sections and joints ONLY for trusses made of two-force members, but for all other multi-force rigid body systems, use FBD's of rigid bodies.

The Process Used to Analyze Frames and Machines:

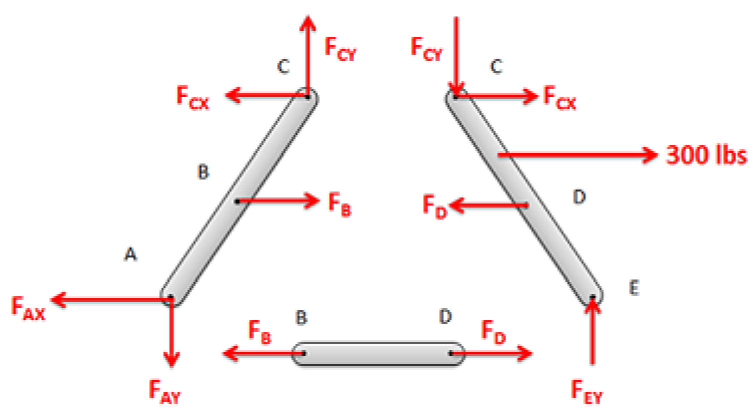
The process used to analyze frames and machines is outlined below:

1. First, determine if we can model the entire structure as a rigid body. To do this, the structure needs to be independently rigid. This means that it would be rigid even if we separated it from its supports. Hint, look for triangles formed among the members, as triangles are an inherently rigid structure. If the structure is independently rigid, then you will be able to model the structure as a single rigid body to determine the reaction forces acting on the structure. If the structure is not independently rigid then skip this step.



[caption] If, and only if, the structure is independently rigid, you should analyze the whole structure as a single rigid body to solve for the reaction forces. Note the triangle formed between pins B, C, and D.

2. Next you will draw a free body diagram for each of the members in the structure. You will need to include all forces acting on each member, including:
    - All applied forces/couples that may be acting at the members.
    - Interaction forces from two-force members. Recall, at connection points with two-force members there will be single force of unknown magnitude but known direction (the forces will act along the line between the two connection points on the member).
    - Next, add in the remaining reaction forces/couples at the connection points between members. For forces with an unknown magnitude and direction (such as in pin joints) the forces are often drawn in as having unknown x and y components.
    - Remember that the interaction forces/couples between connected bodies will be equal and opposite.



[caption] Figure XX: Separate the structure into individual components and draw a free body diagram of each component. It is important to remember that the forces at each connection point are equal and opposite.

3. Write out the equilibrium equations for each FBD.
4. Finally, solve the equilibrium equations for the unknowns. You can do this algebraically, solving for one variable at a time, or you can use matrix equations to solve for everything at once. If your algebra tells you that any force/couple is negative, that indicates that the assumed direction of that term should be flipped in direction.

Ch. 9: Friction will pick up this topic again, with the added interaction terms of static and kinetic friction between bodies. This will add to the forces we have to consider, but also add another equation (F=μN) to the solution set.

Chapter Review: Looking across the various types of equilibrium

We now have covered all of the equilibrium types in the table below. You should be able to compare and contrast the various types of equilibrium, draw the FBD's associated with each, and know which equations allow you to solve for the unknowns of each case.

[caption] **Table 6-xx:** This table compares the various types of equilibrium topics, plus solving techniques and number of equations available for each.

System Type	Definition	Technique to Solve	Equations and Unknowns
Single Particle (Ch 3)	A single particle where the external forces, body forces, and reactions are all collinear and in equilibrium	Collinear forces allow us to isolate particle as FBD (cutting axial forces in 2-force supports) and $\Sigma\mathbf{F}=0$ (Note: $\Sigma\mathbf{M}=0$ not helpful as forces do not create moment about particle)	2D→2 $\Sigma\mathbf{F}=0$ equations = 2 unknowns 3D→3 $\Sigma\mathbf{F}=0$ equations = 3 unknowns
Single Rigid Body (Ch 5)	A multi-force member where external loading and reactions are in equilibrium	Variety of forces and couples (from loading and reactions) require FBD of rigid body and apply $\Sigma\mathbf{F}=0$ and $\Sigma\mathbf{M}=0$ equations.	2D→2 $\Sigma\mathbf{F}=0$ + 1 $\Sigma\mathbf{M}=0$ equations** = 3 unknowns 3D→3 $\Sigma\mathbf{F}=0$ + 3 $\Sigma\mathbf{M}=0$ equations = 6 unknowns
Trusses (Ch 6) 	An engineering structure that is **rigid as a whole,** is assumed to **consist entirely of two-force members**, and only carries forces at joints between members	Two-force members and loading at joints allow FBD of joints to expose axial loads in members	2D→(2 $\Sigma\mathbf{F}=0$ equations)(# joints) = (# reactions) + (# members) 3D→(3 $\Sigma\mathbf{F}=0$ equations)(# joints) = (# reactions) + (# members)

Frames **(Ch 6)** 	An engineering structure that is **rigid as a whole**, but contains at least one member that is **not a two-force member**.	Mix of two-force and multiforce bodies require require FBD's of rigid bodies where interactions between bodies are equal and opposite (as per Newton's 3rd Law). Can apply Σ**F**=0 and Σ**M**=0 equations for each body to solve for total unknowns.	2D→(2 Σ**F**=0 + 1 Σ**M**=0 equations per multiforce body) + (1 Σ**F**=0 equation per two-force body) 3D→(3 Σ**F**=0 + 3 Σ**M**=0 equations per multiforce body) + (1 Σ**F**=0 equation per two-force body)
Machines **(Ch 6)** 	An engineering structure that is **not rigid as a whole**, and contains **at least one member that is not a two-force member**.		

** Other combinations of equilibrium equations are possible see Ch. 5 Rigid Body Equilibrium Section XX for details