Tab 1

Multilingual Learner Math Toolkit of Strategies

Acknowledgments

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California English Language Development Standards

Figure 3.4

Part I: Interacting in Meaningful Ways	
Collaborative (mode of communication)	
1.	Exchanging information and ideas with others through oral collaborative discussions on a range of social and academic topics
2.	Interacting via written English in various communicative forms, including technology
3.	Offering and justifying opinions, negotiating with and persuading others in communicative exchanges
4.	Adapting language choices to various contexts based on task, purpose, audience, and text type
Interpretive (mode of communication)	
5.	Listening actively to spoken English in a range of social and academic contexts
6.	Reading closely literary and informational texts and viewing multimedia to determine how meaning is conveyed explicitly and implicitly through language
7.	Evaluating how well writers and speakers use language to support ideas and arguments with details or evidence depending on modality, text type, purpose, audience, topic, and content area
8.	Analyzing how writers and speakers use vocabulary and other language resources for specific purposes (to explain, persuade, entertain, etc.) depending on modality, purpose, audience, etc.
Productive (mode of communication)	
9.	Expressing information and ideas in formal oral presentations and academic topics
10.	Composing/writing literary and informational text to present, describe, and explain ideas and information, using appropriate technology
11.	Supporting own opinions and evaluating others' arguments in speaking and writing
12.	Selecting and applying varied and precise vocabulary and language structures to effectively convey ideas

Part II: Learning About How English Works	
Structuring Cohesive Texts (mode of communication)	
1.	Understanding text structure and organization based on purpose, text type, and discipline

Multilingual Learner Math Toolkit of Strategies

2.	Understanding cohesion and how language resources across a text contribute to the way a text unfolds and flows
Expanding and Enriching Ideas (mode of communication)	
3.	Using verbs and verb phrases to create precision and clarity in different text types
4.	Using nouns and noun phrases to expand sentences and ideas and provide more detail
5.	Modifying to add details and provide more information and create precision
Connecting and Condensing Ideas (mode of communication)	
6.	Connecting ideas within sentences by combining clauses and joining ideas
7.	Condensing ideas within sentences using a variety of language resources including nominalization

Adapted from the ELD Standards

Instructional Strategies

Number Talk

What Is this Strategy?

Students participate in a short instructional routine focused on developing and articulating efficient mental math strategies, fostering a deeper understanding of mathematical concepts and improving number sense. They solve problems mentally, share their strategies, and engage in meaningful discourse about different approaches. The teacher supports this process by recording strategies, asking probing questions, and creating a safe environment where students feel comfortable exploring and explaining their thinking.

Why Use This Strategy?

Develop Conceptual Understanding: Helps students understand numbers and operations beyond rote memorization.

Encourage Flexible Thinking: Builds students' ability to think flexibly about numbers, use and compare multiple strategies.

Promote Discourse: Provides opportunities for students to explain their thinking and listen to others, strengthening communication and reasoning skills.

Boost Confidence: Encourages risk-taking in a safe environment, helping students gain confidence in their mathematical abilities.

When to Use This Strategy?

The *Number Talk* routine should be used regularly, ideally as a warm-up or starter activity, to set the tone for a math lesson and activate students' thinking. It is particularly effective when introducing new concepts, reviewing foundational skills, or helping students build fluency with operations and number

relationships. Additionally, it can be used as formative assessment to gauge students' understanding and identify gaps in their mental math strategies.

How Do I Plan For this Strategy?

- Choose a problem
  - that aligns to current or past learning
  - that can be solved mentally using a multiple of strategies

- Prepare questions to ask students:
  - How would you solve this problem?
  - What are other ways to solve this problem?
  - How does one strategy compare with another?
  - How are they different? How are they similar?

- Prepare for recording of student ideas and language supports
  - Anticipate student responses and how to script their thinking to display for the whole-class
  - Utilize sentence frames for students to communicate thinking
  - Define or use pictures for specific vocabulary

Steps

1. **Present the Problem**
   a. Write a purposeful computation problem on the board.
   b. Tell students to use hand signals to show:
      i. I'm thinking - indicated by students making a closed fist in front of their chest.
      ii. I have a strategy - indicated by students holding up their thumb.
      iii. I have another strategy - indicated by students holding up a finger in addition to their thumb.
      iv. I agree with another's thinking - indicated by students waving their hands from side to side while holding up their thumb and pinky finger.
   c. The teacher may model or utilize graphics to show each of the hand signals.

2. **Think Time**
   a. Provide appropriate think time by monitoring students' hand signals.
   b. Optional- *have students share their thinking/strategy with a partner to articulate/practice language.*

3. **Share Answer/Solutions**
   a. Accept, respect, and consider all answers. Ask students for their solutions: the numerical answer to the problem.
   b. Record their responses on the board.

c. Refrain from evaluating the responses. Avoid comments such as “that’s correct,” or “think about that some more.”

4. **Share Strategy/Thinking & Encourage Justification**
    a. Choose students to share their strategy/thinking.
    b. Record students' responses.
    c. Prompt students to explain and justify their own or each other’s thinking/strategy.

5. **Compare Strategies**
    a. Highlight similarities, differences, and connections between strategies.
    b. Encourage students to use the hand signal to show agreement.
    c. Sample sentence frames for students:
        i. I agree with _____ because ____. I do not understand _____. Can you explain this again? I disagree with ____ because ____. How did you decide to _____?

**Optional- provide related math problems for students to apply one or more of the strategies.*

6. **Reflect and Close**
    a. Conclude the number talk using one of the options:
        i. Summarize the key mathematical ideas or strategies.
        ii. Students reflect on how they behaved as a mathematician.
        iii. Students reflect on the strategies and choose efficient strategies for specific problems.

Example

1. **Present the Problem**
   "Alright, everyone," here’s the challenge: (Teacher writes 45 + 36 on the board.) Take a moment to think about it in your head—no pencils, no calculators. Just you and your brain."

2. **Think Time**
    a. **Teacher:** *“Use your hand signals when you have an answer. Remember there isn’t just one way to solve this problem. See if you can come up with more than one strategy”.*

3. **Share Answers/Solutions**
    a. **Teacher:** *“Let’s hear from some of you. What did you get?” (Teacher writes responses on board)*
    b. **Student 1**: *“I got 81.”*
    c. **Student 2:** *“I got 81 too.”*
    d. **Teacher:** *“Did anyone get a different answer”?*

4. **Share Strategy/Thinking**
    a. **Teacher:** “*Who is ready to defend/justify their thinking”.*

b. **Student 1:** *"I got 81. I broke the numbers apart: I added 40 + 30 to get 70, then added 5 + 6 to get 11, and finally added 70 + 11 to get 81."*
c. **Student 2:** *"I also got 81, but I did it differently. I rounded 36 up to 40 by adding 4. Then I added 45 + 40 to get 85 and subtracted the 4 to get 81."*

5. **Encourage Justification**
   a. **Teacher:** *"Can someone explain why both of these strategies work? Do you see a connection between them?"*
   b. **Student 3:** *"Both methods are breaking the numbers into smaller parts to make them easier to add. One used rounding, and the other used place value."*

6. **Compare Strategies**
   a. **Teacher:** *"Which method felt easiest for you? Did one of these strategies change the way you thought about the problem?"*
   b. Students share their preferences and reasons, emphasizing mathematical reasoning.

7. **Reflect and Close**
   a. Teacher: *"Great work, everyone! We saw two strong strategies today: breaking numbers into place values and rounding to make the math simpler. Let's take a moment to reflect."*
   b. Teacher: *"In your math journals, write down the strategy you liked best and why. Also, think about how you might use these strategies in the future."*

Source:

Parrish, S. (2010). *Number Talks: Helping Children Build Mental Math and ComputationStrategies, Grades K-5 [with Dvd].* Math Solutions.

Some Considerations for Different English Proficiency Levels
Note: See section titled "Scaffolding" for more information on how to scaffold for different proficiency levels.

Emerging	Expanding	Bridging
● Provide sentence frames at their proficiency level. ● Allow students to share their ideas in their primary language and then translate their ideas into English. ● Allow students to use gestures to aid in expressing their ideas. ● Student responses may be brief (one word responses or short	● Provide sentence frames at their proficiency level. ● Allow students to use gestures to aid in expressing their ideas. ● Students can apply their understanding of sequence to explain how to solve a problem. ● May need moderate support when speaking/presenting.	● Provide sentence frames at their proficiency level. ● Students can begin offering justifications in a logical order for their opinions. ● May need light support when speaking/presenting.

phrases) ● May need moderate support when speaking/presenting.		
Develop essential academic vocabulary for the concept and allow students to practice using it.		

Adapted from *California English Language Development Standards*, ELD Proficiency Level Continuum.

Integrated ELD

The focus will be on the content.

Sample Language Objective

- Students will be able to orally share how to solve a mathematical problem - using transitional words and steps-in-a-process language structures.

Standards Connections (refer to the Figure 3.4 for more detail on the standards):

- Collaborative Mode: Students will exchange information and ideas with others through oral collaborative discussions (by orally sharing how they would solve a problem).
- Collaborative Mode: Students will offer and justify their opinions (by stating which strategy they feel is best and supporting it with reasons).
- Interpretive Mode: Students will listen actively to spoken English (when listening to other students' responses).
- Interpretive Mode: Students will evaluate how well speakers use language to support ideas (when listening to the arguments of others).
- Productive Mode: Students will support their own opinions and evaluate the arguments of others (by sharing their opinions and reasons as well as by listening to the opinions and reasons of classmates).

Supporting Language Development

- Provide differentiated sentence frames.
- Think-Pair-Share
- Total Physical Response (TPR)
- Visuals
- Think Time

Designated ELD

The focus will be on how language works.

- **Into:**
  - **Sample Language Objective:**
    - Students will be able to orally share how to solve a mathematical problem - using transitional words and steps-in-a-process language structures.

    - **Supporting Language Development**
        - Begin with providing a sentence frame that the student can use to state a claim (ex: I think ____.). Then have the students provide their justification with a focus on transitional words (first, next, then, last, so).
        - Begin with providing a sentence frame that the student can use to agree or disagree (I agree with ___.). Then expand to "I agree with ____ because _____."
- **From:**
    - **Sample Language Objective:**
        - Students will orally tell how they solved a mathematical problem - adding details to improve clarity and precision.

    - **Supporting Language Development**
        - Begin with providing a sentence frame that the student can use to agree or disagree (I agree with ___.).
        - Then expand to "I agree with ____ because _____."
            - Continue working at the students' pace to expand to more in depth responses such as:
                - "I disagree with ____. I arrived at a different answer. First, I _____. Then I _____. Next I _____. Finally I arrived at the conclusion of _____. "

Formative Assessment

Using the ELD Proficiency Level Continuum in the CA ELD Standards…..

Integrated

- check that students can exchange information with others orally (ELD.PI.1).
- check that students can offer and justify their opinion (ELD.PI.3).
- check that students can view multimedia closely (ELD.PI.6).
- check that students can listen actively to spoken English. (ELD.PI.5).
- check that students can support their own opinions and evaluate the opinions of others (ELD.PI.11).

Designated

- check that students understand cohesion (ELD.PII.2).
- check that students can connect ideas within sentences (ELD.PII.6).

K-5 Resources and Videos

Coming Soon

6-12 Resources and Videos

Coming Soon

Which One Doesn't Belong?

What Is This Strategy?

Students are shown a group of four or more images (e.g., shapes, numbers, equations, graphs, etc.). They consider "Which one doesn't belong?" and justify their choice, with no wrong answers. A vocabulary list may be provided to guide their thinking. As an extension, students can explore why each image might not belong. This activity supports critical thinking, sharing ideas, learning from others, and using justification language.

Why Use This Strategy?

Encourages Explanation and Justification: Students practice explaining their thinking and providing reasons to support their ideas, promoting critical thinking and reasoning skills.

Open-Ended Engagement: Carefully selected prompts allow multiple interpretations and solutions, ensuring accessibility for all learners and encouraging diverse perspectives.

Focus on Mathematical Language: Emphasizes the use of relevant vocabulary to help students articulate their reasoning clearly and develop academic language skills.

Promotes Reasoning Over Right Answers: Shifts the focus from finding the "right" answer to justifying thinking, fostering confidence and deeper conceptual understanding.

When to Use This Strategy?

WODB may be utilized at different stages of learning to engage students in discourse and justification:

- Introduction of a new concept: launch the learning by providing students an accessible entry point to use their existing knowledge and language.
- Formative assessment during or after learning a new concept: students may be given a list of vocabulary words to use as they justify their choice for which one does not belong. WODB serves as formative assessment to assess how students are applying their learning to justify. Direct students to articulate a rule that the three images follow and the fourth one does not.
- Spiral review for a previous concept: an intentional WODB paired with a list of vocabulary words can support retention of learning as students apply and refresh their prior knowledge to justify their choice for which one doesn't belong.

How Do I Plan For This Strategy

- Choose and present 4 (or more) images to students and ask, “Which one doesn’t belong?”
    - Aligned to a previous or current learning goal
    - Open-ended so that students may select any one image
- Facilitate students discussion
    - Anticipate justification students may provide for any of the four images
    - Emphasize to students that there is reasoning and justification for why each image may not belong
- Prepare language supports
    - Create a list of relevant vocabulary to display for students
    - Prepare Sentence Frames
        - _____ doesn’t belong because _____.

Steps

Step 1: Present 4 images to students and ask, “Which one doesn’t belong?”
- Prompt students to use the vocabulary list as they think about their choice for which one does not belong.
- Students think independently by selecting one image that does not belong and why.

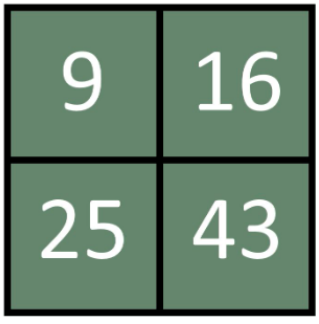
Step 2: Facilitate pair share
- Students pair-share about the image they chose and justify why they chose it.

Step 3: Facilitate class discussion
- Call on volunteers and/or randomly call on students to share their choice for which one doesn’t belong and why using a sentence frame.
- Check off or tally the image students choose.
- Prompt students to provide reasoning and justification for their choice of which one doesn’t belong.
- Check off or tally the vocabulary words students use as they justify.

Optional Step 4: Extend students’ thinking
- Ask students to think about a rule for why two or more of the images belong together and other images do not.
- This step encourages students to focus on all four images and not just the one they chose for which one doesn’t belong.

Example

Step 1: Present the image shown along with the question, "Which one doesn’t belong?” Students think independently.



Step 2: Tell students to take turns sharing their thinking with a partner. Provide the sentence frame for students: "I think ____ doesn't belong because _____."

Step 3: Call on volunteers to share their thinking with the whole class. Encourage students to use the sentence frame to share.

Student 1: *"I think 9 does not belong because it is the only one-digit number."*

Teacher: "*Did we all hear that? Can anyone repeat what you just heard in your own words?"*

Student 2: "*I heard that 9 does not belong because it only has a ones digit and no tens digit like the other numbers."*

Teacher: "*Who chose 9 as their number that doesn't belong? Show me with a thumbs up."*

Several students put their thumbs up.

Teacher: "*Did anyone have a different reason for why 9 doesn't belong?"*

The teacher is having students revoice one of their peer's ideas. Having students revoice ideas is an important talk move to encourage students to listen to and understand others' thinking.

Step 4: Extend students' thinking by asking them to justify why the three numbers they did not choose belong together.

Student 3: *"I chose 43 as a number that does not belong. The other three numbers belong because they are all perfect squares."*

Student 4: *"I chose 9 as the number that doesn't belong. The other three numbers belong because the ones and tens digit in each number adds up to 7."*

Source:

Danielson, C. (2019). *Which one doesn't belong? : playing with shapes*. Charlesbridge.

Some Considerations for Different English Proficiency Levels

Note: See section titled "Scaffolding" for more information on how to scaffold for different proficiency levels.

Emerging	Expanding	Bridging
● Provide sentence frames at their proficiency level. ● Allow students to share their ideas in their primary language and then translate their ideas into English. ● Allow students to use pictures, drawings and	● Provide sentence frames at their proficiency level. ● Allow students to use pictures, drawings, and gestures to aid in expressing their ideas. ● Students will be able to explain the images in terms of categories in	● Provide sentence frames at their proficiency level. ● Students can begin offering justifications in a logical order for their opinions. ● May need light support when speaking/presenting.

gestures to aid in expressing their ideas. ● Students may be able to state and describe how two or more of the images do or do not go together. ● Student responses may be brief (one word responses or short phrases). ● May need moderate support when speaking/presenting.	greater detail. ● May need moderate support when speaking/presenting.	
Develop essential academic vocabulary for the concept and allow students to practice using it.		

Adapted from the ELD Proficiency Level Continuum in the California English Language Development Standards.

Integrated ELD

The focus will be on the content.

Sample Language Objective

- Students will be able to read information by viewing multimedia to determine how meaning is conveyed - using categorizing and classifying language structures.

Standards Connections (refer to the Figure 3.4 for more detail on the standards):

- Collaborative Mode: Students will exchange information and ideas with others through oral collaboration (by orally comparing and contrasting images).
- Collaborative Mode: Students will offer and justify their opinions, negotiating with and persuading others in communicative exchanges (by orally stating which image does not belong and justifying it with reasons).
- Interpretive Mode: Students will listen actively to spoken English in a range of academic contexts (by listening to other students' responses).
- Interpretive Mode: Students will view multimedia to determine how the meaning is conveyed (by determining how to categorize the images).
- Interpretive Mode: Students will evaluate how well speakers use language to support arguments with details or evidence (by listening to the responses and arguments of others).
- Productive Mode: Students will support their own opinions and evaluate the arguments of others (by orally stating their opinion and evaluating the arguments and reasons of classmates).

Supporting Language Development

- Provide differentiated sentence frames.
- Think-Pair-Share
- Visuals
- Total Physical Response (TPR)

- Drawings

Designated ELD

The focus will be on how language works.

- **Into:**
    - **Sample Language Objective:**
        - Students will be able to read information by viewing multimedia to determine how meaning is conveyed - using categorizing and classifying language structures.

    - **Supporting Language Development**
        - Teach different ways to express comparing and contrasting. (ex: similar, same, different, unlike).
            - Include instances of nominalization (ex: different, differences, differentiation).
        - Begin with providing a sentence frame that the student can use to state a claim (ex: I think ____ doesn't belong.). Then work with the students to have them provide their justification. (ex: It does not belong because _____.).
            - An additional piece that could be added as needed would be to discuss contractions (does not → doesn't) and how they can be used for varied language.
        - Begin with providing a sentence frame that the student can use to state a claim (ex: I think ____, _____, and _____ belong together.).
            - Focus on subject verb agreement (The circle, triangle, and square **belong** together. The cube **belongs** by in a different category.)
            - Call attention to commas in a series.
            - Continue by having students justify their answers.
                - Focus on having students add details to increase the precision and clarity of their responses.
        - Begin with providing a sentence frame that the student can use to agree or disagree (I agree with ___.). Then expand to "I agree with ____ because _____."
- **From:**
    - **Sample Language Objective:**
        - Students will orally share their ideas on how they categorized and classified images - using conjunctions and other connective language structures.

        - **Supporting Language Development**
            - Begin with providing a sentence frame that the student can use to state a claim (ex: I think ____ doesn't belong.). Then work with the students to have them provide their justification. (ex: It does not belong because _____).
            - Begin with providing a sentence frame that the student can use to state a claim (ex: I think ____, _____, and _____ belong together.).
                - Continue by having students justify their answers.
                    - Focus on having students add details to increase the

precision and clarity of their responses.

- Begin with providing a sentence frame that the student can use to agree or disagree (I agree with ___.). Then expand to "I agree with ____ because _____."
    - Share other words and phrases that could be used instead of agree/disagree to vary word choice.
- Work with students to practice combining sentences or clauses to make compound/complex sentences. (ex.: I think the circle, triangle, and square belong together. The circle, triangle, and square are all 2 dimensional shapes. →I think the circle, triangle and square belong together because they are all 2 dimensional shapes.)

Formative Assessment

Using the ELD Proficiency Level Continuum in the CA ELD Standards…..

Integrated

- check that students can exchange information with others orally (ELD.PI.1).
- check that students can offer and justify their opinion (ELD.PI.3).
- check that students can listen actively to spoken English (ELD.PI.5).
- check that students can view multimedia closely (ELD.PI.6).
- check that students can evaluate how well speakers use language to support their ideas and arguments (ELD.PI.7).
- check that students can support their own opinions and evaluate the opinions of others (ELD.PI.11).

Designated

- check that students understand cohesion (ELD.PII.2).
- check that students are able to add details to provide more information and create precision (ELD.PII.5).
- check that students can connect ideas within sentences (ELD.PII.6).
- check that students can condense ideas including using nominalization (ELD.PII.7).

K-5 Resources and Videos

Coming Soon

6-12 Resources and Videos

Coming Soon

Stronger and Clearer Each Time

What Is This Strategy?

Students revise and refine their ideas in verbal and written form (Zwiers, 2014). Students begin by individually thinking about and writing a response to a question. They then pair up to share their ideas, switching partners one or more times. With each successive pairing, students refine and clarify their responses through conversation, incorporating new ideas and language. Finally, students revise their original written responses by incorporating new ideas and language they gained through the multiple pair shares.

Why Use This Strategy?

Build on and improve rough draft thinking: students revisit and refine their ideas by giving each other feedback leading to deeper understanding.

Clear individual and collaboration expectations: the routine is structured for students to listen to and build on each other's ideas. After the pair share, students utilize the new learning to revise their individual responses.

When to Use This Strategy?

This strategy may be used during or after the learning of a new concept. During the learning, this strategy serves as formative assessment for both students and the teacher to know how the written response is becoming stronger and clearer with each successive pair share. After the learning, this strategy serves as practice for students to use the language and knowledge they have gained and continue refining it with each successive pair share.

How Do I Plan For This Strategy

- Choose a prompt which guides students to think about a concept or big idea that they can write about.
- Optional: choose or create visuals to accompany the prompt. This will cut down on the need to explain the prompt.
- Provide the relevant phrases, vocabulary words and/or sentence frames for students to use as they write their first draft response to the prompt.
- Prepare to facilitate structured pair sharing three or more times with students switching partners each time.

Steps

1. Provide students with a prompt/question.
2. Provide students with individual think time to write their response or talking points.
3. Ask students to share their ideas with partners.

4. Partners provide each other with feedback to improve their responses.
5. Each student collects information from their partner to add to their response.
6. Students share the refined response with new partners, so that their responses are "Stronger and Clearer Each Time."
7. Repeat steps 3 through 6 as necessary.
8. Students write their final draft based on new learning during successive pair shares. Students may also include pictures or other representations.

Example

1. Provide the prompt.
   Darla decides to buy a sports drink. Her choices are a 20-ounce bottle for $1.49 or a 32-ounce bottle for $2.49. Which is the better value? Explain what you did to get your answer and why.

2. Students respond to the prompt in writing.
   Student's response: "*I think the 32 ounce is a better deal because you get 12 more ounces for a dollar more."*

3. Students share their responses with the first partner

4. The first partner gives feedback. Include dialogue about the feedback

5. Students write a revised response.
   Student's revised response: *"I want to find how many ounces I can get for a dollar and compare them. So I have to divide 20 by 1.49 and 32 by 2.49."*

6. Students share again with a second partner.

7. Students revise their response again based on feedback.
   Student's second revised response: *"First I found how many ounces I can get for a dollar. I get 13 ounces for a dollar by dividing 20 by 1.49. I got 12 ounces for a dollar by dividing 32 by 2.49. The 20 ounce bottle is a better deal because you get 1 more ounce for a dollar."*

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Some Considerations for Different English Proficiency Levels *Note: See section titled "Scaffolding" for more information on how to scaffold for different proficiency levels.*		
Emerging ● Provide sentence frames at their proficiency level. ● Allow students to share their ideas in their primary language and then translate their ideas into English. ● Allow students to use pictures, drawings and gestures to aid in expressing their ideas. ● Student responses may be brief (one word responses or short phrases) ● May need moderate support when speaking/presenting.	**Expanding** ● Provide sentence frames at their proficiency level. ● Allow students to use pictures, drawings, and gestures to aid in expressing their ideas. ● May need moderate support when speaking/presenting.	**Bridging** ● Provide sentence frames at their proficiency level. ● Students should be able to describe their ideas using key details. ● Students should be able to use varied vocabulary and words with similar meanings to convey ● May need light support when speaking/presenting.
Develop essential academic vocabulary for the concept and allow students to practice using it.		

Adapted from the ELD Proficiency Level Continuum in the California English Language Development Standards.

Integrated ELD

The focus will be on the content.

Sample Language Objective

- Students will be able to share written ideas - responding to clarifying questions asked of them and providing clarifying questions to enhance the ideas of others.

Standards Connections (refer to the Figure 3.4 for more detail on the standards):

- Collaborative Mode: Students will exchange information through oral collaborative exchanges (sharing their critiques of the other student's work).
- Collaborative Mode: Students will interact with others via written English (writing their explanation for the answer and how to solve the problem).
- Collaborative Mode: Students will orally negotiate with and persuade others in communicative exchanges (sharing what they feel the other student could do to improve the work).
- Interpretive Mode: Students will listen actively to other students' responses (listening to their partner's critique of their response).
- Interpretive Mode: Students will view text and multimedia to determine the meaning and categorize the images (reading closely what the problem states, reading closely what their

partner's explanation).

- Interpretive Mode: Students will evaluate the responses and arguments of others (when reading their partner's response, when discussing/clarifying someone's critique of their response).
- Interpretive Mode: Students will analyze how other students use vocabulary and other language resources to explain their thinking (when analyzing another student's response for clarity and precision).
- Productive Mode: Students will compose informational text to explain ideas and information (writing their answer and explaining their thought process).
- Productive Mode: Students will support their own opinions and evaluate the arguments of others (justifying their own thinking, evaluating the response of their partner).
- Productive Mode: Students will select and apply precise vocabulary and language structures (when clarifying their responses to make them clearer).

Supporting Language Development

- Provide differentiated sentence frames.
- Think-Pair-Share
- Visuals
- Total Physical Response (TPR)
- Drawings
- Graphic Organizers or Sequence Map (Flow Map) to organize sequence ideas
- Explicit Feedback

Designated ELD

The focus will be on how language works.

- **Into:**
    - **Sample Language Objective:**
        - Students will write the steps to solving a mathematical problem - using transitional words and steps-in-a-process language structures.
    - **Supporting Language Development**
        - Give the students sentence frames and have them share the answer to the problem.
        - Have the students use a Sequencing Map (Flow Map) to show how to solve the problem. Label above each step: first, next, then, finally, etc.
            - Have the students practice reading their own steps in the process out loud. Have students practice reading each other's steps.
- **From:**
    - **Sample Language Objective:**
        - Students will write the steps to solving a mathematical problem - adding or modifying details to create clarity and precision.
        - **Supporting Language Development**
            - Have the students participate in the activities listed in the Into section.
                - Show students sentences where they can use verbs and verb

phrases to create precision and clarity.
    - Show students sentences where they can use nouns and noun phrases to create precision and clarity.
    - Encourage the students to add more details to each of their steps as appropriate to become more precise.
- Select sentences where students can practice connecting ideas by combining sentences and clauses.

Formative Assessment

Using the ELD Proficiency Level Continuum in the CA ELD Standards…..

Integrated

- check that students can exchange information with others orally (ELD.PI.1).
- check that students can interact via written English (ELD.PI.2)
- check that students can offer and justify their opinion (ELD.PI.3).
- check that students can listen actively to spoken English (ELD.PI.5).
- check that students can view multimedia closely (ELD.PI.6).
- check that students can evaluate how well speakers use language to support their ideas and arguments (ELD.PI.7).
- check that students can analyze how writers use vocabulary and other language resources for specific purposes (ELD.PI.8).
- check that students can write informational text to explain ideas and information (ELD.PI.10)
- check that students can support their own opinions and evaluate the opinions of others (ELD.PI.11).
- check that students can select and apply varied and precise vocabulary and language structures to effectively convey ideas (ELD.PI.12).

Designated

- check that students understand text structure (ELD.PII.1).
- check that students understand cohesion (ELD.PII.2).
- check that students are able to use verbs and verb phrases to provide more precision and clarity (ELD.PII.3)
- check that students are able to use nouns and noun phrases to provide more precision and clarity (ELD.PII.4)
- check that students are able to add details to provide more information and create precision (ELD.PII.5).
- check that students can connect ideas within sentences (ELD.PII.6).

K-5 Resources and Videos

Coming Soon

6-12 Resources and Videos

Coming Soon

Collect and Display

What Is This Strategy?

Students' output language is used to develop their mathematical language. During partner talk, the teacher listens and records the language students are using with words, diagrams, and pictures. The teacher gathers and organizes students' ideas, strategies, or responses in a visible format so that all students can refer to, build on, or make connections with during future discussions or writing. Students refer to these written ideas to improve the clarity and focus of their communication.

Why Use This Strategy?

Sense-Making: Encourages students to verbalize their thinking, promoting deeper understanding and helping them make sense of mathematical concepts through discussion and reflection.

Bridging Informal and Academic Language: Supports students in transitioning from everyday language to precise mathematical vocabulary, enhancing their ability to express ideas clearly and accurately.

Equity and Inclusion: Ensures all voices are heard, providing opportunities for diverse learners, including Multilingual Learners, to contribute and participate meaningfully in discussions.

Meta-Awareness of Language and Long-Term Retention: Develops students' awareness of language use, helping them reflect on word choices and retain key concepts over time.

Collaboration and Peer Learning: Promotes teamwork by making ideas visible for group analysis, allowing students to identify patterns, connections, and actionable insights that deepen collective understanding.

When to Use This Strategy?

Collect and display can be used when students are brainstorming or problem solving with a partner/group to capture students' oral words, phrases, and language. This reference can then be used to develop mathematical language and shared content understanding. It can be used at the beginning of a unit, during concept development, introducing vocabulary, discussions/collaboration, etc.

How Do I Plan For This Strategy

- Identify the goal(s) and prepare materials

    - Consider the math language or terms students will develop
    - Choose a prompt that aligns to your goal and can be solved collaboratively in pairs or groups.
    - Sticky notes, index cards, chart paper, padlet, etc.
    - Prepare a display space to organize and showcase ideas
- Plan for the Process/Facilitation
    - How will students share ideas? (anonymously, openly, etc)
    - How will you collect student thinking, language, and ideas as you walk around? Consider how you will categorize or organize by theme, timelines, categories, etc.
- Anticipate Challenges:
    - Plan for large volumes of input-how will you organize quickly
    - Prepare strategies to re-engage participants of the discussion veers off-track or stagnates
- Visualize the Discussion:
    - What guiding questions/prompts will you ask to deepen insights?
        - What did you think about _____? Can you explain your thinking? Tell me more about your thinking.
    - How will you summarize language that emerges?
        - Scribe student language/contributions, Revoice or Reframe as needed
        - Organize responses visibly and meaningfully for all students to see.
- End with Action: Reflect and Extend
    - Prepare a way to document or save the displayed input for future reference if needed (consider an anchor chart).
    - Update or revise the display as student language evolves

Steps

1. **Present the Problem/Visual**
    a. Provide students with a prompt/question
    b. Ask students to collaborate and discuss

2. **Collect**
    a. Circulate and listen to conversations
        i. Collect/write down student phrases, ideas, visuals
        ii. Prompt/Question students as needed to collect information

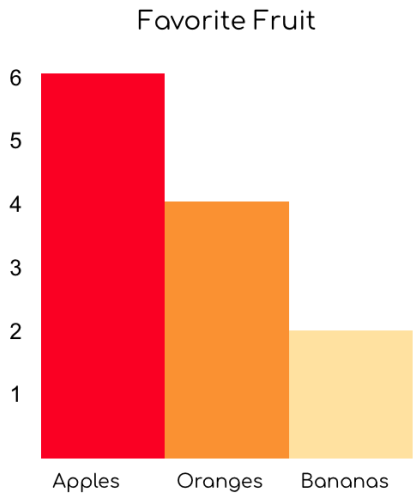
3. **Display**
    a. Call on students to share their thinking
    b. Record phrases/words for everyone to see
    c. Organize and provide connections to the language and representations students are using.

4. **Discuss**

a. Facilitate a discussion around the displayed content
b. Ask questions to guide the conversation (keep the goal(s) in mind) and make connections between student's thinking and language
    i. "What language is similar/different?"
    ii. "What language or phrases are familiar and which ones are unfamiliar?"
    iii. "What connections do you see?""What patterns or themes do you see?"
    iv. "Which ideas stand out to you, and why?"
c. If applicable, refine the displayed content based on the discussion.
d. Display so students can refer to, build on, or make connections for future discussions or writing

Example

1. **Present the Visual (Bar Graph)**
    a. Display a bar graph showing favorite fruit. (apples, oranges, and bananas)
    b. **Teacher:** *"What do you notice and wonder about this bar graph?" "How can we describe it using numbers, comparisons, or patterns?"*

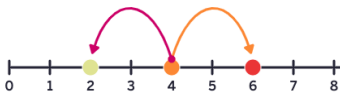


i.

2. **Collect**
    a. **Teacher:** *"Take some time to think and discuss the question with your partner".*
    b. **Teacher:** Circulate and listen to partner conversations. Collect/Write down student phrases, ideas, visuals
        i. **Student 1**: *"Apples are the highest. There are 6 apples and 2 bananas."*
        ii. **Student 2:** *"There are 2 less bananas than oranges"*
        iii. **Student 1:** *"If you add the oranges and apples it equals the apples."*
3. **Display**
    a. **Teacher:** Call on students to share and record their words for everyone to see on the board. Ask additional questions.
        i. *"What do you notice about the number of apples, oranges, or bananas?"*
        ii. *"Can you describe a relationship between the bars?"*

iii. *"Can you explain what you heard in a different way?"*

b. **Teacher:** Organize and provide connections. Teacher displays responses in a table and makes connections to the language, equations, and representations students are using.

Words	Numbers/Number Sente
"Apples are the highest"	6
"Bananas are two less than oranges"	4 - 2 = 2
"If you add bananas and oranges, it equals the apples"	2 + 4 = 6
"Oranges are halfway between bananas and apples" "I can use a number line to show halfway"	2, 4, 6 

c. Teacher draws arrows or brackets on the bar graph to show relationships (e.g., a bracket connecting oranges and bananas with the label "2 less").

4. Discuss

a. Facilitate a discussion around the displayed content.

i. **Teacher:** *"What language or phrases are familiar and which ones are unfamiliar?"*

1. **Student:** *"equals, halfway, highest, number line, number sentences"*

ii. Using student responses, categorize the words/phrases and make connections

iii. **Teacher**: " *Notice how we are using different words/phrases, however they still represent the same thinking. For example when less than or fewer are two words I heard to describe the same thinking. Etc.".*

Source:
Graduate, S., Zwiers, J., Dieckmann, J., Rutherford-Quach, S., Daro, V., Skarin, R., Weiss, S., & Malamut, J. (2017). *Understanding Language/SCALE Principles for the Design of Mathematics Curricula: Promoting Language and Content Development*.
https://ul.stanford.edu/sites/default/files/resource/2021-11/Principles%20for%20the%20Design%20of%20Mathematics%20Curricula_1.pdf

Some Considerations for Different English Proficiency Levels *Note: See section titled "Scaffolding" for more information on how to scaffold for different proficiency levels.*		
Emerging ● Provide sentence frames at their proficiency level.	**Expanding** ● Provide sentence frames at their proficiency level.	**Bridging** ● Provide sentence frames at their proficiency level.

• Allow students to share their ideas in their primary language and then translate their ideas into English. • Allow students to use pictures, drawings and gestures to aid in expressing their ideas. • Student responses may be brief (one word responses or short phrases) • May need moderate support when speaking/presenting.	• Allow students to use pictures, drawings, and gestures to aid in expressing their ideas. • May need moderate support when speaking/presenting.	• Students should be able to describe their ideas using key details. • Students should be able to use varied vocabulary and words with similar meanings to convey • May need light support when speaking/presenting.
Develop essential academic vocabulary for the concept and allow students to practice using it.		

Adapted from the ELD Proficiency Level Continuum in the California English Language Development Standards.

Integrated ELD

The focus will be on the content.

Sample Language Objective

- Students will be able to read multimedia information closely using compare and contrast language structures.

Standards Connections (refer to the Figure 3.4 for more detail on the standards):

- Collaborative Mode: Students will exchange information through oral collaborative exchanges (sharing their interpretations of the data).
- Collaborative Mode: Students will interact with others via written English (writing their ideas of what the information means).
- Collaborative Mode: Students will orally negotiate with and persuade others in communicative exchanges (sharing differing opinions on how to interpret the data).
- Interpretive Mode: Students will listen actively to other students' responses (listening to the responses of their classmates).
- Interpretive Mode: Students will view text and multimedia to determine the meaning and categorize the images (reading closely the data provided).
- Interpretive Mode: Students will evaluate the responses and arguments of others (when listening to how their partners and classmates understand the information).
- Interpretive Mode: Students will analyze how other students use vocabulary and other language resources to explain their thinking (when analyzing another student's response for clarity and precision).
- Productive Mode: Students will compose informational text to explain ideas and information (writing their answer and explaining their thought process).

- Productive Mode: Students will support their own opinions and evaluate the arguments of others (justifying their own thinking, evaluating the response of their partner).

Supporting Language Development

- Provide differentiated sentence frames.
- Think-Pair-Share
- Visuals
- Total Physical Response (TPR)
- Drawings
- Small Groups
- Table or Categorization/Classification Map (Tree Map)

Designated ELD

The focus will be on how language works.

- **Into:**
  - **Sample Language Objective:**
    - Students will orally share their ideas - adding or modifying details to provide clarity and precision.
  - **Supporting Language Development**
    - Provide students with sentence frames that they can use to state what they see in the figure.
      - Ex:
        - Emerging - I see _____. I notice _____. (I see apples.)
        - Expanding - I see that there are _____, _____, and _____. (I see that there are apples, oranges, and bananas.)
        - Bridging - I notice that there are _____, while there are _____. (I notice that there are 6 apples, while there are 4 oranges.)
    - Identify and teach multiple meaning words or other vocabulary words that they might be unfamiliar with (ex: relationship, connections, patterns, themes)
    - Use a table, chart or Categorization/Classification Map (Tree Map) to group words with similar meanings that students can refer back to during the core content lesson (ex: less, fewer, take away)
- **From:**
  - **Sample Language Objective:**
    - Students will orally share their ideas - adding or modifying details to provide clarity and precision.
  - **Supporting Language Development**
    - Provide an example using two objects (apples and oranges) and then an example of using three objects. Model for students when to use comparative adjectives (comparing two objects) and when to use superlative adjectives (comparing three or more objects).
    - Encourage students to add details (based on their proficiency level) to their

responses to improve clarity. (ex: I see apples. → I see 6 apples.)

Formative Assessment

Using the ELD Proficiency Level Continuum in the CA ELD Standards…..

Integrated

- check that students can exchange information with others orally (ELD.PI.1).
- check that students can justify their opinions, negotiate with and persuade others in communicative exchanges (ELD.PI.3).
- check that students can view multimedia closely to determine meaning (ELD.PI.6)
- check that students can support their own opinions and evaluate others' arguments in speaking (ELD.PI.11)

Designated

- check that students understand cohesion (ELD.PII.2).
- check that students can use verbs and verb phrases to create clarity (ELD.PII.3).
- check that students can use noun and noun phrases to create clarity (ELD.PII.4).
- check that students can add details and provide more information to create precision (ELD.PII.5).
- check that students can connect ideas within sentences by combining clauses and joining ideas (ELD.PII.6).

K-5 Resources and Videos

Coming Soon

6-12 Resources and Videos

Coming Soon

Critique, Correct and Clarify

What Is this Strategy?

Students critique the work of others by identifying errors or areas of improvement. They use language to provide feedback, further clarify the ideas, and the meaning of the written work. During this routine, students are given a written/work sample that is not their own to analyze, reflect on, and develop further. Students revise, make corrections, and further enhance the clarity of the sample.

Why Use This Strategy?

Fortify student output: students are building on the rough draft thinking provided to them in the form of a written/work sample. They are further improving their thinking by attending to the language as well as the mathematics.

Engage students in meta-awareness: students are comparing their own understanding of a concept with the written/work sample. As a result, they are thinking about what they know and what's missing from the sample.

When to Use This Strategy?

This routine can be used during or at the end of learning a concept to support students in thinking about how to solve problems, to use subject-specific language, and to improve upon their written work by correcting errors and clarifying meaning.

How Do I Plan For this Strategy?

- Gather or create a written argument, explanation, or strategy that is incorrect or missing relevant information.
- Identify and create any scaffolds or modifications you may need to make to support your students.

Steps

1. **Present the problem and the response**
    a. Present a problem with a response consisting of a partial/broken argument, explanation, or solution method. Teacher can play the role of the student who produced the response, and ask for help in fixing it.
    b. Given response could include a common error, an ambiguous term or phrase, or an

informal way of expressing a mathematical idea.

2. **Clarify and Critique**
    a. Prompt students to identify the error(s) or ambiguity, analyze the response in light of their own understanding of the problem.
    b. Students work individually on an improved response.
3. **Share**
    a. Have students work in pairs to share their draft of the improved response.
4. **Refine**
    a. Students refine their own draft response.

Example

1. **Present the problem**

2nd grade context

Ms Daines needs to drive to San Jose which is 109 miles away. Along the way she stopped in Salinas which is 48 miles away. When she began to drive from Salinas, how far away was Ms. Daines from San Jose?

Solution:

Hundreds	*Tens*	*Ones*
100	*00*	*9*
	40	*8*
100	*40*	*1*

Ms Daines needs to drive 141 miles.

2. **Clarify and Critique**
    a. The teacher prompts students to make sense of the questions and examine the solution for errors.
    Student response: "*This is incorrect. They did not unbundle the 100s. They need to unbundle the 100s into 10s."*
    b. Students work individually on an improved response.

Student response:

Hundreds	*Tens*	*Ones*
~~100~~ →	*100*	*9*
	40	*8*
	60	*1*

Ms Daines has to drive 61 miles.

3. **Share**
    a. Students share their response with a partner and give each other feedback as necessary.
4. **Refine**
    a. Students work individually to improve their response based on the pair share.

Student work:
Utilize a tape diagram to share their thinking.

109 miles to San Jose	
48 miles to Salinas	*? miles left*

61 miles left

Sources:

Mastering Mathematical Language Routine 3: Critique, Correct, and Clarify. (2023, November 23). Nowa Techie. https://nowatechie.com/2023/11/23/mastering-mathematical-language-routine-3-critique-correct-and-clarify/

Zwiers, J., Dieckmann, J., Rutherford-Quach, S., Daro, V., Skarin, R., Weiss, S., & Malamut, J. (2017). Principles for the Design of Mathematics Curricula: Promoting Language and Content Development. Retrieved from Stanford University, UL/SCALE website: http://ell.stanford.edu/content/mathematics-resources-additional-resources

Some Considerations for Different English Proficiency Levels
Note: See section titled "Scaffolding" for more information on how to scaffold for different proficiency levels.

Emerging	**Expanding**	**Bridging**
● Provide sentence frames at their proficiency level. ● Students can respond in simple phrases such as "I think___." ● Students can identify and use language to support opinions with moderate support.	● Provide sentence frames at their proficiency level. ● Students can begin elaborating more to include opinions and reasons such as "I think____ because____." ● Students can identify and use language to support opinions with mild support.	● Provide sentence frames at their proficiency level. ● Students can begin elaborating on their own opinions and compare/contrast their opinions with those of others. ● Students can identify and use language to support opinions with light support.
Develop essential academic vocabulary for the concept and allow students to practice using it.		

Adapted from the ELD Proficiency Level Continuum in the California English Language Development Standards.

Integrated ELD

The focus will be on the content.

Sample Language Objective

- Students will be able to read information - through critiquing others' work and asking clarifying questions.

Standards Connections (refer to the Figure 3.4 for more detail on the standards):

- Collaborative Mode: Students will exchange information through oral collaborative exchanges (when they share what they think the problem/ambiguity was).
- Collaborative Mode: Students will interact with others via written English (when students write their improved responses).
- Collaborative Mode: Students will orally negotiate with and persuade others in communicative exchanges (when discussing with partners and the class where they feel the problem/ambiguity is).
- Interpretive Mode: Students will listen actively to other students' responses (when working with partners or listening to whole-class ideas).
- Interpretive Mode: Students will view text and multimedia to determine the meaning and categorize the images (when students review the given flawed argument, explanation, or solution).
- Interpretive Mode: Students will evaluate the responses and arguments of others (when analyzing the student work for accuracy and clarity).
- Interpretive Mode: Students will analyze how other students use vocabulary and other language resources to explain their thinking (when listening to understand what other students' ideas and explanations are).
- Productive Mode: Students will compose informational text to explain ideas and information (when students write their revised responses).
- Productive Mode: Students will support their own opinions and evaluate the arguments of others (when sharing orally and in writing where they believe the inaccuracy is and why along with their revised responses).
- Productive Mode: Students will select and apply precise vocabulary and language structures (when creating their revised responses).

Supporting Language Development

- Provide differentiated sentence frames.
- Think-Pair-Share
- Visuals
- Total Physical Response (TPR)
- Drawings
- Small Groups
- Tables, Charts, Graphic Organizers
- Word banks/Word Walls

Designated ELD

The focus will be on how language works.

- **Into:**
  - **Sample Language Objective:**
    - Students will write their response to a mathematical solution - through stating an argument and providing relevant evidence.
  - **Supporting Language Development**
    - Provide differentiated sentence frames to help students state agreement or disagreement. (ex: I disagree with _____ because I think _____).
    - Students can use pictures, drawings, charts or other visuals to help them explain their opinion and justification. The teacher can then validate the student's contribution and model language by restating what the student said/gestured.
- **From:**
  - **Sample Language Objective:**
    - Students will write how to solve a mathematical problem - using temporal words and steps-in-a-process language structures.
  - **Supporting Language Development**
    - Provide a graphic organizer or a Sequence Map (Flow Map) to support the students in stating the steps in their process.
      - Provide sentence frames with transitional words to express their steps orally or in writing. (ex: First, I would _____).
    - Have the students practice active listening when participating in partner collaboration time. Model what listening for understanding means. Have students practice restating what they heard in their own words. Have students practice asking questions for clarification when needed.
    - Have the students practice adding details to create precision and clarity when working with their partners.

Formative Assessment

Using the ELD Proficiency Level Continuum in the CA ELD Standards…..

Integrated

- check that students can exchange information with others orally (ELD.PI.1).
- check that students can justify their opinions, negotiate with and persuade others in communicative exchanges (ELD.PI.3).
- check that students can view multimedia closely to determine meaning (ELD.PI.6)
- check that students can support their own opinions and evaluate others' arguments in speaking (ELD.PI.11)

Designated

- check that students can understand text structure and organization (ELD.PII.1).
- check that students understand cohesion (ELD.PII.2).
- check that students can add details and provide more information to create precision (ELD.PII.5).

K-5 Resources and Videos

Coming Soon

6-12 Resources and Videos

Coming Soon

Information Gap

What Is this Strategy?

It is designed to create a need for students to communicate in order to solve a problem, play a game, or complete a task. In this routine, students are given different pieces of information that, when combined, allow them to collaboratively accomplish something they couldn't achieve alone.

Why Use This Strategy?

Promotes Collaboration: Students must rely on one another to gather missing information, fostering teamwork and communication.

Enhances Language Development: Encourages the use of academic vocabulary, as students must articulate their reasoning, ask questions, and justify their answers.

Develops Problem-Solving Skills: Students practice critical thinking by synthesizing information and solving problems together.

Increases Engagement: The interactive nature of this strategy motivates students to participate actively.

When to Use This Strategy?

This strategy can be used to introduce new content by engaging students in a collaborative and interactive way to explore key concepts. It is also effective for reinforcing learning, as it helps deepen understanding through review activities. Additionally, it is ideal for promoting language development, particularly in classes with English learners or those focusing on communication skills. Furthermore, the strategy is well-suited for group problem-solving tasks, especially in lessons that require logical reasoning and collaborative solutions.

How Do I Plan For this Strategy?

- **Choose the Task:** Select a problem, game, or activity that requires multiple pieces of information to solve.
- **Divide the Information:** Prepare materials where each group or partner has access to only part of the required information.
- **Set Clear Objectives:** Define what students must achieve together and what success looks like.
- **Model Expectations:** Demonstrate how to ask for and share information, clarify misunderstandings, and justify ideas.

- **Prepare Prompts:** Create sentence starters or guiding questions to support student conversations. Examples:
    - "Can you explain why...?"
    - "What do you think about...?"
    - "I think this is true because..."

Steps

1. **Place students into groups or pairs:** Organize students to work collaboratively.
2. **Distribute Information:** Give each student a different piece of the necessary information.
3. **Explain the task:** Clearly communicate the goal and ensure students understand the rules.
4. **Model the process:** Role-play with a student or colleague to show how to ask questions, share, and justify information.
5. **Monitor and support:** Circulate around the room to listen, prompt, and provide support as needed.
6. **Debrief and reflect:** After the activity, discuss the process with the class. Highlight effective communication strategies and clarify misunderstandings.

Example

Task: Solve a mystery number using clues.

Objective: Students work in pairs to find the number by sharing information and reasoning together.

Preparation:

- Partner A's card says:
  "The mystery number is a two-digit number divisible by 5."
- Partner B's card says:
  "The mystery number is greater than 40 but less than 60."
- Partner C's card says:
  "The sum of the digits in the mystery number is 9."

Teacher-Student Dialogue:

Teacher: "Today, you'll work with your partners to solve a mystery number. Each of you has a piece of the puzzle, but none of you can solve it alone. Use your clues, ask questions, and explain your reasoning to find the answer. Let's model this together. I'll be Partner A. Who wants to be Partner B?"

Student 1: "I'll be Partner B!"

Teacher: "Great! My clue says the number is divisible by 5. What's your clue?"

Student 1: “Mine says it’s greater than 40 but less than 60.”

Teacher: “Okay, so we know it’s a two-digit number divisible by 5, and it’s between 40 and 60. Can we list possible numbers?”

Student 2: “Yes, maybe 45, 50, or 55?”

Teacher: “Good thinking! Now, let’s bring in Partner C. What’s your clue?”

Student 2: “My clue says the sum of the digits is 9.”

Teacher: “Let’s check each number. Does 4 + 5 equal 9?”

Student 1: “Yes! So the mystery number is 45!”

Teacher: “Exactly! You all shared your clues and reasoned together. Now it’s your turn to try with new clues!”

Source:

Graduate, S., Zwiers, J., Dieckmann, J., Rutherford-Quach, S., Daro, V., Skarin, R., Weiss, S., & Malamut, J. (2017). *Understanding Language/SCALE Principles for the Design of Mathematics Curricula: Promoting Language and Content Development*. https://ul.stanford.edu/sites/default/files/resource/2021-11/Principles%20for%20the%20Design%20of%20Mathematics%20Curricula_1.pdf

Some Considerations for Different English Proficiency Levels *Note: See section titled “Scaffolding” for more information on how to scaffold for different proficiency levels.*		
Emerging ● Provide sentence frames at their proficiency level. ● Students may be able to respond to yes/no questions and use phrases and short sentences.	**Expanding** ● Provide sentence frames at their proficiency level. ● Students may be able to share their ideas and respond to others’ ideas using relevant information.	**Bridging** ● Provide sentence frames at their proficiency level. ● Students can share their own ideas and respond to those of others using coherent, on-topic information.
Develop essential academic vocabulary for the concept and allow students to practice using it.		

Adapted from the ELD Proficiency Level Continuum in the California English Language Development Standards.

Integrated ELD

The focus will be on the content.

Sample Language Objective

- Students will be able to listen to information shared by classmates - listening for key details to solve a mathematical problem.

Standards Connections (refer to the Figure 3.4 for more detail on the standards):

- Collaborative Mode: Students will exchange information through oral collaborative exchanges (when they listen to the clues that their classmates have and when they share their clues with others).
- Collaborative Mode: Students will orally negotiate with and persuade others in communicative exchanges (when discussing with partners what they think the solution is).
- Interpretive Mode: Students will listen actively to other students' responses (when working with partners).
- Interpretive Mode: Students will evaluate the responses and arguments of others (when listening their partner's clues and potential solutions).
- Interpretive Mode: Students will analyze how other students use vocabulary and other language resources to explain their thinking (when listening to understand what other students' ideas and explanations are).
- Productive Mode: Students will support their own opinions and evaluate the arguments of others (when sharing orally and in writing where they believe the inaccuracy is and why along with their revised responses).

Supporting Language Development

- Provide differentiated sentence frames.
- Think-Pair-Share
- Visuals
- Total Physical Response (TPR)
- Drawings
- Small Groups
- Word banks/Word Walls

Designated ELD

The focus will be on how language works.

- **Into:**
  - **Sample Language Objective:**
    - Students will orally exchange information - using statements and questions to collaboratively calculate a solution.
  - **Supporting Language Development**
    - Provide differentiated sentence frames to help students state information and ask for clarification (ex: The clue that I have is _____. Can you repeat that a little slower please?).
    - Students can use pictures, drawings, charts or other visuals to help them explain their opinion and justification. The teacher can then validate the student's

contribution and model language by restating what the student said/gestured.

- **From:**
    - **Sample Language Objective:**
        - Students will orally evaluate the arguments of classmates - using agree/disagree language structures.

    - **Supporting Language Development**
        - Provide students with sentence frames (ex: I agree _____ because. I disagree with _____ because _____.)
        - Have the students practice active listening when participating in partner collaboration time. Model what listening for understanding means. Have students practice restating what they heard in their own words. Have students practice asking questions for clarification when needed.
        - Have the students practice adding details to create precision and clarity when working with their partners.

Formative Assessment

Using the ELD Proficiency Level Continuum in the CA ELD Standards…..

Integrated

- check that students can exchange information with others orally (ELD.PI.1).
- check that students can justify their opinions, negotiate with and persuade others in communicative exchanges (ELD.PI.3).
- check that students can view multimedia closely to determine meaning (ELD.PI.6)
- check that students can support their own opinions and evaluate others' arguments in speaking (ELD.PI.11)

Designated

- check that students understand cohesion (ELD.PII.2).
- check that students can add details and provide more information to create precision (ELD.PII.5).

K-5 Resources and Videos

Coming Soon

6-12 Resources and Videos

Coming Soon

Co Craft Questions and Problems

What Is This Strategy?

Students collaborate with peers to create and refine meaningful questions and/or problems that deepen mathematical thinking and understanding. Students analyze a mathematical situation, brainstorm possible questions or problems, and refine them to align with key learning goals. This strategy allows students to get inside of a context before feeling pressure to produce answers, to create space for students to produce the language of mathematical questions themselves, and to provide opportunities for students to analyze how different mathematical forms can represent different situations.

Why Use This Strategy?

Process the existing information to produce language: students generate, choose, and improve questions aligned to the given information.

Develop meta-awareness: students evaluate the given information to generate questions that can be answered or think of questions that require more information to answer.

When to Use This Strategy?

Use this strategy to empower students to collaborate, think critically, and engage with context in an open-ended manner, encouraging them to behave as mathematicians and develop as independent thinkers.

How Do I Plan For This Strategy

- Identify the Learning Goal
    - Determine the mathematical concept or skill you want students to explore or deepen through the routine.
- Select or Design a Stimulus
    - Co-craft questions-choose a problem or situation that prompts curiosity and allows multiple entry points for questioning.
    - Co-craft problems-plan for interesting topics or tasks so that students can create their own problems.
- Anticipate Student Responses
    - Predict possible questions and problem ideas students might generate to prepare for guiding discussions.
- Prepare Sentence Starters and Supports

- Develop language supports such as sentence frames (e.g., *"I have a question about _____." "I notice _____." "I wonder if..."*) to help students formulate questions and engage in discussions.

- Plan for Collaboration
  - Decide how students will be paired.
- Create Guiding Questions
  - Prepare prompts to help students analyze, evaluate, and refine their questions or problems.
- Consider Extensions and Supports
  - Plan scaffolds for students who may need additional help and extensions for those ready for deeper exploration.

Steps

Step 1: Teacher presents a situation - a context or a stem for a problem, with or without values included.

Step 2: Students write down possible mathematical questions that might be asked about the situation. These should be questions that they think are answerable by doing math. They can also be questions about the situation, information that might be missing, and even about assumptions that they think are important.

Step 3: In pairs, students compare their questions.

Step 4: Students are invited to share their questions, with some brief discussion.

Step 5: The actual questions students are expected to work on are revealed, the students are set to work.

**This can also be done by students co-creating their own problems, solving, and then checking solutions.*

Example

1. **Present the problem**

   6th Grade Ratios and Proportional Relationships: Two different pitchers of lemonade are made using water and lemon juice. The first pitcher has more water than lemon juice. The second pitcher is larger but has the same balance of water and lemon juice as the first pitcher.

2. **Brainstorm possible questions**
   a. Students are prompted to write down questions they have about the situation.
      i. Possible student responses:

1. *Which one would taste the best? Taste the most sour? Taste the most watered down?*
2. *What would happen if we combined the lemonade from both pitchers?*
3. *Can we create a recipe that keeps the balance of water and lemon juice the same no matter the size of the pitcher?*
4. *If we change the amount of water or lemon juice, how does that affect the taste?*
5. *How do we know if the balance of water and lemon juice are the same in both pitchers?*

3. **Pairs Compare**
    a. Students compare their questions (1-2 min)
    b. Students may use language supports or sentence frames.
        i. A question I have is _____.
        ii. I wonder _____.

4. **Share Questions** (whole group)
    a. Students share their questions to the whole group
    b. Improve questions by pushing for clarity. Have a brief discussion about the questions presented (analyze, make predictions, make connections, estimate, refine, etc.)
    c. Revoice oral responses as necessary

5. **Reveal Question(s)**
    a. Reveal Problem with number values included

    Two different pitchers of lemonade are made using water and lemon juice. The first pitcher has 4 cups of water and 2 cups of lemon juice. The second pitcher is larger but has 6 cups of water and 3 cups of lemon juice as the first pitcher.

    b. Original Question: *How do we know if the balance of water and lemon juice are the same in both pitchers?*
    c. Refined Question: *How can we prove that the two pitchers have the same concentration of lemon juice? (now that the values are given)*

6. **Solve and Discuss**
    a. Students collaborate to answer the question(s)
        i. Students use a variety of strategies such as tables, diagrams, double lines, visuals, etc.
        ii. Students share and discuss their reasoning with their peers

7. **Reflect and Extend**
    a. What strategies helped us refine our question(s)?
    b. How did creating your own questions deepen your understanding of ratios?
    c. Where else might we see ratios used in real life?

Source:

Graduate, S., Zwiers, J., Dieckmann, J., Rutherford-Quach, S., Daro, V., Skarin, R., Weiss, S., & Malamut, J. (2017). *Understanding Language/SCALE Principles for the Design of Mathematics Curricula: Promoting Language and Content Development*. https://ul.stanford.edu/sites/default/files/resource/2021-11/Principles%20for%20the%20Design%20of%20Mathematics%20Curricula_1.pdf

Some Considerations for Different English Proficiency Levels *Note: See section titled "Scaffolding" for more information on how to scaffold for different proficiency levels.*		
Emerging ● Provide sentence frames at their proficiency level. ● Students may be able to generate questions regarding the topic using short phrases and sentences and some domain-specific words. ● Students may be able to persuade others in conversations using words and learned phrases.	**Expanding** ● Provide sentence frames at their proficiency level. ● Students may be able to generate questions regarding the topic using increasingly detailed sentences and an increasing variety of domain specific words. ● Students may be able to persuade others with an expanded set of learned phrases.	**Bridging** ● Provide sentence frames at their proficiency level. ● Students may be able to generate questions regarding the topic using a variety of well-articulated sentences and domain-specific academic vocabulary. ● Students may be able to persuade others using appropriate register, indirect reported speech, and open responses.
Develop essential academic vocabulary for the concept and allow students to practice using it.		

Adapted from the ELD Proficiency Level Continuum in the California English Language Development Standards.

Integrated ELD

The focus will be on the content.

Sample Language Objective

- Students will be able to orally exchange information - using interrogative language structures (wh-, how, I wonder____).

Standards Connections (refer to the Figure 3.4 for more detail on the standards):

- Collaborative Mode: Students will exchange information through oral collaborative exchanges (when they share their questions with partners and the class).
- Collaborative Mode: Students will interact with others via written English (when students write their questions).
- Collaborative Mode: Students will orally negotiate with and persuade others in communicative exchanges (when sharing with a partner how he/she can improve the clarity of his/her question).
- Interpretive Mode: Students will listen actively to other students' responses (when working with partners or listening to whole-class ideas).
- Interpretive Mode: Students will view text and multimedia to determine the meaning and categorize the images (when students review the given prompt).
- Interpretive Mode: Students will evaluate the responses and arguments of others (when

analyzing the student work for accuracy and clarity).

- Interpretive Mode: Students will analyze how other students use vocabulary and other language resources to explain their thinking (when listening to understand what other students' questions are).
- Productive Mode: Students will compose informational text to explain ideas and information (when students write their original and revised questions).
- Productive Mode: Students will select and apply precise vocabulary and language structures (when creating their revised responses).

Supporting Language Development

- Provide differentiated sentence frames.
- Think-Pair-Share
- Visuals
- Total Physical Response (TPR)
- Drawings
- Small Groups
- Tables, Charts, Graphic Organizers
- Word banks/Word Walls

Designated ELD

The focus will be on how language works.

- **Into:**
  - **Sample Language Objective:**
    - Students will be able to orally exchange information - using interrogative language structures (wh-, how, I wonder____).
  - **Supporting Language Development**
    - Look for opportunities to highlight grammatical or syntactical features. Discuss how or why that feature works (ex: less than, fewer → uses a comparative as opposed to a superlative because only 2 things are being used and a superlative would be used for 3 or more items).
    - Practice asking questions.
      - Some students, especially younger students, confuse asking questions and answering them. Highlight that asking questions is when you want to know something. Time could be spent asking questions for different purposes (ex: asking for information, asking for permission, asking to persuade) prior to beginning the Math example.

- **From:**
  - **Sample Language Objective:**
    - Students will orally exchange information - using language structures for interrogative and declarative sentences.
  - **Supporting Language Development**
    - Provide question starters (Wh-, How, I wonder) and have the students practice

creating questions.

- Have the students practice active listening when participating in partner collaboration time. Model what listening for understanding means. Have students practice restating what they heard in their own words. Have students practice asking questions for clarification when needed.
- Have the students practice adding details to create precision and clarity when working with their partners.

- Practice asking questions.
    - Some students, especially younger students, confuse asking questions and answering them. Highlight that asking questions is when you want to know something.
    - An additional activity could be to: Write a statement on a sentence frame. Cut the sentence into individual word cards. Change the order of the word cards to create a question. Do the activity again, turning a question into a statement. Discuss what additional words might be needed or which words might need to be removed to complete the sentences.
    - Ex: The water and the lemon juice are balanced in the pitcher
        - Is the pitcher balanced with water and lemon juice?
        - How do we know that the water and lemon juice are balanced?
    - Ex: Which pitcher would taste the best, the first pitcher or the second pitcher?
        - The second pitcher would taste the best.

Formative Assessment

Using the ELD Proficiency Level Continuum in the CA ELD Standards…..

Integrated

- check that students can exchange information with others orally (ELD.PI.1).
- check that students can adapt their language choices based on the task, purpose and audience (ELD.PI.4).
- check that students can listen actively to spoken English in a range of academic contexts (ELD.PI.5).
- check that students can view text and multimedia closely to determine meaning (ELD.PI.6).
- check that students can analyze how writers and speakers use vocabulary and other resources for specific purposes (ELD.PI.8).
- check that students can select and apply varied and precise vocabulary and language structures to effectively convey ideas (ELD.PI.12).

Designated

- check that students can understand text structure and organization (ELD.PII.1).
- check that students understand cohesion (ELD.PII.2).
- check that students can add details and provide more information to create precision (ELD.PII.5).

K-5 Resources and Videos

Multilingual Learner Math Toolkit of Strategies

Coming Soon

6-12 Resources and Videos

Coming Soon

Three Reads

What Is This Strategy?

Students approach word problems and other mathematical texts by reading the problem three times with a focus on comprehension before beginning the computation. The first reading helps them understand the context of the problem. The second reading allows them to interpret the question and clarify what is being asked. The third reading involves identifying important details and planning a solution.

Why Use This Strategy?

Support understanding of word problems: guide students through structured reading to understand the combination of narrative and expository text present in math word problems. Narrative text tells a story and expository text has the information. The purpose of reading a word problem is to answer the question by understanding both the narrative and expository text. Therefore, word problems must be read several times, each time with a focus on a different aspect of the word problem (Kelemanik, Lucenta & Creighton, 2016). This strategy provides students a sequence to understand and use the context to solve the problem.

When to Use This Strategy?

Three Reads is ideal for solving word problems through a structured process to comprehend the context as well as understand the mathematics.

How Do I Plan For This Strategy

- Choose a task/word problem
  - that aligns to your math and language goals
  - that requires students to analyze context and information to make sense of the situation
  - that can be displayed or viewed by the students i.e., text, slide, chart paper, handout
- Prepare the 3 questions you will use with your students
  - What is the problem about?
  - What is the question?
  - What is the important information?
- Prepare language supports
  - Utilize sentence frames for students to communicate thinking
  - Define or use pictures for specific vocabulary

Steps

FIRST READ

1. **Individual Think Time**
    a. Students read the problem for the first time and think, “What is the problem about?”
    b. Students may read the problem on their own, the teacher may read the problem aloud to the class or have a student read it aloud.
    c. After the reading, provide individual think time as students make sense of the context. Students may record or draw their ideas.

2. **Whole class share-out**
    a. Teacher calls on students by asking, “What is the story about?” or a similar open-ended question.
    b. Provide students with sentence frames i.e., The story/problem is about _____. I agree with _____ because _____.
    c. Teacher records words and/or phrases to highlight the context and the math using the table as a template.
    d. Teacher clarifies math vocabulary as needed

Context	Math

SECOND READ

3. **Think Pair Share**
    a. Students read the problem for the second time and think, “What is the question we are asked to answer?”
    b. Students may read the problem on their own, the teacher may read the problem aloud to the class or have a student read it aloud.
    c. After the reading, provide individual think time as students make sense of the question. Students may record or draw their ideas.
    d. Students work with a partner to articulate the question in their own words.

4. **Whole class share-out**
    a. Teacher calls on students by asking, “What is the question we are asked to answer?” or “What are we trying to find out?” or a similar open-ended question.
    b. Provide students with sentence frames i.e., The question is _____, We need to find a solution for _____.
    c. Teacher calls on additional students to rephrase each other’s ideas.
    d. Teacher records different ways that students have phrased the question.

THIRD READ

5. **Pair Share**
    a. Students read the problem for the third time and think, “What is the important

information needed to answer the question?"

b. After the reading, students talk with a partner to identify mathematically important information.
c. Teacher listens and notices the avenues of thinking such as quantities, relationships, structure, and repeated reasoning.

6. **Whole class share-out**
    a. Teacher calls on students by asking, "What is the important information needed to answer the question?" or a similar open-ended question.
    b. Provide students with sentence frames i.e., The quantities are _____. The information we need is_____.
    c. Teacher calls on additional students to rephrase each other's ideas.
    d. Teacher records students thinking about the important information and has a discussion around the quantities, relationships, structure, and repeated reasoning.
    e. The teacher may follow up with additional questions: What are the quantities in this situation? What can be counted? What did you notice about the quantities? Do these quantities help answer the question? Which quantities will help us answer the question? Is there other information that is important to answering the question? Is there information we don't need?

Teacher connects the learning from each of the three reads so that students may start to solve the problem.

Example

FIRST READ

1. **Individual Think Time**
    a. **Teacher:** *"Let's read the problem together. The fourth grade students are going on a field trip to one of the California Missions. There are 5 fourth grade classes and each class has 34 students. If each bus holds 30 students, how many buses do they need?* **Teacher:** What is the problem about?"

2. **Whole class share-out**
    a. **Teacher:** "*Use the sentence frames 'The problem is about _____. I agree with _____ because _____.' "*
    b. **Student 1:** *"The story/problem is about 4th grade students going on a field trip."*
    c. **Student 2:** *"The story is about a field trip to the mission."*
    d. **Student 3:** *"The problem is about a field trip and they need to know how many buses they need."*
    e. **Teacher:** *"Let's record our ideas. (teacher uses a table to record students' share-outs)."*

Teacher Recordings	
Context	Math

Fourth Grade California Missions Field Trip	5 fourth grade classes 34 students in each class 30 students fit in each bus

SECOND READ

3. **Think-Pair -Share**
    a. **Teacher:** "*Now let's read the problem again".* (provide time). *"What is the question we are asked to answer"?*
    b. **Teacher:** *"Discuss with your partner what you have identified as the question".*
4. **Whole Class Share Out**
    a. **Teacher:** *"What is the question we are asked to answer?" Feel free to use the sentence frames when you share out. For example, The question is _____, We need to find a solution for _____."*
    b. Teacher records student responses:
        i. **Student 1:** *"How many buses do we need?"*
        ii. **Student 2:** *"How many buses do we need so that all the 4th graders can go?"*
    c. Teacher calls on additional students to rephrase each other's ideas.

THIRD READ

5. **Pair Share**
    a. **Teacher:** "*Now, let's read the problem a third time".* (provide time). "What is the important information needed to answer the question *If each bus holds 30 students, how many buses do they need*?"
    b. **Teacher:** *"At this time, take some time to talk with a partner to identify mathematically important information."*
    c. Teacher listens and notices students' thinking.
6. **Whole class share-out**
    a. **Teacher:** *What is the important information needed to answer the question? "If ea*ch *bus holds 30 students, how many buses do they need*?" *Use sentence frames. For example "The quantities are _____. The information we need is_____."*
    b. Teacher records student responses:
        i. **Student 1:** *"Important information is that there are five fourth grade classes."*
        ii. **Student 2**: *"Each class has 34 students."*
        iii. **Student 3:** *"30 students fit in a bus."*
        iv. **Student 4:** *"How many buses do we need?"*
    c. **Teacher:** *"_____, can you rephrase what _____ said."*
    d. Teacher highlights important information and discusses the quantities, relationships, structure, and repeated reasoning.
    e. The teacher may follow up with additional questions.

Teacher connects the learning from each of the three reads so that students may start to solve the problem.

Source:
Multilingual Learner Math Toolkit of Strategies

Graduate, S., Zwiers, J., Dieckmann, J., Rutherford-Quach, S., Daro, V., Skarin, R., Weiss, S., & Malamut, J. (2017). *Understanding Language/SCALE Principles for the Design of Mathematics Curricula: Promoting Language and Content Development*. https://ul.stanford.edu/sites/default/files/resource/2021-11/Principles%20for%20the%20Design%20of%20Mathematics%20Curricula_1.pdf

Kelemanik, G., Lucenta, A., Susan Janssen Creighton, & Lampert, M. (2016). *Routines for reasoning : fostering the mathematical practices in all students*. Heinemann.

Some Considerations for Different English Proficiency Levels
Note: See section titled "Scaffolding" for more information on how to scaffold for different proficiency levels.

Emerging	Expanding	Bridging
• Provide sentence frames at their proficiency level. • Students may be able to describe text elements with substantial support. • Students may be able to use frequently used affixes, context and visual cues to determine the meaning of unknown words on familiar topics.	• Provide sentence frames at their proficiency level. • Students may be able to describe text elements with moderate support. • Students may be able to use morphology, context and visual cues to determine the meaning of unknown words on familiar topics.	• Provide sentence frames at their proficiency level. • Students may be able to describe text elements with light support. • Students may be able to use morphology, context and visual cues to determine the meaning of unknown words on familiar and new topics.
Develop essential academic vocabulary for the concept and allow students to practice using it.		

Adapted from the ELD Proficiency Level Continuum in the California English Language Development Standards.

Integrated ELD

The focus will be on the content.

Sample Language Objective

- Students will listen to a text - listening for a different purpose each time to determine the meaning.

Standards Connections (refer to the Figure 3.4 for more detail on the standards):

- Collaborative Mode: Students will exchange information through oral collaborative exchanges (when they share their questions with partners and the class).
- Collaborative Mode: Students will orally negotiate with and persuade others in communicative exchanges (when sharing with a partner responses to the three different questions).
- Interpretive Mode: Students will listen actively to other students' responses (when working with partners or listening to whole-class ideas).

- Interpretive Mode: Students will view text and multimedia to determine the meaning (when students read the given prompt).
- Interpretive Mode: Students will evaluate the responses and arguments of others (when listening to their partners ideas and reasons).
- Interpretive Mode: Students will analyze how other students use vocabulary and other language resources to explain their thinking (when listening to understand what other students' ideas are).
- Productive Mode: Students will compose informational text to explain ideas and information (when students write their personal notes for each of the three reads).
- Productive Mode: Students will select and apply precise vocabulary and language structures (when determining their shared responses).

Supporting Language Development

- Provide differentiated sentence frames.
- Think-Pair-Share
- Visuals
- Picture Vocabulary
- Think Time
- Total Physical Response (TPR)
- Student drawings/sketches
- Small Groups
- Tables, Charts, Graphic Organizers
- Word banks/Word Walls
- Guided Questions

Designated ELD

The focus will be on how language works.

- **Into:**
  - **Sample Language Objective:**
    - Students will listen to a text - listening for a different purpose each time to determine the meaning.
  - **Supporting Language Development**
    - Read through the prompt together. Look for any figurative language that needs further explanation.
- **From:**
  - **Sample Language Objective:**
    - Students will listen to a text - determining if there are areas that need clarification (more precise nouns or verbs, or if more details are needed).
  - **Supporting Language Development**
    - Reread the prompt. Select one sentence and look at the syntax.
      - Discuss what verbs were used. Are there any other verbs that could have been used that would have made the problem clearer?
      - Discuss the nouns that were used. Are there any nouns that were used

that could have made the problem clearer.

- Look at the entire prompt. Are there any areas where additional details could have been added to increase clarity? Are there any sentences that could have been combined to condense ideas without taking away from the meaning? Are there any pieces of information that didn't need to be in the prompt at all?

Formative Assessment

Using the ELD Proficiency Level Continuum in the CA ELD Standards…..

Integrated

- check that students can exchange information with others orally (ELD.PI.1).
- check that students can read closely literary and informational texts and view multimedia to determine meaning (ELD.PI.6).
- check that students can evaluate how well writers use language to support their ideas (ELD.PI.7).
- check that students can express information and ideas in formal presentations (ELD.PI.9).
- Check that students can support their own opinions and evaluate others' arguments in speaking and writing (ELD.PI.11).

Designated

- check that students understand text structure based on purpose, text type, and discipline (ELD.PII.2)
- check that students understand cohesion (ELD.PII.2).
- check that students can use verbs/verb phrases to create clarity (ELD.PII.3).
- check that students can use nouns/noun phrases to create clarity (ELD.PII.4).
- check that students can add details to create precision (ELD.PII.5).
- check that students can connect ideas within sentences by combining clauses and joining ideas (ELD.PII.6).
- check that students can condense ideas within sentences (ELD.PII.7).

K-5 Resources and Videos

Coming Soon

6-12 Resources and Videos

Coming Soon

Compare and Connect

What Is This Strategy?

Students analyze and discuss mathematical ideas by comparing and connecting different methods, representations, and solutions. These methods may be student-created or teacher-provided. This approach deepens conceptual understanding and encourages the use of precise mathematical language. By comparing different strategies, students reflect on their reasoning and make connections between ideas. The routine promotes mathematical communication, critical thinking, and problem-solving skills.

Why Use This Strategy?

Encourages meta-cognitive and meta-linguistic awareness. Students reflect on their own thinking processes and how language shapes mathematical understanding.

Promotes critical thinking and reasoning skills. Students analyze and justify different approaches, fostering logical reasoning.

Helps students recognize multiple ways to approach and solve problems. Exploring varied strategies highlights flexibility and adaptability in problem-solving.

Enhances mathematical communication and discourse. Students practice articulating ideas clearly, improving their ability to explain and defend their reasoning while learning from one another.

When to Use This Strategy?

Use this routine to compare multiple strategies, analyze and correct errors, and deepen understanding to consolidate learning and connect ideas. This routine gives students the opportunity to communicate their thinking, justify their reasoning and ask questions.

How Do I Plan For This Strategy

1. **Select mathematical tasks that allow for multiple approaches or representations.**
    - i.e. numerical, graphical, or algebraic methods, to provide students with diverse entry points.
2. **Prepare visual aids or examples that highlight different methods or perspectives.**
    - Use charts, diagrams, and models to illustrate key concepts and emphasize connections between strategies.

3. **Develop guiding questions to prompt reflection and discussion.**
    - "What is similar and different about these approaches?"
    - "Why might someone choose this method over another?"
    - "How does this representation help us understand the problem?"
    - "What connections can we make between these strategies?"
4. **Create sentence starters to scaffold student responses.** Provide prompts such as:
    - "I noticed that both methods..."
    - "This approach is similar because..."
    - "One difference between these strategies is..."
    - "I prefer this method because..."
5. **Plan for both individual and group work**
    - Start with individual reflection before transitioning to group discussions, ensuring all students have time to formulate ideas and contribute.
    - Determine how you will pair/group students (i.e. randomized groups of 3)

Steps

1. **Present the Problem**
    a. Present a problem that allows for two or more mathematical methods, representations, or examples.
2. **Think Time**
    a. Students work on problem independently
3. **Pair/Group Students**
    a. Review Norms
    b. Provide students with the task: compare and connect their strategies
        i. "What is similar and different about these approaches?"
        ii. "Why might someone choose this method over another?"
        iii. "How does this representation help us understand the problem?"
        iv. "What connections can we make between these strategies?"
4. **Facilitate Student Discussion**
    a. Encourage students to describe, compare, and contrast approaches using mathematical language.
    b. Provide sentence starters
        i. "I noticed that both methods..."
        ii. "This approach is similar because..."

iii. "One difference between these strategies is..."

iv. "I prefer this method because..."

c. Utilize guiding questions

5. **Facilitate Group Discussion**
    a. Highlight the mathematics
        i. Relationships, operations, quantities, values, etc.
    b. Utilize Questions to compare ideas and/or language, emphasizing comparative language
    c. Summarize key observations and emphasize connections between methods.
6. **Reflect and Close**
        i. Students reflect on how they behaved as a mathematician.
        ii. Students reflect on the strategies and how they are connected.

Example

1. **Present the Problem**
    a. **Teacher:** *Let's look at this problem together: 345 + 219.*
    b. **Teacher:** *Take a moment to think about how you might solve it. What are some different ways we can solve this problem?*
2. **Think Time**
    a. **Teacher:** *"Here is a list of strategies you can choose from. Try using more than one strategy".* Teacher provides a list of strategies students have the option to choose from.
3. **Pair/Group Students**
    a. **Teacher:** "Let's review our norms for partner talk. Remember to take turns, listen and ask questions".
    b. **Teacher:** Now, discuss your strategies with a partner. Compare and connect your methods.
    c. *Students pair up and share their strategies, taking turns and asking questions.*
4. **Facilitate Student Discussion**
    a. ***Teacher:*** *Now, discuss your strategies with a partner. Compare and connect your methods.*
    b. *Students pair up and share their strategies, taking turns and asking questions.*
    c. ***Teacher:*** *What did you notice about your strategies? How are they similar or different?*
    d. ***Student 1:*** *We both got the same answer and used place value, but I decomposed the numbers into hundreds, tens, and ones, while she used a number line.*
    e. ***Student 2:*** *Yes! I started with 345 on the number line, then jumped 200, then 10, then 9. So, we both used place value, but he did it for both numbers, and I only broke apart the second number.*
    f. ***Teacher:*** *How does using a number line help us understand the problem?*
    g. ***Student 1:*** *A number line helps us break the problem into smaller chunks and makes it easier to see how numbers come together to get the final answer.*

h. ***Teacher:*** *Can you revoice what he said?*
i. ***Student 2:*** *Sure! A number line helps break the problem into steps, making it easier to see how numbers add up to the final answer.*

*Teacher continues to walk around and encourage students to describe, compare, and contrast approaches using mathematical language.

5. **Facilitate Group Discussion**
    a. **Teacher:** *Let's come back together and share one method your group discussed. How do these methods connect?*
    b. **Teacher:** *What's the benefit of breaking apart the numbers by place value before adding? How does the standard algorithm help keep track of the place values? How are the number line and base ten blocks connected?*
    c. Students respond and discuss
    d. **Teacher**: *Now, let's think about the mathematics a little more. When we break numbers apart by place value (decomposing), this can help us understand the math behind the algorithm.*
6. **Reflect and Close**
    a. **Teacher Prompts:**
        i. *How did you behave like a mathematician today? Did you explain your thinking clearly?*
        ii. *How do different methods in math help you understand the concept at a deeper level?*
        iii. *How did using clear mathematical language help you compare and connect the strategies?*
        iv. *What did you like about working with a partner*?

*Students reflect and share their thoughts.

Source:
Graduate, S., Zwiers, J., Dieckmann, J., Rutherford-Quach, S., Daro, V., Skarin, R., Weiss, S., & Malamut, J. (2017). *Understanding Language/SCALE Principles for the Design of Mathematics Curricula: Promoting Language and Content Development*. https://ul.stanford.edu/sites/default/files/resource/2021-11/Principles%20for%20the%20Design%20of%20Mathematics%20Curricula_1.pdf

Some Considerations for Different English Proficiency Levels *Note: See section title "Scaffolding" for more information on how to scaffold for different proficiency levels.*		
Emerging ● Provide sentence frames at their proficiency level. ● Allow students to share their ideas in their primary language and	**Expanding** ● Provide sentence frames at their level. ● Students may be able to respond in complete sentences. ● Students may be able to	**Bridging** ● Provide sentence frames at their level. ● Begin encouraging students to add more details. ● Begin encouraging

then translate their ideas into English. ● Allow students to use pictures and gestures to aid in expressing their ideas. ● Student responses may be brief (one word responses or short phrases)	respond more independently stating both an opinion and supporting reasons.	students to connect ideas by combining clauses.
Develop essential academic vocabulary for the concept and allow students to practice using it.		

Adapted from the ELD Proficiency Level Continuum in the California English Language Development Standards.

Integrated ELD

The focus will be on the content.

Sample Language Objective

- Students will be able to orally discuss different strategies for reaching the solution of a mathematical problem - using compare and contrast language structures.

Standards Connections (refer to the Figure 3.4 for more detail on the standards):

- Collaborative Mode: Students will collaborate by orally comparing and contrasting strategies.
- Collaborative Mode: Students will orally state their opinion (which strategy they feel is better) and justify it with reasons.
- Interpretive Mode: Students will listen carefully to other students' responses in order to be able to make connections and compare and contrast.
- Productive Mode: Students will support their own opinions and evaluate the arguments of others.

Supporting Language Development

- Provide differentiated sentence frames.
- Think-Pair-Share
- Visuals
- Total Physical Response (TPR)
- Drawings

Designated ELD

The focus will be on how language works.

- **Into:**
    - **Sample Language Objective:**
        - Students will demonstrate their understanding of text structure by being able to arrange the words in order to create a complete sentence.

- **Supporting Language Development**
  - Cut the teacher questions into individual word cards. Have the students work in pairs or groups to arrange the word cards (for one of the questions) into complete sentences.
  - Discuss the various terms that could be used for compare and contrast (compare/similar, contrast/different).
  - Discuss what is meant by the terms students may not be familiar with or may have multiple meanings (connect, represent).

- **From:**
  - **Sample Language Objective:**
    - Students will demonstrate their understanding of text structure by being able to arrange the words in order to create a complete sentence.

  - **Supporting Language Development**
    - Collect samples of student answers given during Math core time.
      - Have students arrange the cards in order to create a complete sentence. Discuss the order of the words. Explore if the words can be moved around without changing the meaning (ex: I think that my strategy is the best *because* I had the right answer. → I had the right answer, *so* I think that my answer is the best).
      - Model how students can connect ideas by combining clauses and joining ideas. (ex: I had the right answer. I think my strategy is the best. → I had the right answer, so I think my strategy is the best.)

Formative Assessment

Using the ELD Proficiency Level Continuum in the CA ELD Standards…..

Integrated

- check that students can exchange information with others orally (ELD.PI.1).
- check that students can offer and justify their opinion (ELD.PI.3).
- check that students can listen actively to spoken English. (ELD.PI.5).
- check that students can support their own opinions and evaluate the opinions of others (ELD.PI.11)

Designated

- check that students understand cohesion (ELD.PII.2).
- check that students can connect ideas within sentences (ELD.PII.6).

K-5 Resources and Videos

Coming Soon

6-12 Resources and Videos

Coming Soon

KWL Chart

What Is This Strategy?

A KWL chart is a graphic organizer that helps students record and organize their learning process in three stages. The first stage, labeled "K," represents what students already know about a topic. The second stage, "W," is for what students want to know or what questions they have. The final stage, "L," is used to record what students have learned after the lesson or activity. This tool encourages active engagement, critical thinking, and communication.

Why Use This Strategy?

Using a KWL chart in a math classroom supports both content mastery and language development. It allows students to connect prior knowledge to new concepts, set learning goals, and reflect on their understanding. This approach fosters a growth mindset and helps students recognize their progress in mastering math skills.

Connect Prior Knowledge to New Concepts: Activates and organizes existing knowledge, providing a foundation for learning new math concepts. Encourages the use of academic language to articulate what they already know.

Set Learning Goals: Guides students to ask meaningful, content-related questions, helping them develop critical thinking and inquiry skills. Promotes the use of precise mathematical vocabulary and sentence structures as they formulate their "W" section.

Reflect on Their Understanding: Provides an opportunity for students to summarize their learning, reinforcing new vocabulary and language structures. Encourages discussion and collaboration, supporting oral language skills.

Enhances mathematical communication and discourse. Promotes collaboration and discussion, helping students explain and refine their thinking.

When to Use This Strategy?

A KWL chart is especially effective at different stages of learning and is a valuable tool when developing language skills alongside math concepts. At the start of a new unit, it helps activate prior knowledge, allowing students to connect what they already know to the new topic while encouraging the use of academic vocabulary. During the lesson, the chart serves as a tool to monitor learning goals and fosters communication as students articulate questions and ideas. After the lesson, it provides a structured way to assess understanding, consolidate learning, and practice summarizing concepts using precise mathematical language.

How Do I Plan For This Strategy

1. **Identify Goals for Math and Language**
    - Define the specific math concepts students will learn and the language skills they need, such as using academic vocabulary or forming questions.
2. **Anticipate Prior Knowledge and Misconceptions**
    - Reflect on what students might already know and any common misconceptions they may have about the topic.
3. **Prepare Supporting Materials**
    - Chart Paper and Markers to create chart (whole class) or blank paper for students to create their own.
    - Gather resources like visuals, manipulatives, or real-world examples to support understanding and engagement.
4. **Provide Scaffolds**
    - Create sentence starters, word banks, or graphic supports.
        i. I know that _____. (K)
        ii. I think _____ about this topic. (K)
        iii. I want to know _____. (W)
        iv. What is the difference between _____ and _____? (W)
        v. I learned that _____. (L)
        vi. I understand _____ now. (L)
5. **Plan Time for Reflection**
    - Include time in your lesson plan for students to revisit their KWL chart and reflect on their learning to reinforce both content and language development.

Steps

1. **Introduce the KWL Chart:**
    a. Explain the purpose/structure of the chart and how this will help organize thoughts before, during, and after the lesson.
    b. Students listen and ask clarifying questions if needed.
2. **Complete the "K" Section:**
    a. Ask students to brainstorm what they already know about the topic.
    b. Ask guiding questions to prompt students to recall prior knowledge.
    c. Use sentence frames to support responses:
        i. "I know that _____."
        ii. "I think _____ about this topic."
    d. Students reflect on what they already know, share ideas, and use sentence frames to articulate.

e. Teacher writes responses in the K column or students write in their K column. (varies from whole group to individual)

3. **Fill Out the "W" Section:**
    a. Have students think about questions or goals they want to learn. (they can also write them down)
    b. Guide students to think about what they still need to know. Prompt them to formulate questions or identify learning goals:.
    c. Provide sentence frames to support responses
        i. "What do you want to learn about _____?"
        ii. "How does _____ work?"
        iii. "What is the relationship between _____?"
    d. Teacher writes responses in the W column or students write in their W column. (varies from whole group to individual)
4. **Students Learn Content**
    a. Students learn new content with an emphasis on developing language goals
        i. Teacher Led: Model, Think-Aloud, etc.
        ii. Student Centered: Exploration/Investigation, Thinking Task
    b. Students actively listen and try to connect new content with what they already know
    c. Teacher encourages students to ask questions and use academic language
5. **Complete the "L" Section:**
    a. After the lesson, guide students to reflect on what they learned about the topic
    b. Provide sentence frames to help students articulate their thinking/learning:
        i. "I learned that _____."
        ii. "Now I understand _____."
        iii. "This is how _____ works."
    c. Teacher writes responses in the L column or students write in their L column. (varies from whole group to individual)
6. **Review and Discuss:**
    a. Discuss insights from the chart as a class to reinforce learning
        i. Encourage students to share their reflections or lingering questions
        ii. Reinforce the math and language objectives
        iii. Students listen, offer additional thoughts, and make connections

Example

Standard: 6.RP.A.3.C Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent.

1. **Introduce the KWL Chart:**

a. *"This chart has three sections: 'K' for what we already know, 'W' for what we want to learn, and 'L' for what we learned. We'll fill out each section as we go through the lesson. First, let's think about what we already know about percents."*

2. **Complete the "K" Section:**
    a. **Teacher:** *"Let's brainstorm together. What do you already know about percents? Think about how you might have seen percentages in real life, like at a store or in sports. Use these sentence frames: 'I know that _____' or 'I think _____ about this topic.'"*
    b. **Student 1:** *"I know that percent means out of 100."*
    c. **Teacher:** *"Great! Let's write that down. Percent means out of 100"*
    d. **Student 2:** "*I think 50% means half of a whole"*
    e. **Teacher:** *"Yes, that's right. 50% is the same as one-half, or 50 out of 100. I'll write that down as well. Keep thinking, what else do you know?"*
    f. **Student 3:** *"I remember using percents when we had sales, like 30% off something"*
    g. **Teacher:** "*Perfect, let's write down that percents are used for sales and discounts"*
3. **Fill Out the "W" Section:**
    a. *Now that we have some ideas about what we know, let's think about what we still want to learn. What do you want to know more about? Use the sentence frame: 'I want to know _____' or 'How does _____ work?'"*"
    b. **Student 1:** *"How do you find a percent of a number?"*
    c. **Teacher:** *"Great question. We'll definitely be learning that. Let's write that down in the W column".*
    d. **Student 2:** "What is the difference between percentages and fractions?"
    e. ***Teacher:*** *"Excellent questions. Let's add that to our chart".*
    f. **Student 3:** *"I want to know how to figure out a percent if I know part of the whole, like 20% of something."*
    g. **Teacher:** "Another great question. Let's add that one".
4. **Students Learn Content**
    a. "Let's dive into learning! Today, we're going to talk about how to find percentages of numbers and how percents relate to our daily lives."

*Learning can vary (student exploration, use of ration tables, double number lines, or tape diagrams, teacher led instruction, etc)

7. **Complete the "L" Section:**
    a. **Teacher:** *"Now that we've learned about how to find a percentage, let's fill out the 'L' section of the chart. What did we learn today? Use these sentence frames: 'I learned that _____' or 'Now I understand _____.'"*
    b. **Student 1:** *"I learned that 20% of 150 is 30, and I can find it by using a tape diagram to visually see the amount."*

c. T**eacher:** *“That’s right! I’ll add that to the 'L' section.”*
Student 2: *“I learned that to find a percent of a number, you can use a tape diagram to show/determine the part of the whole.”*
d. **Student 3:** *"Now I understand that you can find percent using a variety of methods.”*
e. **Teacher:** *"Excellent! Let’s add that to the 'L' column: 'Now I understand that percents can be found using a variety of methods such as a tape diagram, double number line, equation, and ratio table.”*
f. **Teacher:** *(Writes responses in the L column)*

8. **Review and Discuss:**
   a. **Teacher:** *“Let’s take a moment to look at everything we’ve written down. What did you learn today? Does anyone have any lingering questions or thoughts?”*

Source:

Balisar, N., & Situmorang, R. (2018). The Effectiveness Of K-W-L Technique To Increase Students’ Reading Comprehension Achievement Through Reading Descriptive Text For Grade 8 of SMP Negeri 1 Cisarua. https://files.eric.ed.gov/fulltext/EJ1304478.pdf

Some Considerations for Different English Proficiency Levels *Note: See section titled “Scaffolding” for more information on how to scaffold for different proficiency levels.*		
Emerging ● Provide sentence frames at their proficiency level. ● Students may be able to express ideas by asking and answering yes/no and wh- questions and responding with simple phrases.	**Expanding** ● Provide sentence frames at their proficiency level. ● Students may be able to ask relevant questions, affirm others, and add relevant information.	**Bridging** ● Provide sentence frames at their proficiency level. ● Students may be able to build upon the responses of others and provide useful feedback to others.
Develop essential academic vocabulary for the concept and allow students to practice using it.		

Adapted from the ELD Proficiency Level Continuum in the California English Language Development Standards.

Integrated ELD

The focus will be on the content.

Sample Language Objective

- Students will listen to an exploration of a topic by actively listening to what their classmates know and want to know about the topic.

Standards Connections (refer to the Figure 3.4 for more detail on the standards):

- Collaborative Mode: Students will exchange information through oral collaborative exchanges (when they share their prior knowledge, questions, and new learning with partners and the

class).
- Interpretive Mode: Students will listen actively to other students' responses (when working with partners or listening to whole-class ideas).
- Interpretive Mode: Students will evaluate the responses and arguments of others (when listening to their partners ideas and reasons).
- Interpretive Mode: Students will analyze how other students use vocabulary and other language resources to explain their thinking (when listening to understand what other students' ideas are).

Supporting Language Development

- Provide differentiated sentence frames.
- Think-Pair-Share
- Visuals
- Picture Vocabulary
- Think Time
- Total Physical Response (TPR)
- Student drawings/sketches
- Small Groups
- Tables, Charts, Graphic Organizers
- Word banks/Word Walls
- Guided Questions

Designated ELD

The focus will be on how language works.

- **Into:**
  - **Sample Language Objective:**
    - Students will listen to an exploration of a topic by actively listening to what their classmates know and want to know about the topic.
  - **Supporting Language Development**
    - Students can use pictures, drawings, charts or other visuals to help them explain their opinion and justification. The teacher can then validate the student's contribution and model language by restating what the student said/gestured.
- **From:**
  - **Sample Language Objective:**
    - Students will orally investigate a topic - using statements tied to prior knowledge and experiences as well as questions about the topic.
  - **Supporting Language Development**
    - Provide differentiated sentence frames to help students state what they know about the topic (ex: I know _____).
    - Provide differentiated sentence frames to help students state what they would like to know about the topic (ex: I would like to know_____. I wonder _____? Why____?).
    - Provide differentiated sentence frames to help students state what they have

learned about the topic (ex: I learned ____. I now know ____. Based on the text, I know _____ and _____).

Formative Assessment

Using the ELD Proficiency Level Continuum in the CA ELD Standards…..

Integrated

- check that students can exchange information with others orally (ELD.PI.1).
- check that students can adapt their language choices based on the task, purpose and audience (ELD.PI.4).
- check that students can listen actively to spoken English in a range of academic contexts (ELD.PI.5).
- check that students can view text and multimedia closely to determine meaning (ELD.PI.6).
- check that students can select and apply varied and precise vocabulary and language structures to effectively convey ideas (ELD.PI.12).

Designated

- check that students can understand text structure and organization (ELD.PII.1).
- check that students understand cohesion (ELD.PII.2).
- check that students can add details and provide more information to create precision (ELD.PII.5).

K-5 Resources and Videos

Coming Soon

6-12 Resources and Videos

Coming Soon

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