JEE MAINS PATTERN REVISION CLASS XI

SARASWATI CLASSES

Section A

Q1) Let *S* be the locus of a moving point *z* in the Argand plane satisfying the equation:

$|z - 2|^2 + |z + 2|^2 = 10$

Let z_0 be a specific point on the locus *S* situated in the second quadrant. A tangent is drawn from a point z_1 lying on the circle *C*: $|z| = 2$ to the curve *S* such that the point of contact is exactly z_0.

Find the value of $|z_0 - z_1|$.

a) $2\sqrt{3}$ b) $\sqrt{5}$ c) 1 d) $\sqrt{3}$

Q2)

Let $\{a_n\}$ be a sequence such that $a_0 = 1, a_1 = 0, a_n = 3a_{n-1} - 2a_{n-2}$. The correct statement is: SS

(A) $\sum_{i=1}^{50} a_i = 2(51 - 2^{50})$ (B) $a_{51} = 2^{25} - 2$ (C) $a_{48} = 2(1 - 2^{47})$ (D) $\sum_{i=1}^{49} a_i = (2^{49} - 50)$

Q3) Let m,n be natural numbers such that the roots of the equation $nx^2-mx+1=0$ always lie between m-1 and n+1. Then the minimum value of n is

a)1 b) 2 c) 5 d) 4

Q4) Let ABC be a triangle such that the centre of circle passing through the vertices A,B,C bisects the side BC. If the coordinates of the vertex A and centre of the circle are (0,0) and (3,4) respectively, then the coordinates of the point of intersection of CM and BN where M and N are midpoints of the sides opposite to vertex they origin from is

a) (6/5, 8/5) b) (2, 8/3) c) (5/2, 7/2) d) (1, 2)

Q5) Sum of all possible values of ordinate of centre of the circle which touches x axis at a distance of 3 from origin and intercepts a length of $2\sqrt{7}$ is

a) 0 b) 2 c) 3 d) 1

Q6)

7 Let a, b and c be real numbers, each greater than 1, such that

$$\frac{2}{3}\log_b a + \frac{3}{5}\log_c b + \frac{5}{2}\log_a c = 3.$$

If the value of b is 9, then the value of a must be

(A) $\sqrt[3]{81}$ (B) $\frac{27}{2}$ (C) 18 (D) 27.

If $e^{f(x)} = \dfrac{10 + x}{10 - x}$, $x \in (-10, 10)$ and

$f(x) = kf\left(\dfrac{200x}{100 + x^2}\right)$, then $k =$

(A) 0.5 (B) 0.6

(C) 0.7 (D) 0.8

Q7)

Q8)

$A_r, r = 1, 2, 3, \ldots, n,$ are n points on the parabola $y^2 = 4x$ in the first quadrant. If $A_r = (x_r, y_r)$, where $x_1, x_2, x_3, \ldots, x_n$ are in GP and $x_1 = 1$, $x_2 = 2$, then y_n is equal to

(a) $y_{11} = 2^6$ (b) $y_{17} = 2^9$ (c) $y_{22} = 2^{\frac{23}{2}}$ (d) $y_{18} = 2^9$

(please solve for yn is not equal to)

Q9)

For each integer $n \geq 2$, let $A(n)$ be the area of the region in the coordinate plane defined by the inequalities $1 \leq x \leq n$ and $0 \leq y \leq x\left\lfloor\sqrt{x}\right\rfloor$, where $\left\lfloor\sqrt{x}\right\rfloor$ is the greatest integer not exceeding $\sqrt{x}$. Find the number of values of n with $2 \leq n \leq 1000$ for which $A(n)$ is an integer.

a) 289 b) 196 c) 483 d) 144

Q10) The value of x for which the 4th term in the expansion of

$$\left[\left(5^{\frac{1}{3}}\right)^{-\frac{1}{2}\log_{10}\left(6-\sqrt{8}x\right)}+\left(\frac{5^{\log_{10}(x-1)}}{25^{\log_{10}5}}\right)^{\frac{1}{6}}\right]^{m}$$

is
equal to 84/5, if it is known that 14/9 of binomial coefficient of 3rd term, binomial coefficient of 4th term and binomial coefficient of 5th term constitute a GP

a) 4 b) 3 c) 2 d) 12

Q11) A tangent is drawn at any point P on the parabola $y^2=8x$. If from a point Q on it tangents are dropped to circle $x^2+y^2=4$ then the correct statements are

1. The number of points from which perpendicular tangents can be dropped to both the circle and the parabola is 1
2. The number of points from which perpendicular tangents can be dropped to both the circle and parabola is 2
3. For a fixed point P the locus of circumcentre is a circle

The correct statements are

a) Only 1 b) Only 1 and 3 c) Only 2 d) Only 2 and 3

Q12)

Let $\theta_1, \theta_2, \ldots, \theta_{10}$ be positive valued angles (in radian) such that $\theta_1 + \theta_2 + \cdots + \theta_{10} = 2\pi$. Define the complex numbers $z_1 = e^{i\theta_1}$, $z_k = z_{k-1}e^{i\theta_k}$ for $k = 2, 3, \ldots, 10$, where $i = \sqrt{-1}$. Consider the statements P and Q given below:

$$P : |z_2 - z_1| + |z_3 - z_2| + \cdots + |z_{10} - z_9| + |z_1 - z_{10}| \leq 2\pi$$

$$Q : |z_2^2 - z_1^2| + |z_3^2 - z_2^2| + \cdots + |z_{10}^2 - z_9^2| + |z_1^2 - z_{10}^2| \leq 4\pi$$

Then,

(A) P is **TRUE** and Q is **FALSE**

(B) Q is **TRUE** and P is **FALSE**

(C) both P and Q are **TRUE**

(D) both P and Q are **FALSE**

Q13) A sequence of numbers a,b,c is said to form a mountain if a<b and b>c. Then for the sequence 5,3,4,2,9,1,6,17,8,3, number of subsequences that forms a mountain are

a) 43 b) 40 c) 41 d) 44

Q14) F_1 and F_2 are the right and left foci respectively for the ellipse $x^2/16+y^2/64=1$. When angle F_1PF_2 is maximum, the value of ratio PF_1/PF_2 if P lies on the line $x-\sqrt{3}y+8+2\sqrt{3}=0$

a) 3 b) $\sqrt{3}$ c) 2 d) $\sqrt{2}$

Q15)

If $x = \sqrt[3]{49} + \sqrt[3]{42} + \sqrt{36}$, then the value of $x - \frac{1}{x^2}$ is

a) $2\sqrt[3]{42}$

b) $3\sqrt[4]{42}$

c) $\sqrt[4]{42}$

d) $4\sqrt[3]{42}$

Section B

Q1)

11. There are several tea cups in the kitchen, some with handles and the others without handles. The number of ways of selecting two cups without a handle and three with a handle is exactly 1200. What is the maximum possible number of cups in the kitchen?

Q2)

If the value of $$\frac{\sin x}{\cos 3x} + \frac{\sin 3x}{\cos 9x} + \frac{\sin 9x}{\cos 27x}$$ = k(tan(bx)-tan(ax)) then the value of a+b+k is

Q3)

A light source at the point $(0, 16)$ in the coordinate plane casts light in all directions. A disc (a circle along with its interior) of radius 2 with center at $(6, 10)$ casts a shadow on the X axis. The length of the shadow can be written in the form $m\sqrt{n}$ where m, n are positive integers and n is square-free. Find $m + n$.

Q4)

(16 marks) In a plane rectangular coordinate system, F_1 and F_2 are the foci of hyperbola $\Gamma : \frac{x^2}{3} - y^2 = 1$. Point P on Γ satisfies $\overrightarrow{PF_1} \cdot \overrightarrow{PF_2} = 1$. Find the sum of the distances from P to the two asymptotes of Γ.

Q5)

(20 marks) Suppose positive numbers a_1, a_2, a_3, b_1, b_2 and b_3 satisfy: a_1, a_2 and a_3 form an arithmetic sequence with common difference b_1, b_1, b_2 and b_3 form a geometric sequence with common ratio a_1, and $a_3 = b_3$. Find the minimum value of a_3, and determine the value of $a_2 b_2$ when a_3 takes the minimum value.

Then the sum of both of these values is

Q6)
Let C(n,r) be the number of ways for choosing r objects from n.
Then the value of C(10,0)C(20,10) -C(10,1)C(18,9) +
C(10,2)C(16,10)....-C(10,5)C(10,10)

Q7) If c,d are 2 AM’s inserted between a and b and the equations
ax2+x+b=0 and cx2+x+d=0 have a negative root in common then the value of a+b is

Q8) If the reflected ray and incident ray are respectively 11x+2y-41=0 and 3x-4y+7=0 then the distance of the point (14,5/2) from the mirror is

Q9) If $\sqrt{3\sqrt{2} + \sqrt{10}}$ is of the form

$a + \sqrt{b}$ where a and b are coprime numbers then the value of a+b is

Q10) Given

$$\sqrt{x^2 + y^2 - 6x + 8y + 40} + \sqrt{x^2 + y^2 + 10x - 12y + 61} = 20$$

Then the radius of circle at which all perpendicular tangents to (x,y) meet at is