1.3 Rates of Change in Linear & Quadratic Functions

TEACHER ANSWER KEY | Classifying functions

1. Could the function be linear, quadratic, or neither? Justify.

x	2	5	8	11	14
f(x)	4	-2	-2	4	16

Compute Δy divided by Δx for each adjacent pair.

Average rates: -2, 0, 2, 4.

Changes in rates: 2, 2, 2.

The input intervals are equal and the changes in average rates are constant and nonzero.

ANSWER: Quadratic.

2. Could the function be linear, quadratic, or neither? Justify.

x	2	6	18	54	162
f(x)	4	7	10	13	16

Compute Δy divided by Δx for each adjacent pair.

Average rates: $\frac{3}{4}$, $\frac{1}{4}$, $\frac{1}{12}$, $\frac{1}{36}$.

Changes in rates: $\frac{-1}{2}$, $\frac{-1}{6}$, $\frac{-1}{18}$.

The average rates are not constant, and their changes do not fit a quadratic model.

ANSWER: Neither.

3. Could the function be linear, quadratic, or neither? Justify.

x	-3	0	2	6	11
f(x)	-7	2	8	20	35

Compute Δy divided by Δx for each adjacent pair.

Average rates: 3, 3, 3, 3.

Changes in rates: 0, 0, 0.

All average rates are equal.

ANSWER: Linear.

1.3 Rates of Change in Linear & Quadratic Functions

TEACHER ANSWER KEY | Classifying functions

4. Could the function be linear, quadratic, or neither? Justify.

x	-6	-3	0	3	6
f(x)	1	3	11	29	61

Compute Δy divided by Δx for each adjacent pair.

Average rates: $\frac{2}{3}$, $\frac{8}{3}$, 6, $\frac{32}{3}$.

Changes in rates: 2, $\frac{10}{3}$, $\frac{14}{3}$.

The average rates are not constant, and their changes do not fit a quadratic model.

ANSWER: Neither.

5. Could the function be linear, quadratic, or neither? Justify.

x	2	6	10	14	18
f(x)	7	7	7	7	7

Compute Δy divided by Δx for each adjacent pair.

Average rates: 0, 0, 0, 0.

Changes in rates: 0, 0, 0.

All average rates are equal; a constant function also fits the linear model.

ANSWER: Linear.

6. Could the function be linear, quadratic, or neither? Justify.

x	1	5	9	13	17
f(x)	3	7	5	-3	-17

Compute Δy divided by Δx for each adjacent pair.

Average rates: 1, $\frac{-1}{2}$, -2, $\frac{-7}{2}$.

Changes in rates: $\frac{-3}{2}$, $\frac{-3}{2}$, $\frac{-3}{2}$.

The input intervals are equal and the changes in average rates are constant and nonzero.

ANSWER: Quadratic.

1.3 Rates of Change in Linear & Quadratic Functions

TEACHER ANSWER KEY | Concavity from tables

7. Could the function be concave up, concave down, or neither? Justify.

x	2	5	8	11	14
f(x)	-14	-5	1	4	5

Compute Δy divided by Δx for each adjacent pair.

Average rates: 3, 2, 1, $\frac{1}{3}$.

Changes in rates: -1, -1, $\frac{-2}{3}$.

Average rates strictly decrease.

ANSWER: Concave down.

8. Could the function be concave up, concave down, or neither? Justify.

x	2	4	6	8	10
f(x)	15	11	7	3	-1

Compute Δy divided by Δx for each adjacent pair.

Average rates: -2, -2, -2, -2.

Changes in rates: 0, 0, 0.

Average rates are constant or fail to change consistently in one direction.

ANSWER: Neither.

9. Could the function be concave up, concave down, or neither? Justify.

x	2	6	10	14	18
f(x)	0	4	12	28	40

Compute Δy divided by Δx for each adjacent pair.

Average rates: 1, 2, 4, 3.

Changes in rates: 1, 2, -1.

Average rates are constant or fail to change consistently in one direction.

ANSWER: Neither.

1.3 Rates of Change in Linear & Quadratic Functions

TEACHER ANSWER KEY | Average rate of change

10. Find the average rate of change of $f(x) = 4x - 9$ on $[-4, 3]$. Evaluate both endpoints: $f(-4) = -25$, $f(3) = 3$. Subtract outputs and divide by the change in inputs: $(3 - (-25)) \div (3 - (-4)) = 4$. ANSWER: 4.	11. Find the average rate of change of $f(x) = -3x - 10$ on $[4, 8]$. Evaluate both endpoints: $f(4) = -22$, $f(8) = -34$. Subtract outputs and divide by the change in inputs: $(-34 - (-22)) \div (8 - (4)) = -3$. ANSWER: -3.
12. Find the average rate of change of $f(x) = x^2 + 5x - 7$ on $[-3, 5]$. Evaluate both endpoints: $f(-3) = -13$, $f(5) = 43$. Subtract outputs and divide by the change in inputs: $(43 - (-13)) \div (5 - (-3)) = 7$. ANSWER: 7.	13. Find the average rate of change of $f(x) = 3x^2 - 8x + 6$ on $[-4, -2]$. Evaluate both endpoints: $f(-4) = 86$, $f(-2) = 34$. Subtract outputs and divide by the change in inputs: $(34 - (86)) \div (-2 - (-4)) = -26$. ANSWER: -26.

1.3 Rates of Change in Linear & Quadratic Functions

TEACHER ANSWER KEY | Rate of the average rates

14. Find the rate of change of the average rates of change of $f(x) = 17x - 4$ over consecutive equal-length intervals.

Use unit intervals [0, 1] and [1, 2]. The outputs are -4, 13, 30.

Average rates: 17 and 17. Their change per unit input is $17 - (17) = 0$.

In general this rate is $2a$ for $ax^2 + bx + c$, and 0 for a linear function.

ANSWER: 0.

15. Find the rate of change of the average rates of change of $f(x) = -2x + 11$ over consecutive equal-length intervals.

Use unit intervals [0, 1] and [1, 2]. The outputs are 11, 9, 7.

Average rates: -2 and -2. Their change per unit input is $-2 - (-2) = 0$.

In general this rate is $2a$ for $ax^2 + bx + c$, and 0 for a linear function.

ANSWER: 0.

16. Find the rate of change of the average rates of change of $f(x) = -2x^2 + 5x - 9$ over consecutive equal-length intervals.

Use unit intervals [0, 1] and [1, 2]. The outputs are -9, -6, -7.

Average rates: 3 and -1. Their change per unit input is $-1 - (3) = -4$.

In general this rate is $2a$ for $ax^2 + bx + c$, and 0 for a linear function.

ANSWER: -4.

17. Find the rate of change of the average rates of change of $f(x) = 4x^2 + 3x - 2$ over consecutive equal-length intervals.

Use unit intervals [0, 1] and [1, 2]. The outputs are -2, 5, 20.

Average rates: 7 and 15. Their change per unit input is $15 - (7) = 8$.

In general this rate is $2a$ for $ax^2 + bx + c$, and 0 for a linear function.

ANSWER: 8.

1.3 Rates of Change in Linear & Quadratic Functions

TEACHER ANSWER KEY | Concavity justification

18. Assume concavity does not change. Is the function concave up or concave down? Justify.

x	2	6	10	14	18
f(x)	48	28	16	8	4

Compute Δy divided by Δx for each adjacent pair.

Average rates: -5, -3, -2, -1.

Changes in rates: 2, 1, 1.

Average rates strictly increase.

ANSWER: Concave up.

19. Assume concavity does not change. Is the function concave up or concave down? Justify.

x	-6	-2	2	6	10
f(x)	120	112	92	60	12

Compute Δy divided by Δx for each adjacent pair.

Average rates: -2, -5, -8, -12.

Changes in rates: -3, -3, -4.

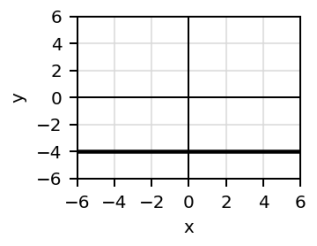
Average rates strictly decrease.

ANSWER: Concave down.

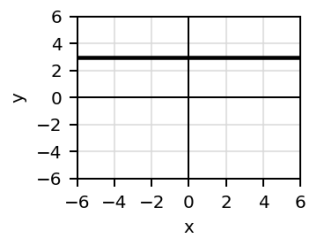
1.3 Rates of Change in Linear & Quadratic Functions

TEACHER ANSWER KEY | Graph matching

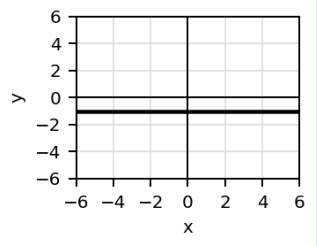
A



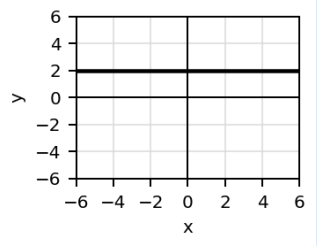
B



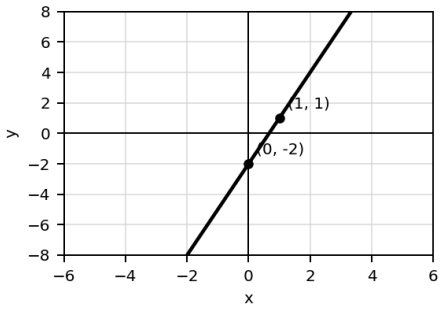
C



D



20. Find the average rate of change of the line. Match it to constant graph A–D (see choices on this page).

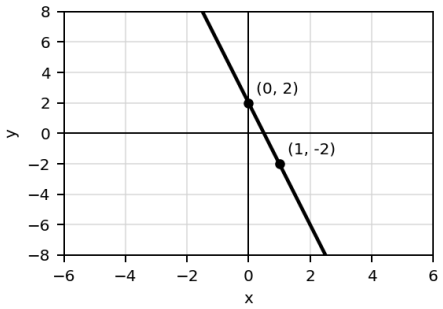


Use the marked points (0, -2) and (1, 1). Slope = (1 – (-2)) ÷ (1 – 0) = 3.

The horizontal graph at y = 3 represents this constant rate.

ANSWER: 3; choice B.

21. Find the average rate of change of the line. Match it to constant graph A–D (see choices on this page).

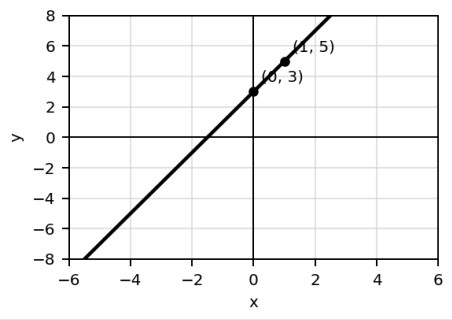


Use the marked points (0, 2) and (1, -2). Slope = (-2 – (2)) ÷ (1 – 0) = -4.

The horizontal graph at y = -4 represents this constant rate.

ANSWER: -4; choice A.

22. Find the average rate of change of the line. Match it to constant graph A–D (see choices on this page).

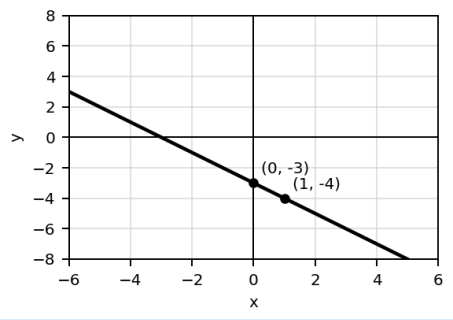


Use the marked points (0, 3) and (1, 5). Slope = (5 – (3)) ÷ (1 – 0) = 2.

The horizontal graph at y = 2 represents this constant rate.

ANSWER: 2; choice D.

23. Find the average rate of change of the line. Match it to constant graph A–D (see choices on this page).



Use the marked points (0, -3) and (1, -4). Slope = (-4 – (-3)) ÷ (1 – 0) = -1.

The horizontal graph at y = -1 represents this constant rate.

ANSWER: -1; choice C.

1.3 Rates of Change in Linear & Quadratic Functions

TEACHER ANSWER KEY | Secant line equations

24. Find the average rate of change of $f(x) = \frac{7}{2}x - 5$ on $[-4, 2]$. Write the secant line equation.

Evaluate both endpoints: $f(-4) = -19$, $f(2) = 2$.

Subtract outputs and divide by the change in inputs: $(2 - (-19)) \div (2 - (-4)) = \frac{7}{2}$.

Use $y - y_1 = m(x - x_1)$: $y - (-19) = \frac{7}{2}(x - (-4))$. Simplify.

ANSWER: $m = \frac{7}{2}$; $y = \frac{7}{2}x - 5$.

25. Find the average rate of change of $f(x) = x^2 - 4x$ on $[2, 7]$. Write the secant line equation.

Evaluate both endpoints: $f(2) = -4$, $f(7) = 21$.

Subtract outputs and divide by the change in inputs: $(21 - (-4)) \div (7 - (2)) = 5$.

Use $y - y_1 = m(x - x_1)$: $y - (-4) = 5(x - (2))$. Simplify.

ANSWER: $m = 5$; $y = 5x - 14$.

26. Find the average rate of change of $f(x) = -3x^2 + 7$ on $[2, 4]$. Write the secant line equation.

Evaluate both endpoints: $f(2) = -5$, $f(4) = -41$.

Subtract outputs and divide by the change in inputs: $(-41 - (-5)) \div (4 - (2)) = -18$.

Use $y - y_1 = m(x - x_1)$: $y - (-5) = -18(x - (2))$. Simplify.

ANSWER: $m = -18$; $y = -18x + 31$.

27. Find the average rate of change of $f(x) = x^2 - 5x + 4$ on $[3, 6]$. Write the secant line equation.

Evaluate both endpoints: $f(3) = -2$, $f(6) = 10$.

Subtract outputs and divide by the change in inputs: $(10 - (-2)) \div (6 - (3)) = 4$.

Use $y - y_1 = m(x - x_1)$: $y - (-2) = 4(x - (3))$. Simplify.

ANSWER: $m = 4$; $y = 4x - 14$.

1.3 Rates of Change in Linear & Quadratic Functions

TEACHER ANSWER KEY | Difference quotients

28. For a general function y = f(x), what does $\frac{f(8)-f(3)}{8-3}$ represent?

The numerator is the change in output and the denominator is the change in input. Their quotient measures the slope of the secant line.

ANSWER: Average rate of change on [3, 8].

29. If $f(x) = x^2 + 4x + 3$, evaluate $\frac{f(7)-f(3)}{7-3}$.

$f(7) = 49 + 28 + 3 = 80$.

$f(3) = 9 + 12 + 3 = 24$.

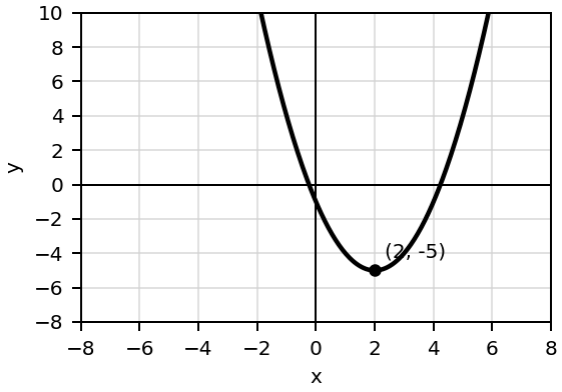
The quotient is $(80 - 24) \div 4 = 14$.

ANSWER: 14.

1.3 Rates of Change in Linear & Quadratic Functions

TEACHER ANSWER KEY | Increasing and decreasing parabolas

30. Use the graph of $y = x^2 - 4x - 1$ to estimate intervals of increase, decrease, and constant behavior. Support numerically. The parabola continues beyond the viewing window.

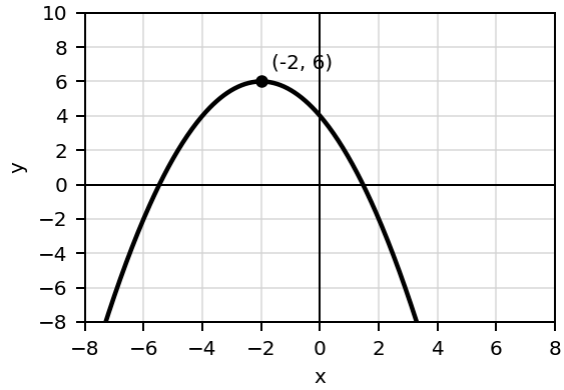


The vertex is (2, -5). Compare f(0) = -1, f(2) = -5, f(4) = -1.

The adjacent secant slopes are -2 and 2. The parabola opens up.

ANSWER: Increasing $(2, \infty)$; decreasing $(-\infty, 2)$; constant: none.

31. Use the graph of $y = -\frac{1}{2}x^2 - 2x + 4$ to estimate intervals of increase, decrease, and constant behavior. Support numerically. The parabola continues beyond the viewing window.



The vertex is (-2, 6). Compare f(-4) = 4.0, f(-2) = 6, f(0) = 4.0.

The adjacent secant slopes are 1 and -1. The parabola opens down.

ANSWER: Increasing $(-\infty, -2)$; decreasing $(-2, \infty)$; constant: none.

1.3 Rates of Change in Linear & Quadratic Functions

TEACHER ANSWER KEY | Missing table values

32. The average rate of change on [1, 9] is 7. Find the missing table value.

x	1	3	5	7	9
f(x)	4	10	19	32	?

Use average rate = (ending output − starting output) ÷ (ending input − starting input).

7 = (? − (4)) ÷ 8.

Multiply by 8 and isolate the unknown.

ANSWER: 60.

33. The average rate of change on [-3, 5] is -6. Find the missing table value.

x	-3	-1	1	3	5
f(x)	?	32	21	10	-8

Use average rate = (ending output − starting output) ÷ (ending input − starting input).

-6 = (-8 − (?)) ÷ 8.

Multiply by 8 and isolate the unknown.

ANSWER: 40.

34. The average rate of change on [-8, 16] is 4. Find the missing table value.

x	-8	-2	4	10	16
f(x)	?	14	22	-7	-30

Use average rate = (ending output − starting output) ÷ (ending input − starting input).

4 = (-30 − (?)) ÷ 24.

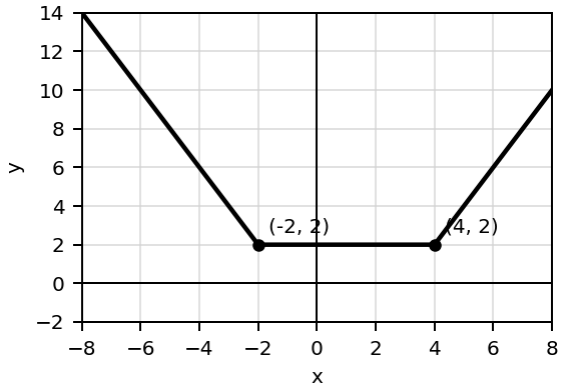
Multiply by 24 and isolate the unknown.

ANSWER: -126.

1.3 Rates of Change in Linear & Quadratic Functions

TEACHER ANSWER KEY | Intervals from graphs

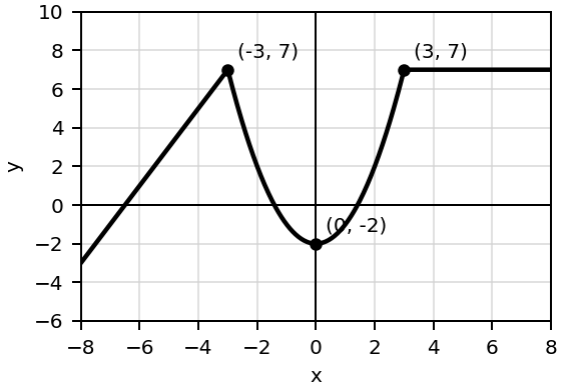
35. Find intervals of increase, decrease, and constant behavior. Justify using average rates of change. Outer rays continue indefinitely.



Left ray slope = −2; middle slope = 0; right ray slope = 2. Negative, zero and positive rates indicate decrease, constant behavior and increase.

ANSWER: Decreasing $(-\infty, -2)$; constant $(-2, 4)$; increasing $(4, \infty)$.

36. Find intervals of increase, decrease, and constant behavior. Justify using average rates of change. Outer rays continue indefinitely.



The left ray has slope 2. From $x = -3$ to 0, the average rate is $(-2 - 7) \div 3 = -3$. From 0 to 3, it is $(7 - (-2)) \div 3 = 3$. The right ray is horizontal.

ANSWER: Increasing $(-\infty, -3)$ and $(0, 3)$; decreasing $(-3, 0)$; constant $(3, \infty)$.

1.3 Rates of Change in Linear & Quadratic Functions

TEACHER ANSWER KEY | Classifying functions

37. Could the function be linear, quadratic, or neither? Justify.

x	2	5	8	11	14
f(x)	-4	2	14	32	56

Compute Δy divided by Δx for each adjacent pair.

Average rates: 2, 4, 6, 8.

Changes in rates: 2, 2, 2.

The input intervals are equal and the changes in average rates are constant and nonzero.

ANSWER: Quadratic.

38. Could the function be linear, quadratic, or neither? Justify.

x	-6	-3	0	3	6
f(x)	3	5	14	24	70

Compute Δy divided by Δx for each adjacent pair.

Average rates: $\frac{2}{3}$, 3, $\frac{10}{3}$, $\frac{46}{3}$.

Changes in rates: $\frac{7}{3}$, $\frac{1}{3}$, 12.

The average rates are not constant, and their changes do not fit a quadratic model.

ANSWER: Neither.

39. Could the function be linear, quadratic, or neither? Justify.

x	-11	-7	-4	0	5
f(x)	44	28	16	0	-20

Compute Δy divided by Δx for each adjacent pair.

Average rates: -4, -4, -4, -4.

Changes in rates: 0, 0, 0.

All average rates are equal.

ANSWER: Linear.

40. Could the function be linear, quadratic, or neither? Justify.

x	2	4	6	8	10
f(x)	12	16	12	0	-20

Compute Δy divided by Δx for each adjacent pair.

Average rates: 2, -2, -6, -10.

Changes in rates: -4, -4, -4.

The input intervals are equal and the changes in average rates are constant and nonzero.

ANSWER: Quadratic.

1.3 Rates of Change in Linear & Quadratic Functions

TEACHER ANSWER KEY | Concavity from tables

41. Could the function be concave up, concave down, or neither? Justify.

x	2	6	10	14	18
f(x)	8	6	4	2	0

Compute Δy divided by Δx for each adjacent pair.

Average rates: $\frac{-1}{2}, \frac{-1}{2}, \frac{-1}{2}, \frac{-1}{2}$.

Changes in rates: 0, 0, 0.

Average rates are constant or fail to change consistently in one direction.

ANSWER: Neither.

42. Could the function be concave up, concave down, or neither? Justify.

x	2	4	6	8	10
f(x)	3	7	19	39	67

Compute Δy divided by Δx for each adjacent pair.

Average rates: 2, 6, 10, 14.

Changes in rates: 4, 4, 4.

Average rates strictly increase.

ANSWER: Concave up.

43. Could the function be concave up, concave down, or neither? Justify.

x	2	5	8	11	14
f(x)	-8	-5	1	13	22

Compute Δy divided by Δx for each adjacent pair.

Average rates: 1, 2, 4, 3.

Changes in rates: 1, 2, -1.

Average rates are constant or fail to change consistently in one direction.

ANSWER: Neither.

44. Could the function be concave up, concave down, or neither? Justify.

x	1	4	7	10	13
f(x)	24	51	54	33	-12

Compute Δy divided by Δx for each adjacent pair.

Average rates: 9, 1, -7, -15.

Changes in rates: -8, -8, -8.

Average rates strictly decrease.

ANSWER: Concave down.

1.3 Rates of Change in Linear & Quadratic Functions

TEACHER ANSWER KEY | Application: height

45. An object's height is recorded below. Assume the height remains concave down through $t = 4.0$ s.

t (s)	3.5	3.6	3.7	3.8	3.9
h (ft)	176.40	177.92	179.12	180.00	180.56

A(i) Find the average rate on [3.8, 3.9], including units.

A(ii) Write the secant line equation.

B(i) Estimate the height at $t = 4.0$ using the secant line.

B(ii) Is this an overestimate or underestimate? Justify.

A(i) $(180.56 - 180.00) \div (3.9 - 3.8) = 5.6$ ft/s.

A(ii) Point-slope form: $h - 180 = 5.6(t - 3.8)$. Thus $h = 5.6t + 158.72$.

B(i) $h(4.0) = 5.6(4) + 158.72 = 181.12$ ft.

B(ii) The preceding average rates are 15.2, 12, 8.8, 5.6 ft/s. Since concavity remains down, the rate after 3.9 is smaller; the extended secant lies above the actual height.

ANSWER: 5.6 ft/s; $h = 5.6t + 158.72$; 181.12 ft; overestimate.

1.3 Rates of Change in Linear & Quadratic Functions

TEACHER ANSWER KEY | Application: profit

46. A community festival models profit in dollars by $P(t) = -25t^2 + 900t + 150$, where t is the ticket price in dollars.

A. From $t = 0$ to $t = 18$, how much does profit change on average per $1 increase in price?

B. What ticket price maximizes profit? Explain.

C. Find the maximum profit.

D. If tickets cost $24, what is the average profit loss per $1 from $18 to $24?

A. $P(0) = 150$; $P(18) = 8250$. Average rate = $(8250 - 150) \div 18 = 450$ dollars per dollar.

B. The parabola opens downward because $a = -25$. Its vertex occurs at $t = -b \div (2a) = -900 \div (-50) = 18$.

C. $P(18) = -25(324) + 900(18) + 150 = 8250$ dollars.

D. $P(24) = -25(576) + 900(24) + 150 = 7350$. Average rate = $(7350 - 8250) \div 6 = -150$.

ANSWER: A. $450 per $1; B. $18; C. $8,250; D. $150 loss per $1 (total loss $900).