

## Eleanor's Conjecture Blog Post

If  $J$  is odd, not a factor or multiple of a factor of  $DN$ , and larger than 1, all the dots will be touched by lines before dot 0 is reached again.

That probably means nothing to you. Here, I'll explain:

Connect the Dots is a procedure, often performed using a code in the program NetLogo. In this procedure, there is a series of evenly spaced dots (you can change how many) around a circle, (you will what me refer to the number of dots as "dot number" and " $DN$ ") and lines of equal length are drawn between the dots, and you can change the number of dots you skip (jump size,  $J$ ).<sup>6</sup>

I began by experimenting with Connect the Dots, and noticed that some of the arrangements touched all of the dots while others didn't. Then I noticed that all of the ones that touched all the dots were odd numbers, and the ones that didn't were even.<sup>7,8</sup>

At that point I was only looking at circles with a dot number of 16, but a little while later I started looking at circles with other dot numbers. And when I looked at those, my observation held up.<sup>7,8</sup>

But then I noticed that if the jump size was a factor of the dot number, then my conjecture didn't work. So I modified my conjecture slightly to include these exceptions. But that modification posed another problem: 1 is a factor of every number, so it was included as something that did not work, but it did. So I modified the conjecture again. But as I was running through all the jump sizes for one dot number<sup>2,3,4,5</sup>, I found that some of the jump sizes -- which weren't factors of the dot number -- still worked. But in Connect the Dots, once the jump size reaches half of the dot number, it repeats itself in reverse. So with a dot number of 12, the jump sizes of 5 and 7 will make the same final pattern, and so on. So I figured out that the dot numbers were not hitting all the dots because they created the patterns of factors. So that's why I added the reverse factor part. A reverse factor, as defined by me, is a number that when subtracted from the dot number forms a factor of the dot number.

Originally I was just picking random jump sizes and dot numbers,<sup>1</sup> but then Alex encouraged me to try every jump size for a dot number. I tried that, and my conjecture held up.<sup>2,3,4,5</sup>

One thing that I really liked about the CTD process is that often, the patterns look really cool, though it always takes a bit of time to make them. Drawing them by hand takes a while, and NetLogo runs very slowly.

Note: Pictures and referenced pictures can be found in the gallery below.

## Gallery

1.

| dn  | j  | Does it work? |
|-----|----|---------------|
| 35  | 3  | yes           |
| 35  | 73 | yes           |
| 35  | 27 | yes           |
| 50  | 27 | yes           |
| 50  | 21 | yes           |
| 50  | 17 | yes           |
| 100 | 77 | yes           |
| 68  | 77 | yes           |
| 36  | 4  | no            |
| 36  | 2  | no            |
| 36  | 6  | no            |
| 36  | 3  | no            |
| 36  | 9  | no            |
| 36  | 12 | no            |
| 36  | 18 | no            |
| 50  | 1  | no            |
| 50  | 2  | no            |
| 50  | 5  | no            |
| 50  | 10 | no            |
| 50  | 25 | no            |
| 50  | 50 | no            |

2.

| d  | j  | dots all touched? | skipping:          |
|----|----|-------------------|--------------------|
| 12 | 1  | yes               |                    |
| 12 | 2  | no                | odd numbers        |
| 12 | 3  | no                | all but 0, 3, 6, 9 |
| 12 | 4  | no                | all but 0, 4, 8    |
| 12 | 5  | yes               |                    |
| 12 | 6  | no                | all but 0,6        |
| 12 | 7  | yes               |                    |
| 12 | 8  | no                | all but 0, 4, 8    |
| 12 | 9  | no                | all but 0, 3, 6, 9 |
| 12 | 10 | no                | odd numbers        |
| 12 | 11 | yes               |                    |
| 12 | 12 | no                |                    |

3.

| dn | j | dots all touched? | skipping:          |
|----|---|-------------------|--------------------|
| 15 | 1 | yes               |                    |
| 15 | 2 | yes               |                    |
| 15 | 3 | no                | all but 0,3,6,9,12 |
| 15 | 4 | yes               |                    |
| 15 | 5 | no                | all but 0,5,10     |

4.

| dn | j | all dots touched? | skipping:     |
|----|---|-------------------|---------------|
| 9  | 1 | yes               |               |
| 9  | 2 | yes               |               |
| 9  | 3 | no                | all but 0,3,6 |
| 9  | 4 | yes               |               |
| 9  | 5 | yes               |               |
| 9  | 6 | no                | all but 0,3,6 |
| 9  | 7 | yes               |               |
| 9  | 8 | yes               |               |
| 9  | 9 | no                | all but 0     |

5.

| dn | j  | all dots touched? | skipping:         |
|----|----|-------------------|-------------------|
| 10 | 1  | yes               |                   |
| 10 | 2  | no                | all but 0,2,4,6,8 |
| 10 | 3  | yes               |                   |
| 10 | 4  | no                | all but 0,2,4,6,8 |
| 10 | 5  | no                | all but 0,5       |
| 10 | 6  | no                | all but 2,4,6,8   |
| 10 | 7  | yes               |                   |
| 10 | 8  | no                | all but 0,2,4,6,8 |
| 10 | 9  | yes               |                   |
| 10 | 10 | no                |                   |

6.

12

dotnum

4

jumpsize

CLEAR

DRAW

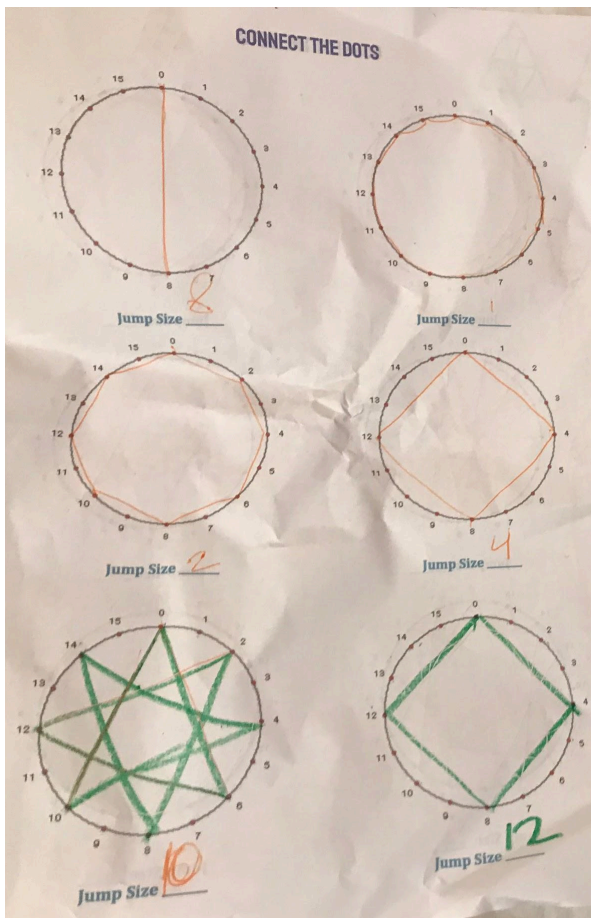
☒ Draw-one-se...

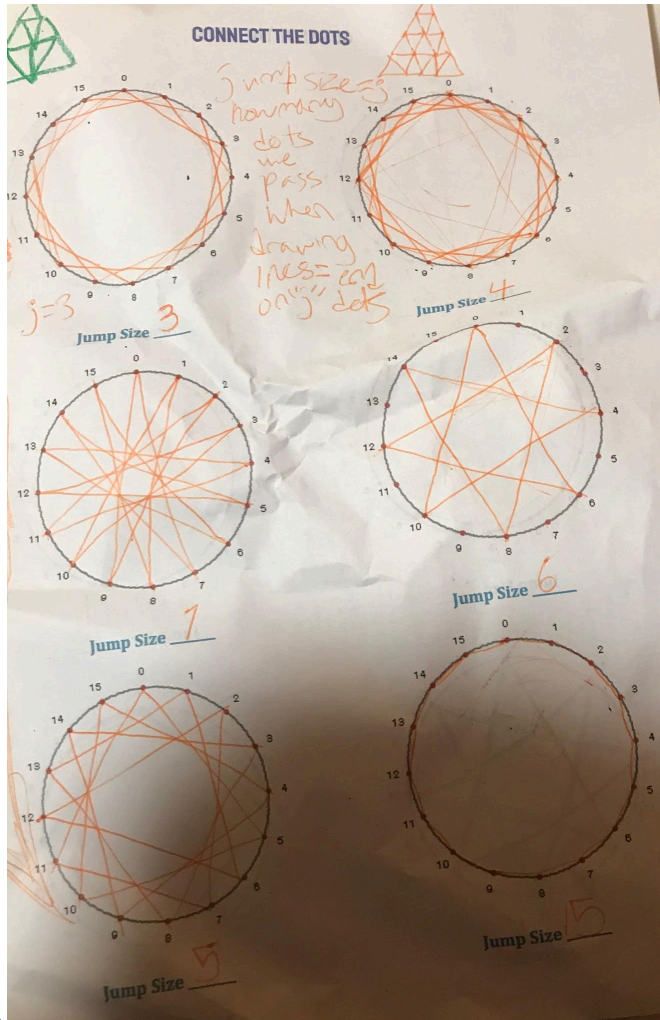
☒ Show-labels

Dots Hit For This Shape:

☐ Sorted

7.





8.

9.

<https://docs.google.com/spreadsheets/d/1ajGLCu1jtP7ikgiaTG5tRmc3zm6mZw8utp8DDva2fl4/edit?usp=sharing>