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K-12 Mathematics Curriculum

from the Colorado Academic Standards for Mathematics

Kindergarten

In Kindergarten, instructional time should focus on two critical areas: (1) representing, relating, and operating on whole numbers, initially with sets of objects; (2) describing shapes and space. More learning time in Kindergarten should be devoted to number than to other topics.

Key Fluency:

- Add and subtract within 5

1st Grade

In Grade 1, instructional time should focus on four critical areas: (1) developing understanding of addition, subtraction, and strategies for addition and subtraction within 20; (2) developing understanding of whole number relationships and place value, including grouping in tens and ones; (3) developing understanding of linear measurement and measuring lengths as iterating length units; and (4) reasoning about attributes of, and composing and decomposing geometric shapes.

Key Fluency:

- Add and subtract within 10

2nd Grade

In Grade 2, instructional time should focus on four critical areas: (1) extending understanding of base-ten notation; (2) building fluency with addition and subtraction; (3) using standard units of measure; and (4) describing and analyzing shapes.

Key Fluencies:

- Add and subtract within 20 using mental strategies. By the end of Grade 2, know from memory all sums of two one-digit numbers.
- Add and subtract within 100

3rd Grade

In Grade 3, instructional time should focus on four critical areas: (1) developing understanding of multiplication and division and strategies for multiplication and division within 100; (2) developing understanding of fractions, especially unit fractions (fractions with numerator 1); (3) developing understanding of the structure of rectangular arrays and of area; and (4) describing and analyzing two-dimensional shapes.

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Key Fluencies:

- Multiply and divide within 100. By the end of Grade 3, know from memory all products of two one-digit numbers.
- Add and subtract within 1,000

4th Grade

In Grade 4, instructional time should focus on three critical areas: (1) developing understanding and fluency with multi-digit multiplication, and developing understanding of dividing to find quotients involving multi-digit dividends; (2) developing an understanding of fraction equivalence, addition and subtraction of fractions with like denominators, and multiplication of fractions by whole numbers; (3) understanding that geometric figures can be analyzed and classified based on their properties, such as having parallel sides, perpendicular sides, particular angle measures, and symmetry.

Key Fluency:

- Add and subtract multi-digit whole numbers using the standard algorithm

5th Grade

In Grade 5, instructional time should focus on three critical areas: (1) developing fluency with addition and subtraction of fractions, and developing understanding of the multiplication of fractions and of division of fractions in limited cases (unit fractions divided by whole numbers and whole numbers divided by unit fractions); (2) extending division to 2-digit divisors, integrating decimal fractions into the place value system and developing understanding of operations with decimals to hundredths, and developing fluency with whole number and decimal operations; and (3) developing understanding of volume.

Key Fluency:

- Multiply multi-digit whole numbers using the standard algorithm

6th Grade

In Grade 6, instructional time should focus on four critical areas: (1) connecting ratio and rate to whole number multiplication and division and using concepts of ratio and rate to solve problems; (2) completing understanding of division of fractions and extending the notion of number to the system of rational numbers, which includes negative numbers; (3) writing, interpreting, and using expressions and equations; and (4) developing understanding of statistical thinking.

Key Fluencies:

- Divide multi-digit numbers using the standard algorithm

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- Add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.

7th Grade

In Grade 7, instructional time should focus on four critical areas: (1) developing understanding of and applying proportional relationships; (2) developing understanding of operations with rational numbers and working with expressions and linear equations; (3) solving problems involving scale drawings and informal geometric constructions, and working with two- and three-dimensional shapes to solve problems involving area, surface area, and volume; and (4) drawing inferences about populations based on samples.

8th Grade

In Grade 8, instructional time should focus on three critical areas: (1) formulating and reasoning about expressions and equations, including modeling an association in bivariate data with a linear equation, and solving linear equations and systems of linear equations; (2) grasping the concept of a function and using functions to describe quantitative relationships; (3) analyzing two- and three-dimensional space and figures using distance, angle, similarity, and congruence, and understanding and applying the Pythagorean Theorem.

Algebra 1

Algebra 1 focuses on four critical areas: (1) using units and relationships between quantities; (2) reasoning with equations and expressions; (3) analyzing and using linear, exponential, and quadratic functions; and (4) interpreting and displaying data using descriptive statistics.

Geometry

Geometry focuses on three critical areas: (1) exploring congruence, similarity, and right-triangle relationships; (2) proving properties of triangles, quadrilaterals, and circles; and (3) using area and volume to model and solve problems.

Algebra 2

Algebra 2 focuses on four critical areas: (1) analyzing and using polynomial, rational, radical, exponential, logarithmic, and trigonometric functions; (2) modeling sequences and series with recursive and explicit functions; and (3) making statistical inferences from data and applying probability to events of chance.

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Typical Mathematics Course Pathways

Grade	Traditional Pathway	Advanced Pathway	Accelerated Pathway
5th Grade	5th Grade Mathematics	Advanced 5th Grade Mathematics	
6th Grade	6th Grade Mathematics	7th Grade Mathematics	7th/8th Grade Mathematics
7th Grade	7th Grade Mathematics	8th Grade Mathematics	Algebra 1
8th Grade	8th Grade Mathematics	Algebra 1	Geometry*
9th Grade	Algebra 1	Geometry*	Algebra 2*
10th Grade	Geometry*	Algebra 2*	Precalculus w/ Trigonometry*
11th Grade[+]	Algebra 2*	Precalculus w/ Trigonometry*	AP Calculus[#] or AP Statistics
12th Grade	Precalculus w/ Trigonometry*[x] or AP Statistics	AP Calculus[#] or AP Statistics	AP or Concurrent Enrollment**

* Honors opportunities available for weighted credit.

[+] Three years of high school mathematics are required for graduation

[x] High schools may offer different math elective combinations of semester courses after Algebra 2 (i.e. Statistics, Trigonometry, Finite Mathematics).

[#] Advanced Placement (AP) Calculus BC is not offered at every high school. Consult schools directly to see which Calculus courses are available.

** Courses include Calculus II, Calculus III, Differential Equations, and Linear Algebra. Consult the Front Range Community College (FRCC) academic course catalog for specific course prerequisites.

THREE CREDITS OF HIGH SCHOOL MATHEMATICS (ALGEBRA 1 AND HIGHER) ARE REQUIRED FOR GRADUATION FROM ST. VRAIN VALLEY SCHOOLS. ADVANCED MATHEMATICS COURSES TAKEN IN MIDDLE SCHOOL (ALGEBRA 1, GEOMETRY, ETC.) <u>DO NOT</u> COUNT TOWARD HIGH SCHOOL CREDITS.

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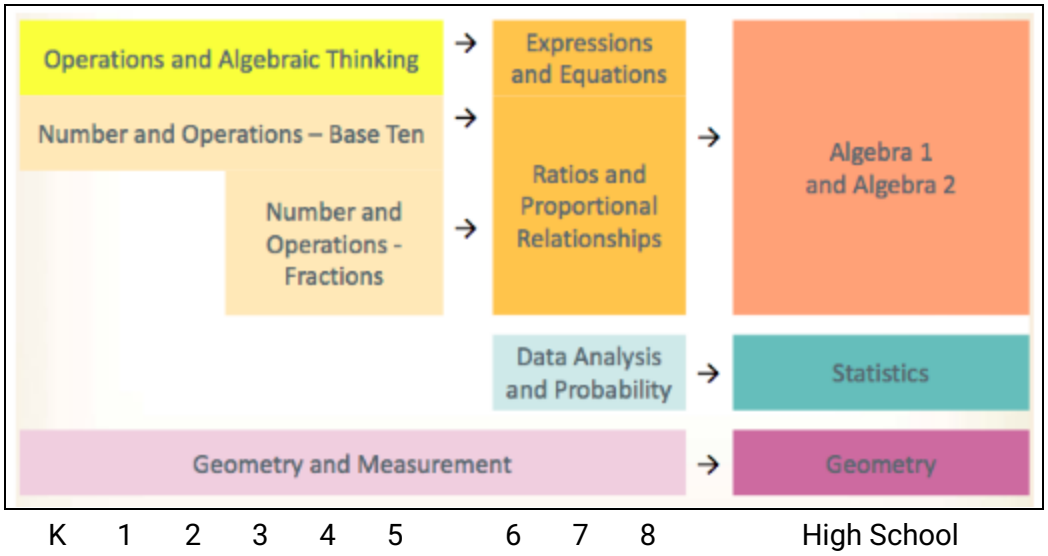
Focus Areas by Grade & Key Shifts in Mathematics

from achievethecore.org



Graphic from Colorado Department of Education

Grade	Focus Areas in Support of Rich Instruction and Expectations of Fluency and Conceptual Understanding
K–2	Addition and subtraction – concepts, skills, and problem solving and place value
3–5	Multiplication and division of whole numbers and fractions – concepts, skills, and problem solving
6	Ratios and proportional reasoning; early expressions and equations
7	Ratios and proportional reasoning; arithmetic of rational numbers
8	Linear algebra and linear functions

College- and Career-Ready Shifts in Mathematics

Focus, Coherence, Rigor

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Colorado Essential Skills

from the Colorado Department of Education

The framework for the Essential Skills document was influenced by the groundwork laid by the in-demand skills identified in the 2015 Colorado Talent Pipeline Report. The Pipeline report was authored by the Colorado Workforce Development Council in partnership with a number of state agencies. The goal of this publication was to explore Colorado jobs that have high growth rates in an effort to better align student skills with behaviors necessary for successful employment in Colorado. To that end, the group "… identified 20 core skills necessary to enter the workforce or continue education beyond high school; these include skills such as critical thinking, creativity, self-direction, cultural awareness, time management and self-advocacy" (CWDC, 2016, p. 5). These skills were grouped under four core categories of Entrepreneurial, Personal, Civic/Interpersonal and Professional skills to add greater clarity to the dispositions that all graduates should demonstrate. This Essential Skills document uses these core skills categories to group the essential P–12 academic skills from CAP4K to enable focus on the development of postsecondary and workforce skills that Colorado graduates should demonstrate.

Personal Skills

A Colorado graduate demonstrates personal skills through self-awareness, initiative and self-direction, personal responsibility and self-management, adaptability and flexibility, and perseverance and resilience.

Entrepreneurial Skills

A Colorado graduate demonstrates entrepreneurial skills through critical thinking and problem-solving, creativity and innovation, inquiry and analysis, and risk-taking.

Civic/Interpersonal Skills

A Colorado graduate demonstrates civic/interpersonal skills through collaboration and teamwork, strong communication skills, global and cultural awareness, civic engagement and strong character.

Professional Skills

A Colorado graduate demonstrates civic/interpersonal skills through collaboration and teamwork, strong communication skills, global and cultural awareness, civic engagement and strong character.

Visit the Colorado Department of Education Essential Skills page for specific student actions and attributes associated with each of these essential skills.

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Standards for Mathematical Practice

from the Common Core State Standards

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report Adding It Up: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

1. Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2. Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities,

not just how to compute them; and knowing and flexibly using different properties of operations and objects.

3. Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4. Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5. Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a

graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6. Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7. Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 × 8 equals the well remembered 7 × 5 + 7 × 3, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2 × 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 – 3(x – y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

8. Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y – 2)/(x – 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x – 1)(x + 1)$, $(x – 1)(x^2 + x + 1)$, and $(x – 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem,

mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Connecting the Standards for Mathematical Practice to the Standards for Mathematical Content

The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years. Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to mathematical content in mathematics instruction. The Standards for Mathematical Content are a balanced combination of procedure and understanding. Expectations that begin with the word “understand” are often especially good opportunities to connect the practices to the content. Students who lack understanding of a topic may rely on procedures too heavily. Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview, or deviate from a known procedure to find a shortcut. In short, a lack of understanding effectively prevents a student from engaging in the mathematical practices. In this respect, those content standards which set an expectation of understanding are potential “points of intersection” between the Standards for Mathematical Content and the Standards for Mathematical Practice. These points of intersection are intended to be weighted toward central and generative concepts in the school mathematics curriculum that most merit the time, resources, innovative energies, and focus necessary to qualitatively improve the curriculum, instruction, assessment, professional development, and student achievement in mathematics.