PRACTICE 4.3 – Further Rational Functions

* Full, worked solutions can be found in the folder linked on the Course Website ☺

Exercise 4E

1 For each function, find the equations of the horizontal and vertical asymptotes, then write down the domain and range.

a $y = \dfrac{x+1}{x-1}$ **b** $y = \dfrac{2x+3}{x+1}$ **c** $y = \dfrac{6x-1}{2x+4}$

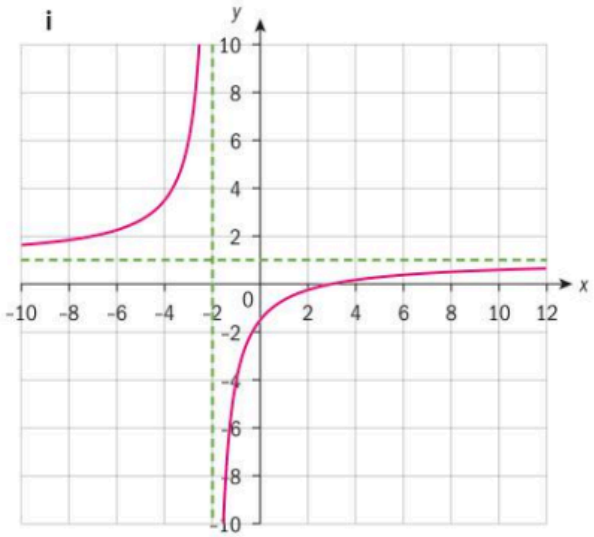
d $y = \dfrac{2-3x}{5-4x}$ **e** $y = \dfrac{9x-2}{6-3x}$

2 Match these equations to their graphs and give reasons for your answer.

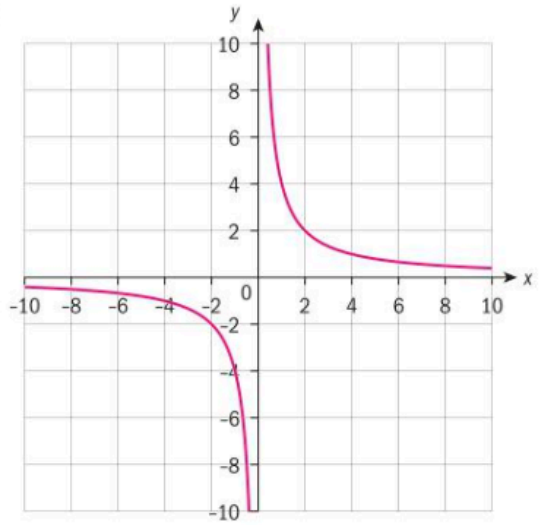
a $y = \dfrac{4}{x}$ **b** $y = \dfrac{x-3}{x+2}$ **c** $y = \dfrac{2x-3}{x+2}$

d $y = \dfrac{3-2x}{x+2}$

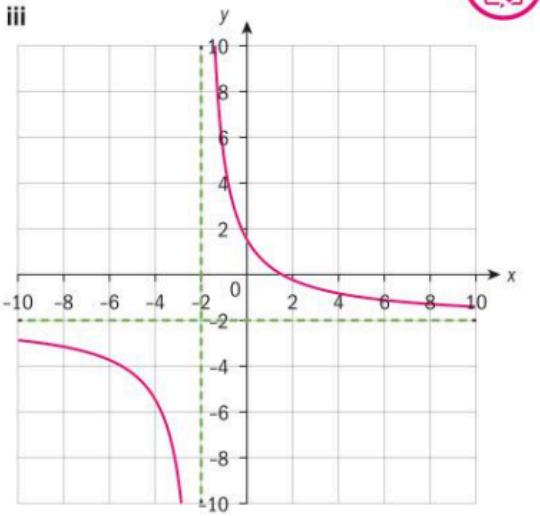
i



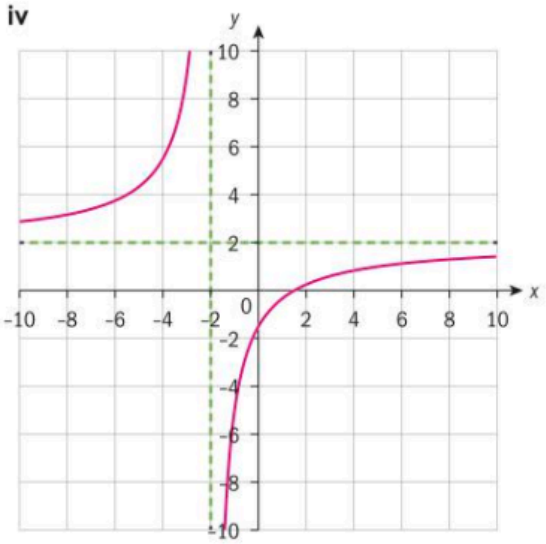
ii



iii



iv



3 Find the asymptotes, domain, and range of the rational function $y = \dfrac{x-p}{x-q}$.

4 Sketch each function. Show the asymptotes as dotted lines.

a $y = \dfrac{x+2}{x-4}$ **b** $y = \dfrac{2x+5}{x+1}$

c $y = \dfrac{x-7}{2x-1}$ **d** $y = \dfrac{1-x}{1+x}$

5 Solve:

a $\dfrac{5}{2x} + \dfrac{x+7}{x+4} = 2$ **b** $\dfrac{2x-3}{x+1} = \dfrac{x+6}{x-2}$

c $7 - \dfrac{5}{x-2} = \dfrac{10}{x+2}$ **d** $\dfrac{x+5}{x+8} = 1 + \dfrac{6}{x+1}$

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6 Will solved $\frac{2}{x-3} = \frac{x}{x-3}$ to get $x = 2$ and $x = 3$. One of these solutions is called *extraneous*. Extraneous solutions are solutions that do not satisfy the original form of the equation because they make the denominator equal to 0.

What is the solution to Will's equation?

7 Find the inverse of each function.

a $f(x) = \frac{x+3}{x-2}$ **b** $f(x) = \frac{7-2x}{x}$

c $f(x) = \frac{1+7x}{9-x}$ **d** $f(x) = \frac{5-11x}{x+6}$

8 Emily is setting up a company to make football shirts. It costs \$500 to purchase the equipment, and costs a further \$10 per shirt to purchase material. The average production cost (\$$M$) to produce s shirts can be modelled by the equation $M(s) = \frac{10s+500}{s}$.

a Sketch a graph of M against s for the first 50 shirts.

b Draw the line $M(s) = 35$ to find the number of shirts that Emily needs to make so that the average production cost per shirt is \$35.

c Find the number of shirts Emily needs to make so that the average production cost per shirt is \$20?

9 An Internet security service charges a 20 AUD set-up fee and then a monthly charge of 10 AUD.

a Construct a model to show the average monthly cost.

b Sketch a graph of the model.

c Use your model to find how long you have to use the service to average 15 AUD per month.

d Tim uses the service for many years at the same price. What will his average cost per month get closer and closer to?

10 Let $f(x) = m + \frac{6}{x-n}$. The line $x = 5$ is an asymptote to the graph of f.

a Write down the value of n.

The graph passes through the point (7, 7).

b Find the value of m.

c Write down the equation of the horizontal asymptote.

11 Consider the function $y = \frac{4}{x-2} + 3$.

a Write down the equation of the horizontal asymptote.

b Find the vertical asymptote.

c Find the coordinates of the axial intercepts.

d Hence, sketch the function. Label the asymptotes and axes intercepts.

12 Let $f(x) = \frac{2x+1}{x-1}$.

a Sketch the graph of f for $-4 \leq x \leq 5$.

b Write down the equations of the asymptotes.

c Find the x-intercept of f.

13 $f(x) = \frac{x+2}{x+3}$ and $g(x) = \frac{1}{x}$:

a Find $(g \circ f)(x)$.

b Plot $(g \circ f)(x)$ and $f(x)$ on your GDC, and use this to solve $f(x) = (g \circ f)(x)$.