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B.Sc. (Hons) Math (Semester – 4th)
DIFFERENTIAL EQUATIONS - II
Subject Code: BMATS 1401
Paper ID: [19131216]

Time: 03 Hours **Maximum Marks: 60**

Instruction for candidates:

1. Section A is compulsory. It consists of 10 parts of two marks each.
2. Section B consist of 5 questions of 5 marks each. The student has to attempt any 4 questions out of it.
3. Section C consist of 3 questions of 10 marks each. The student has to attempt any 2 questions.

Section – A **(2 marks each)**

Q1. Attempt the following:

a) Find order and degree of the PDE: $\left(\frac{\partial z}{\partial x}\right)^2 + \frac{\partial^3 z}{\partial x^3} = 2x\frac{\partial z}{\partial x}$.

b) Form a PDE by eliminating arbitrary constants *a* and *b* from the equation: $z = ax + by$.

c) Classify the differential equation $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} = 0$ according to its nature.

d) Solve the differential equation $D^2 - 2DD' = 0$

e) Form the PDE by eliminating the arbitrary function from $z = f\left(\frac{xy}{z}\right)$.

f) Write down the steps for reducing a parabolic equation to canonical form.

g) Write one dimensional heat as well as Laplace equation.

h) Is the differential equation $z_{xx} + x^2 z_{yy} = 0$ hyperbolic, parabolic or elliptic?

i) Write the Laplacian operator.

j) Express the vector $zi - 2xj + yk$ in cylindrical coordinates.

Section – B **(5 marks each)**

Q2. Examine whether the following system of partial differential equations is compatible or not? If compatible, then find their common solution:

$$\frac{\partial z}{\partial x} = 7x + 18y - 1,\ \frac{\partial z}{\partial y} = 9x + 11y - 2$$

Q3. Solve the second order different equation: $\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = e^{x+2y}$.

Q4. Solve $yzp + zxq = xy$ by Lagrange's method.

Q5. Solve the differential equation $\frac{\partial^2 u}{\partial x^2} = \frac{1}{c^2}\frac{\partial u}{\partial t}$ for the boundary conditions:

$u(0,t) = u(1,t) = 0$ and $\lim_{t\to\infty} u(x,t) = 0$.

Q6. Using spherical polar coordinates, prove that $\nabla(\cos\theta) \times \nabla\phi = \nabla(1/r),\ r \neq 0$.

Section – C **(10 marks each)**

Q7. (a) Apply Charpit's method to find the complete integral of $z = px + qy + p^2 + q^2$.

(b) Find the characteristic curves of $\frac{\partial^2 u}{\partial x^2} - 2\frac{\partial^2 u}{\partial x \partial y} - 8\frac{\partial^2 u}{\partial y^2} = 0$.

Q8. (a) Solve by using Cauchy's method of characteristics $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = x + y$; $u(x,0) = 0$.

(b) Find the solution of the PDE $\frac{\partial^2 u}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2}$ such that $u = B_0 \cos bt$ (B_0 is constant), when $x = a$ and $u = 0$ when $x = b$.

Q9. (a) Using the method of separation of variables, solve $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u,\ u(x,0) = 6e^{-3x}$.

(b) Solve the boundary value problem $y'' - y + x = 0,\ 0 \le x \le 1,\ y(0) = y(1) = 0$, by any suitable method.