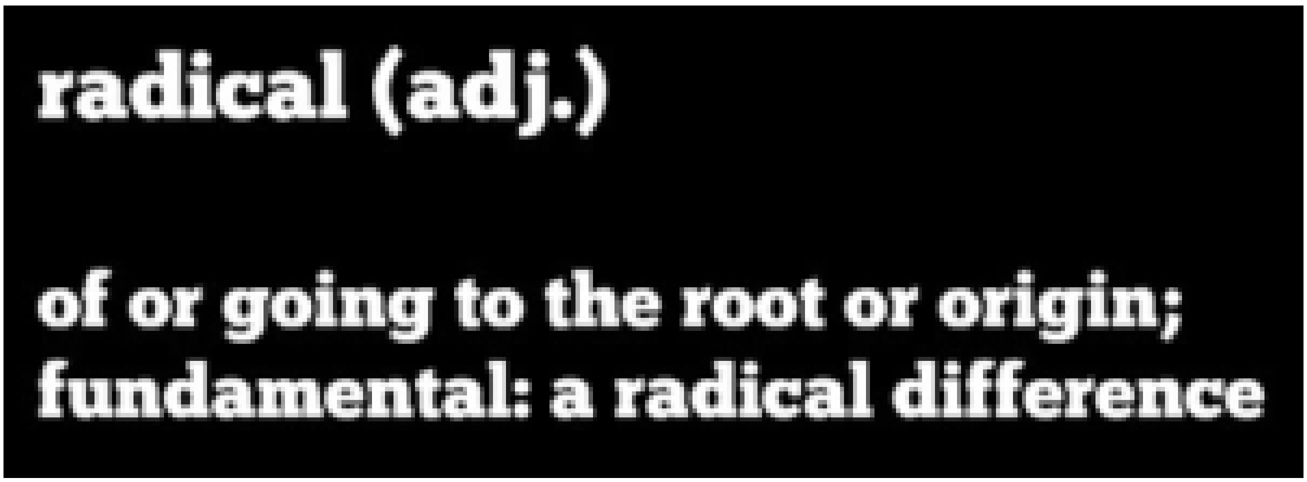
See this page in the course material.

Learning Outcomes

- Simplify square roots with variables
- Recognize that by definition [latex]\sqrt{x^{2}}[/latex] is always nonnegative

Radical expressions are expressions that contain radicals. Radical expressions come in many forms, from simple and familiar, such as[latex] \sqrt{16}[/latex], to quite complicated, as in [latex] \sqrt[3]{250{{x}^{4}}y}[/latex]. Using factoring, you can simplify these radical expressions, too.



Radical

Simplifying Square Roots

Radical expressions will sometimes include variables as well as numbers. Consider the expression [latex] \sqrt{9{{x}^{6}}}[/latex]. Simplifying a radical expression with variables is not as straightforward as the examples we have already shown with integers.

Consider the expression [latex] \sqrt{{{x}^{2}}}[/latex]. This looks like it should be equal to *x*, right? Let's test some values for *x* and see what happens.

In the chart below, look along each row and determine whether the value of *x* is the same as the value of [latex] \sqrt{{{x}^{2}}}[/latex]. Where are they equal? Where are they not equal?

After doing that for each row, look again and determine whether the value of [latex] \sqrt{{{x}^{2}}}[/latex] is the same as the value of [latex]\left|x\right|[/latex].

[latex]x[/latex]	[latex]x^{2}[/latex]	[latex]\sqrt{x^{2}}[/latex]	[latex]\left|x\right|[/latex]
[latex]−5[/latex]	[latex]25[/latex]	[latex]5[/latex]	[latex]5[/latex]
[latex]−2[/latex]	[latex]4[/latex]	[latex]2[/latex]	[latex]2[/latex]
[latex]0[/latex]	[latex]0[/latex]	[latex]0[/latex]	[latex]0[/latex]
[latex]6[/latex]	[latex]36[/latex]	[latex]6[/latex]	[latex]6[/latex]
[latex]10[/latex]	[latex]100[/latex]	[latex]10[/latex]	[latex]10[/latex]

Notice—in cases where *x* is a negative number, [latex]\sqrt{x^{2}}\neq{x}[/latex]! (This happens because the process of squaring the number loses the negative sign, since a negative times a negative is a positive.) However, in all cases [latex]\sqrt{x^{2}}=\left|x\right|[/latex]. You need to consider this fact when simplifying radicals that contain variables, because by definition [latex]\sqrt{x^{2}}[/latex] is always nonnegative.

Taking the Square Root of a Radical Expression

When finding the square root of an expression that contains variables raised to a power, consider that [latex]\sqrt{x^{2}}=\left|x\right|[/latex].

Examples: [latex]\sqrt{9x^{2}}=3\left|x\right|[/latex], and [latex]\sqrt{16{{x}^{2}}{{y}^{2}}}=4\left|xy\right|[/latex]

Let’s try it.

The goal is to find factors under the radical that are perfect squares so that you can take their square root.

Example

Simplify. [latex] \sqrt{9{{x}^{6}}}[/latex]

Show Solution

Factor to find identical pairs.

[latex] \sqrt{3\cdot 3\cdot {{x}^{3}}\cdot {{x}^{3}}}[/latex]

Rewrite the pairs as perfect squares, note how we use the power rule for exponents to simplify [latex]x^6[/latex] into a square: [latex]{x^3}^2[/latex]

[latex] \sqrt{{{3}^{2}}\cdot {{\left({{x}^{3}} \right)}^{2}}}[/latex]

Separate into individual radicals.

[latex] \sqrt{{{3}^{2}}}\cdot \sqrt{{{\left({{x}^{3}} \right)}^{2}}}[/latex]

Simplify, using the rule that [latex] \sqrt{{{x}^{2}}}=\left|x\right|[/latex].

[latex] 3\left|{{x}^{3}}\right|[/latex]

Answer

[latex] \sqrt{9{{x}^{6}}}=3\left|{{x}^{3}}\right|[/latex]

Variable factors with even exponents can be written as squares. In the example above, [latex]{{x}^{6}}={{x}^{3}}\cdot{{x}^{3}}={\left|x^3\right|}^{2}[/latex] and

[latex] {{y}^{4}}={{y}^{2}}\cdot{{y}^{2}}={\left(|y^2\right|)}^{2}[/latex].

Try It

See this interactive in the course material.

Let’s try to simplify another radical expression.

Example

Simplify. [latex] \sqrt{100{{x}^{2}}{{y}^{4}}}[/latex]

Show Solution

Separate factors; look for squared numbers and variables. Factor 100 into [latex]10\cdot10[/latex].

[latex] \sqrt{10\cdot 10\cdot {{x}^{2}}\cdot {{y}^{4}}}[/latex]

Factor [latex]y^{4}[/latex] into [latex]\left(y^{2}\right)^{2}[/latex].

[latex] \sqrt{10\cdot 10\cdot {{x}^{2}}\cdot {{({{y}^{2}})}^{2}}}[/latex]

Separate the squared factors into individual radicals.

[latex] \sqrt{{{10}^{2}}}\cdot \sqrt{{{x}^{2}}}\cdot \sqrt{{{({{y}^{2}})}^{2}}}[/latex]

Take the square root of each radical . Since you do not know whether *x* is positive or negative, use [latex]\left|x\right|[/latex] to account for both possibilities, thereby guaranteeing that your answer will be positive.

[latex]10\cdot\left|x\right|\cdot{y}^{2}[/latex]

Simplify and multiply.

[latex]10\left|x\right|y^{2}[/latex]

Answer

[latex] \sqrt{100{{x}^{2}}{{y}^{4}}}=10\left| x \right|{{y}^{2}}[/latex]

You can check your answer by squaring it to be sure it equals [latex] 100{{x}^{2}}{{y}^{4}}[/latex].

Example

Simplify. [latex] \sqrt{49{{x}^{10}}{{y}^{8}}}[/latex]

Show Solution

Look for squared numbers and variables. Factor 49 into [latex]7\cdot7[/latex], [latex]x^{10}[/latex] into [latex]x^{5}\cdot{x}^{5}[/latex], and [latex]y^{8}[/latex] into [latex]y^{4}\cdot{y}^{4}[/latex].

[latex] \sqrt{7\cdot 7\cdot {{x}^{5}}\cdot{{x}^{5}}\cdot{{y}^{4}}\cdot{{y}^{4}}}[/latex]

Rewrite the pairs as squares.

[latex] \sqrt{{{7}^{2}}\cdot{{({{x}^{5}})}^{2}}\cdot{{({{y}^{4}})}^{2}}}[/latex]

Separate the squared factors into individual radicals.

[latex] \sqrt{7^2}\cdot\sqrt{({x^5})^2}\cdot\sqrt{({y^4})^2}[/latex]

Take the square root of each radical using the rule that [latex] \sqrt{{{x}^{2}}}=\left|x\right|[/latex].

[latex] 7\cdot\left|{{x}^{5}}\right|\cdot{{y}^{4}}[/latex]

Multiply.

[latex] 7\left|{{x}^{5}}\right|{{y}^{4}}[/latex]

Answer

[latex] \sqrt{49{{x}^{10}}{{y}^{8}}}=7\left|{{x}^{5}}\right|{{y}^{4}}[/latex]

You find that the square root of [latex] 49{{x}^{10}}{{y}^{8}}[/latex] is [latex]7\left|{{x}^{5}}\right|{{y}^{4}}[/latex]. In order to check this calculation, you could square [latex]7\left|{{x}^{5}}\right|{{y}^{4}}[/latex], hoping to arrive at [latex] 49{{x}^{10}}{{y}^{8}}[/latex]. And, in fact, you would get this expression if you evaluated [latex] {\left({7\left|{{x}^{5}}\right|{{y}^{4}}}\right)^{2}}[/latex].

In the video that follows we show several examples of simplifying radicals with variables.

Video Link

Example

Simplify. [latex] \sqrt{{{a}^{3}}{{b}^{5}}{{c}^{2}}}[/latex]

Show Solution

Factor to find variables with even exponents.

[latex] \sqrt{{{a}^{2}}\cdot a\cdot {{b}^{4}}\cdot{b}\cdot{{c}^{2}}}[/latex]

Rewrite [latex]b^{4}[/latex] as [latex]\left(b^{2}\right)^{2}[/latex].

[latex] \sqrt{{{a}^{2}}\cdot a\cdot {{({{b}^{2}})}^{2}}\cdot{ b}\cdot{{c}^{2}}}[/latex]

Separate the squared factors into individual radicals.

[latex] \sqrt{{{a}^{2}}}\cdot\sqrt{{{({{b}^{2}})}^{2}}}\cdot\sqrt{{{c}^{2}}}\cdot \sqrt{a\cdot b}[/latex]

Take the square root of each radical. Remember that [latex] \sqrt{{{a}^{2}}}=\left| a \right|[/latex].

[latex] \left| a \right|\cdot {{b}^{2}}\cdot\left|{c}\right|\cdot\sqrt{a\cdot b}[/latex]

Simplify and multiply. The entire quantity [latex] a{{b}^{2}}c[/latex] can be enclosed in the absolute value sign because [latex]b^2[/latex] will be positive anyway.

[latex] \left| a{{b}^{2}}c \right|\sqrt{ab}[/latex]

Answer

[latex] \sqrt{{{a}^{3}}{{b}^{5}}{{c}^{2}}}=\left| a{{b}^{2}}c\right|\sqrt{ab}[/latex]

In the next section, we will explore cube roots, and use the methods we have shown here to simplify them. Cube roots are unique from square roots in that it is possible to have a negative number under the root, such as [latex]\sqrt[3]{-125}[/latex].

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