More Answers for Practice in Logic and HW 1

This is an expanded version showing additional right and wrong answers.

I. Practice in 1st-order predicate logic – with answers.

1. Mary loves everyone. [assuming D contains only humans]

   $\forall x$ **love** (**Mary**, x)

   Note: No further parentheses are needed here, and according to the syntax on the handout, no further parentheses are possible. But "extra parentheses" are in general considered acceptable, and if you find them helpful, I have no objection. So I would also count as correct any of the following:

   $\forall x$ (**love** (**Mary**, x)), ($\forall x$ **love** (**Mary**, x)), ($\forall x$ (**love** (**Mary**, x)))

2. Mary loves everyone. [assuming D contains both humans and non-humans, so we need to be explicit about 'everyone' as 'every *person*']

   $\forall x$ (**person**(x) $\rightarrow$ **love** (**Mary**, x))

   A wrong answer: $\forall x$ (**person**(x) & **love** (**Mary**, x)) This says that everything in the universe is a person and loves Mary.

3. No one talks. [assume D contains only humans unless specified otherwise.]

   $\neg\exists x$ **talk**(x) **or equivalently,** $\forall x\neg$**talk**(x)

4. Everyone loves himself.

   $\forall x$ **love** (x, x)

5. Everyone loves everyone.

   $\forall x\forall y$ **love** (x, y)

6. Everyone loves everyone except himself. (= Everyone loves everyone else.)

   $\forall x\forall y(\neg\, x = y \rightarrow$ **love** (x, y)) **or** $\forall x\forall y(\, x \neq y \rightarrow$ **love** (x, y))

   Or maybe it should be this, which is not equivalent to the pair above:

   $\forall x\forall y(\neg\, x = y \leftrightarrow$ **love** (x, y)) **or** $\forall x\forall y(\, x \neq y \leftrightarrow$ **love** (x, y))

   The first pair allows an individual to also love himself; the second pair doesn't.

7. Every student smiles.

   $\forall x$ (**student**(x) $\rightarrow$ **smile**(x))

8. Every student except George smiles.

   $\forall x$ ((**student**(x) & $x \neq$ **George**) $\rightarrow$ **smile**(x))

   That formula allows the possibility that George smiles too; if we want to exclude it (this depends on what you believe about *except*; there are subtle differences and perhaps some indeterminacy among *except, besides, other than* and their nearest equivalents in other languages), then it should be the following, or something equivalent to it:

$\forall \boldsymbol{x}$ ((**student**($\boldsymbol{x}$) → ($\boldsymbol{x} \neq$ **George** ↔ **smile**($\boldsymbol{x}$)))

9. Everyone walks or talks.

$\forall \boldsymbol{x}$ (**walk** ($\boldsymbol{x}$) ∨ **talk** ($\boldsymbol{x}$))

10. Every student walks or talks.

$\forall x$ (**student**(x) → (**walk** (x) ∨ **talk** (x)))

11. Every student who walks talks.

$\forall x$ ((**student**(x) & **walk** (x)) → **talk** (x))) **or**

$\forall x$ (**student**(x) → (**walk** (x) → **talk** (x)))

12. Every student who loves Mary is happy.

$\forall x$ ((**student**(x) & **love** (x, **Mary**)) → **happy** (x)))

13. Every boy who loves Mary hates every boy who Mary loves.

$\forall x$((**boy**(x) & **love** (x, **Mary**)) → $\forall y$((**boy**(y) & **love**(**Mary**, y))→ **hate** (x,y)))

14. Every boy who loves Mary hates every other boy who Mary loves.

(So if John loves Mary and Mary loves John, sentence 13 requires that John hates himself, but sentence 14 doesn't require that.)

$\forall x$((**boy**(x) & **love** (x, **Mary**)) → $\forall y$((**boy**(y) & **love**(**Mary**, y) & $y \neq x$) →

hate (x,y)))

II. Homework #1, with answers.

1. Everyone loves Mary.

$\forall x$ **love** (x, **Mary**)

2. John does not love anyone. (Not ambiguous, but there are two equivalent and equally good formulas for it, one involving negation and the existential quantifier, the other involving negation and the universal quantifier. Give both.)

$\neg\exists x$ **love**(**John**, x) **or equivalently,** $\forall x\neg$ **love**(**John**, x)

Wrong: $\exists x\neg$ **love**(**John**, x) :That says there is someone John doesn't love.

Wrong: $\neg\forall x$ **love**(**John**, x): That says John doesn't love everyone; it's equivalent to the preceding formula.

3. Everyone who sees Mary loves Mary.

$\forall x$ (**see** (x, **Mary**) → **love** (x, **Mary**))

4. Everyone loves someone. (Ambiguous)

(i) $\forall x\exists y$ **love** (x, y) (For every person x, there is someone whom x loves.)

(ii) $\exists y\forall x$ **love** (x, y) (There is some person y whom everyone loves, i.e. everyone loves some one specific person.)

5. Someone loves everyone. (Ambiguous)

(i) $\exists x \forall y$ **love** (x, y) (There is some person x who loves everyone.)

(ii) $\forall y \exists x$ **love** (x, y) (For every person y, there is someone who loves them – i.e., no one is totally unloved. This second reading is probably dispreferred for the active sentence. It's the preferred reading for the passive sentence "Everyone is loved by someone" and it's the only reading for the agentless passive "Everyone is loved.")

6. Someone walks and talks.

$\exists x$(**walk** (x) & **talk** (x))

7. Someone walks and someone talks.

($\exists x$ **walk** (x) & $\exists x$ **talk** (x)) **or** ($\exists x$ **walk** (x) & $\exists y$ **talk** (y))

Because neither quantifier is inside the scope of the other – i.e. their scopes are independent – it doesn't matter whether we use different variables here or use the same variable twice. But if one quantifier is inside the scope of the other, then it matters a great deal. When one quantifier is inside the scope of another, as in questions 4 and 5 above, always give them different variables!

Also equivalent: $\exists x \exists y$(**walk** (x) & **talk** (y))

8. Everyone who walks is calm.

$\forall x$ (**walk**(x) $\rightarrow$ **calm**(x))

9. No one who runs walks. (Not ambiguous, but same note as for number 2.)

(i) $\neg\exists x$ (**run** (x) & **walk** (x)) **or equivalently,**

(ii) $\forall x$ (**run**(x) $\rightarrow$ $\neg$ **walk**(x)) **or equivalently,**

(iii) $\forall x \neg$(**run** (x) & **walk** (x))

A wrong answer: $\forall x$ ($\neg$**run**(x) $\rightarrow$ **walk**(x)) What does this one say?

Another wrong answer: $\neg\exists x$ (**run** (x) $\rightarrow$ **walk** (x)) This one doesn't correspond to any English sentence; see notes to questions 11 and 6' below.

10. Everyone who Mary loves loves someone who is happy.

$\forall x$(**love** (**Mary**, x)$\rightarrow$ $\exists y$(**love**(x,y) & **happy**(y)))

Also correct: $\forall x \exists y$ (**love** (**Mary**, x)$\rightarrow$ (**love**(x,y) & **happy**(y)))

But I recommend keeping each quantifier as close as possible to the noun it quantifies, or to its surface position. The more you move quantifiers around, the easier it is to make mistakes.

11. If anyone cheats, he suffers. (English paraphrases: Anyone who cheats suffers. Everyone who cheats suffers. On the subtle difference between these two, see (Kadmon and Landman 1993).)

$\forall x$ (**cheat**(x) $\rightarrow$ **suffer**(x))

A wrong answer: $\exists x$(**cheat**(x) $\rightarrow$ **suffer**(x)) A wide scope $\exists x$ like this creates too weak a statement. If $\exists x$ were given scope only over the antecedent, as in: $\exists x$**cheat**(x)

→ **suffer**(***x***), then that error would be corrected but there would be a new problem because the second ***x*** would not be bound.

Note on *any*: Sometimes *anyone* corresponds to ∃ and sometimes to ∀; you have to think about the meaning of the whole sentence. Many papers have been written exploring the issue of how best to account for the distribution of meanings of *any,* and whether it does or doesn't require lexical ambiguity as part of the account. A few classics include (Carlson 1980, Carlson 1981, Haspelmath 1997, Hintikka 1980, Kadmon and Landman 1993, Kratzer and Shimoyama 2002, Ladusaw 1980, Linebarger 1987, Vendler 1962). See also the note about *any* in the next item.

12. If anyone cheats, everyone suffers.

$\forall x$ (**cheat**(x) → $\forall y$ **suffer**(y))

Equivalent: $\forall x \forall y$ (**cheat**(x) → **suffer**(y)) Also

equivalent: $\forall y$ $\forall x$ (**cheat**(x) → **suffer**(y))

Also equivalent: $\exists x$ **cheat**(x) → $\forall y$ **suffer**(y) (Each quantifier has narrow scope here.)

Also equivalent: $\exists x$ **cheat**(x) → $\forall x$ **suffer**(x) (If each quantifier has narrow scope, then they don't need to involve different variables. If one is inside the scope of the other, then they do.)

Also equivalent: $\forall y$($\exists x$ **cheat**(x) → **suffer**(y))

A wrong answer: $\forall y \exists x$ (**cheat**(x) → **suffer**(y)) This has no natural English paraphrase.

A different wrong answer: $\forall y$($\forall x$ **cheat**(x) → **suffer**(y)) This is one way of saying "If everyone cheats, then everyone suffers."

Another note about *any*: As the equivalent answers above illustrate, *any* in this case can be viewed either as a wide-scope universal (with scope over the *if*-clause) or as a narrow-scope existential (with scope inside the *if*-clause). The fact that these are equivalent, at least in this case, is part of the source of debates about *any.* In example 11, we didn't have that choice, because if *any* were treated as a narrow-scope existential in that case, it couldn't bind the second occurrence of the variable *x* corresponding to the pronoun *he.* The same is true for *anyone* in the next example, which has to be treated as a wide-scope universal in order to bind *himself.*

13. Anyone who loves everyone loves himself.

$\forall x$($\forall y$ **love** (x,y)→ (**love**(x,x))

note: Not this: $\forall x \forall y$ (**love** (x,y)→ **love**(x,x)) What this one says is "Anyone who loves anyone loves himself" What the correct one says is IF you love everyone, THEN you love yourself. So the $\forall y$ quantifier has to be inside the scope of the →.

Another wrong answer: $\exists x \forall y$(**love** (x,y)→ **love**(x,x)) This has no natural English paraphrase. *Any* may sometimes be a wide-scope universal, and sometimes a narrow-scope existential, but it is never a "wide-scope existential."

14. Mary loves everyone except John. (For this one, you need to add the two-place predicate of identity, "=". Think of "everyone except John" as "everyone who is not identical to John".)

$\forall x$ (¬ x = **John** → **love** (**Mary**, x)) **or equivalently**

$\forall \boldsymbol{x}\ (\boldsymbol{x} \neq \mathbf{John} \rightarrow \mathbf{love}\ (\mathbf{Mary}, \boldsymbol{x}))$

As in the case of some earlier examples, this is a ‘weak’ reading of *except*, allowing the possibility of Mary loving John. To get a ‘strong’ reading of *except*, ruling out that possibility, replace $\rightarrow$ above by $\leftrightarrow$, or add a conjunct “ & ¬ **love** (**Mary**, **John**)” at the end.

15. Redo the translations of sentences 1, 4, 6, and 7, making use of the predicate **person,** as we would have to do if the domain D contains not only humans but cats, robots, and other entities.

1'. Everyone loves Mary.

$\forall x$ (**person**(x) → **love** (x, **Mary**))

4'. Everyone loves someone. (Ambiguous)

(i) $\forall x$(**person**(x) → $\exists y$(**person**(y) & **love** (x, y))) (For every person x, there is some person y whom x loves.)

(ii) $\exists y$(**person**(y) & $\forall x$(**person**(x) → **love** (x, y))) (There is some person y whom every person x loves.)

An equivalent correct answer for (i): $\forall x \exists y$ (**person**(x) → (**person**(y) & **love** (x, y)))
But I don't recommend moving the second quantifier, because then it's too easy to come up with the following **wrong** answer for (i): $\forall x \exists y$ ((**person**(x) & **person**(y)) → **love** (x, y)). It's always safer to keep a quantifier and its "restrictor" (in this case *person*) as close together as possible, and both of them as close to their surface position as possible.

6'. Someone walks and talks.

$\exists x$(**person**(x) & **walk** (x) & **talk** (x))

Note: technically, we need more parentheses – either

$\exists x$(**person**(x) & (**walk** (x) & **talk** (x))) or

$\exists x$((**person**(x) & **walk** (x)) & **talk** (x))

But since it's provable that & is associative, i.e. the grouping of a sequence of &'s doesn't make any difference, it is customary to allow expressions like (p & q & r). And similarly for big disjunctions, (p ∨ q ∨ r). But not with → !

Wrong: $\exists x$(**person**(x) → (**walk** (x) & **talk** (x))) This has weird truth-conditions, which you can see if you remember that p → q is equivalent to ¬p ∨ q. You will never really want to combine ∃ with → -- it always makes a statement that is too weak.

7'. Someone walks and someone talks.

($\exists x$ (**person**(x) & **walk** (x)) & $\exists x$(**person**(x) & **talk** (x))) **or equivalently**

($\exists x$ (**person**(x) & **walk** (x)) & $\exists y$ (**person**(y) & **talk** (y)))

Note: both in the original 7 and in this 7', it would be OK and customary to drop outermost parentheses, i.e. the very first left parenthesis and the very last right parenthesis may be dropped. (But no parentheses can be dropped in 6; they are not really "outermost". Only when a pair of parentheses contains the entire formula can it be dropped under the "drop outermost parentheses" convention.

Also correct: ∃*x*∃*y* (person(*x*) & walk (*x*) & person(*y*) & talk (*y*))

References

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