Linear Algebra
Lesson 14
Multiplying Matrices

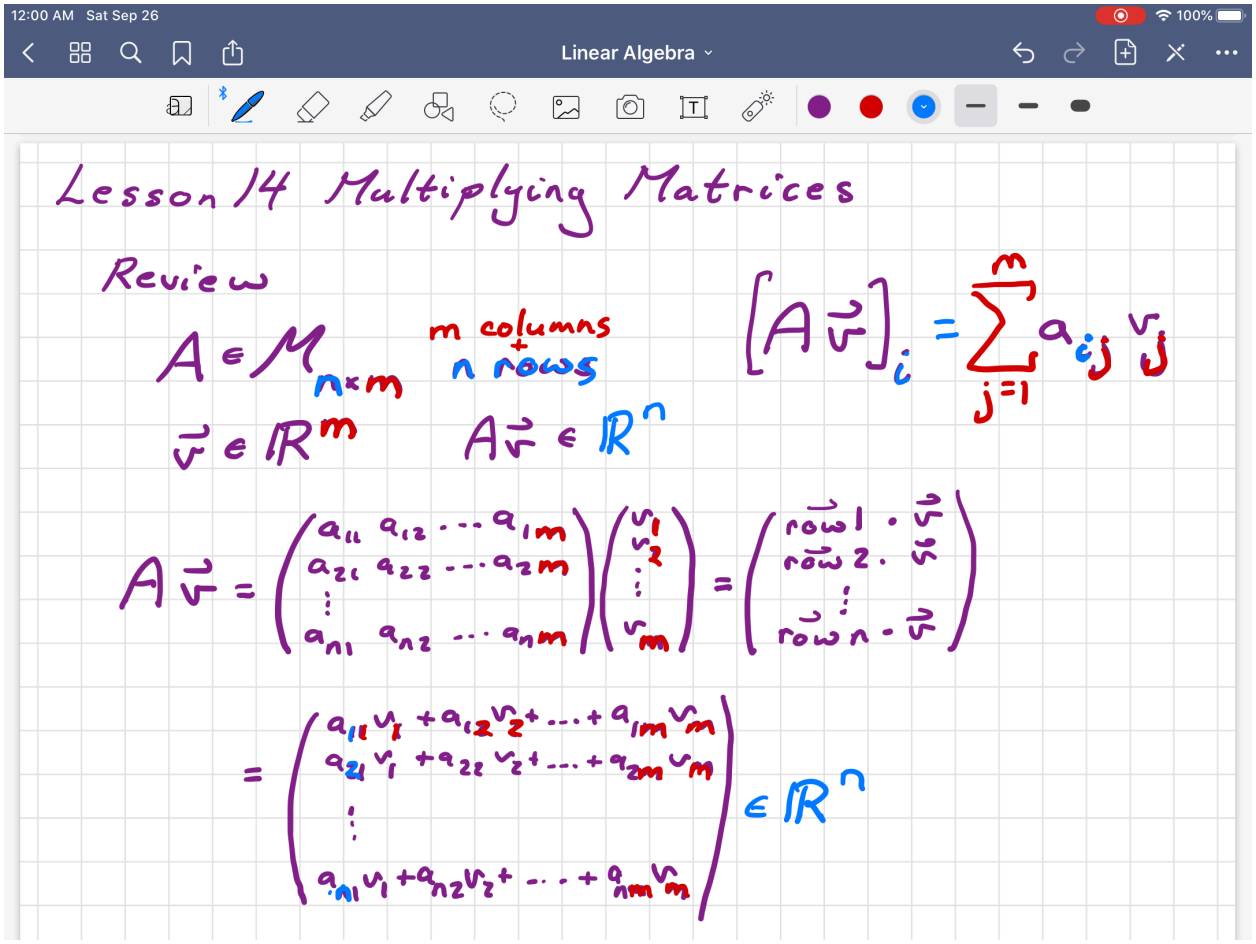
Please be sure to mark down the date and time that you start this lesson. Carefully take notes on pencil and paper while watching the lesson videos. Pause the lesson to try classwork before watching the video going over that classwork. If you work with any classmates, be sure to write their names on the problems you completed together. Please wear masks when meeting with classmates even if you meet off campus.

You will cut and paste the photos of your notes and completed classwork and a selfie taken holding up the first page of your work in a googledoc entitled:

MAT313F21-lesson14-lastname-firstname

and share editing of that document with me sormanic@gmail.com and with our graders. If you have a question, type QUESTION in your googledoc next to the point in your notes that has a question and email me with the subject MAT313 QUESTION. I will answer your question by inserting a photo into your googledoc or making an extra video.

Watch the Playlist 313F20-Lesson14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Linear Algebra
Lesson 14 Multiplying Matrices
Review
$A \in \mathcal{M}_{n \times m}$
m columns
n rows
$\vec{v} \in \mathbb{R}^m$
$A\vec{v} \in \mathbb{R}^n$
$[A\vec{v}]_i = \sum_{j=1}^{m} a_{ij} v_j$
$A\vec{v} = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1m} \\ a_{21} & a_{22} & \cdots & a_{2m} \\ \vdots & & & \\ a_{n1} & a_{n2} & \cdots & a_{nm} \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \\ \vdots \\ v_m \end{pmatrix} = \begin{pmatrix} \vec{row}\,1 \cdot \vec{v} \\ \vec{row}\,2 \cdot \vec{v} \\ \vdots \\ \vec{row}\,n \cdot \vec{v} \end{pmatrix}$
$= \begin{pmatrix} a_{11}v_1 + a_{12}v_2 + \cdots + a_{1m}v_m \\ a_{21}v_1 + a_{22}v_2 + \cdots + a_{2m}v_m \\ \vdots \\ a_{n1}v_1 + a_{n2}v_2 + \cdots + a_{nm}v_m \end{pmatrix} \in \mathbb{R}^n$

Lesson 14 Multiplying Matrices

Review

$A \in \mathcal{M}_{n \times m}$ (m columns, n rows)

$B \in \mathcal{M}_{m \times \ell}$

$A \times B \in \mathcal{M}_{n \times \ell}$

$$[A \times B]_{ik} = \sum_{j=1}^{m} a_{ij} b_{jk}$$

$$A \times B = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1m} \\ a_{21} & a_{22} & \cdots & a_{2m} \\ \vdots & & & \\ a_{n1} & a_{n2} & \cdots & a_{nm} \end{pmatrix} \begin{pmatrix} b_{11} & b_{12} & b_{13} & \cdots & b_{1\ell} \\ b_{21} & b_{22} & \cdots & & \vdots \\ \vdots & & & & \\ b_{m1} & & & & b_{m\ell} \end{pmatrix} = ?$$

Consider $B = \begin{pmatrix} \vec{B}_1 & \vec{B}_2 & \cdots & \vec{B}_\ell \end{pmatrix} \in \mathcal{M}_{m \times \ell}$

$\vec{B}_1, \ldots \vec{B}_\ell \in \mathbb{R}^m$

$A\vec{B}_1, \ldots A\vec{B}_\ell \in \mathbb{R}^n$

$$A \times B = \begin{pmatrix} A\vec{B}_1 & A\vec{B}_2 & \cdots & A\vec{B}_\ell \end{pmatrix} \in \mathcal{M}_{n \times \ell}$$

(n rows)

$m \times \ell$ $\quad$ $n \times \ell$

$$A \times B = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1m} \\ a_{21} & a_{22} & \cdots & a_{2m} \\ \vdots & & & \\ a_{n1} & a_{n2} & \cdots & a_{nm} \end{pmatrix} \begin{pmatrix} b_{11} & b_{12} & b_{13} & \cdots & b_{1\ell} \\ b_{21} & b_{22} & b_{23} & & \\ \vdots & & \vdots & & \\ b_{m1} & & b_{m3} & & b_{m\ell} \end{pmatrix} = ?$$

j counter goes across → the i^{th} row of A

j counter goes down in the k^{th} column of B

$$[A \times B]_{ik} = \text{dot product of } i^{th} \text{ row of } A \text{ and } k^{th} \text{ column of } B$$

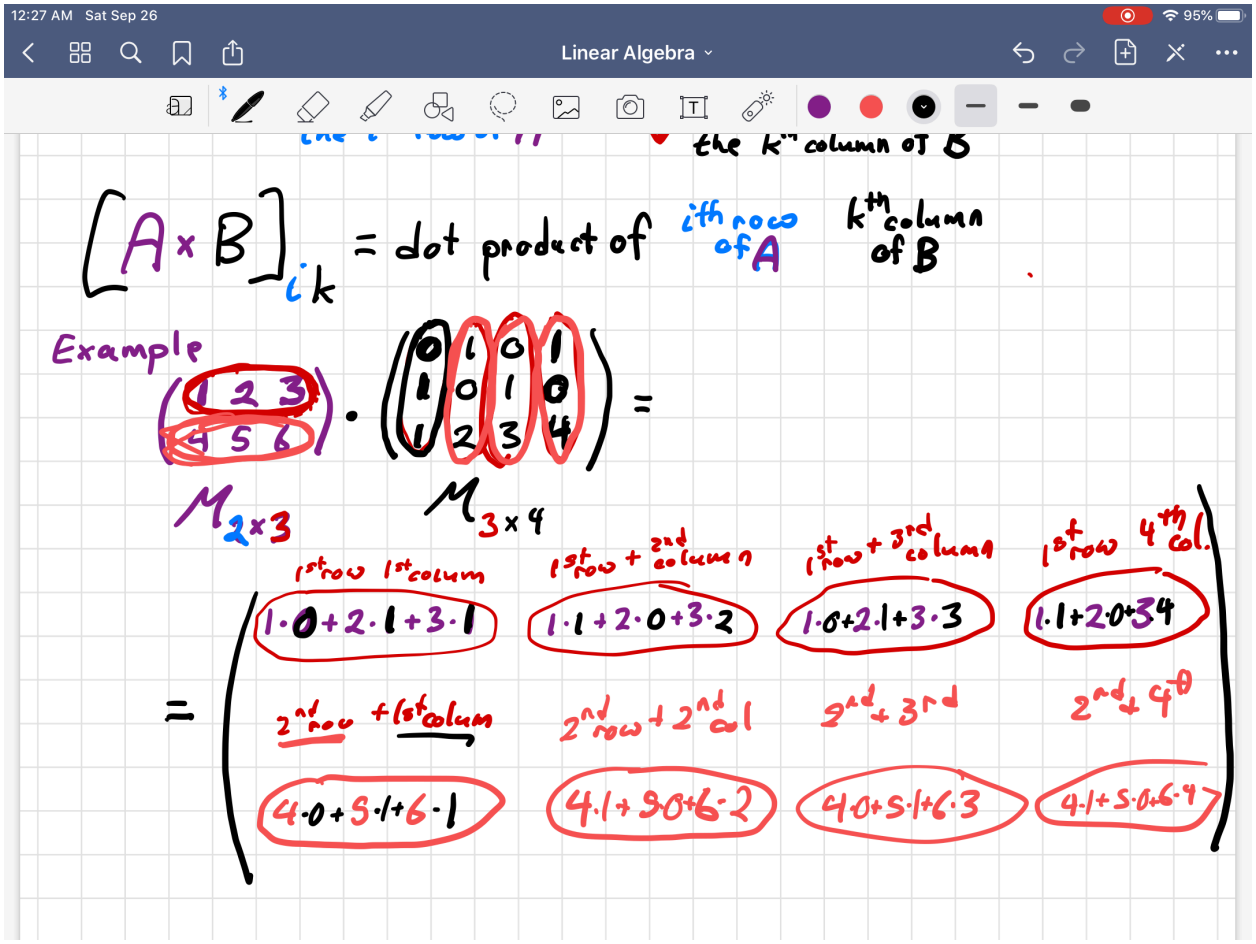
Example

$$\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix} \cdot \begin{pmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 2 & 3 & 4 \end{pmatrix} = \begin{pmatrix} 1\cdot 0+2\cdot 1+3\cdot 1 & 1\cdot 1+2\cdot 0+3\cdot 2 & & \\ & & & \\ & & & \end{pmatrix}$$

1st row 1st column $\quad$ 1st row 2nd column

$M_{2\times 3}$ $\quad$ $M_{3\times 4}$ $\quad$ $M_{2\times 4}$

↑ fill in + watch next video



Practice multiplying matrices at this website which has examples and complete solutions:

https://www.mathsisfun.com/algebra/matrix-multiplying.html

Then do the following ten homework problems:
(if you need help watch the *silent* homework hints Video 313F20-14-hw-hints).

Homework Multiply the Following Matrices:

HW1 $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \times \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

HW2 $\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

HW3 $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} a & b & c \\ d & e & f \end{pmatrix}$

HW4 $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} a & b & c \\ d & e & f \end{pmatrix}$

HW5 $\begin{pmatrix} k & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \end{pmatrix}$ What row action is this?

HW6 $\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{pmatrix}$ What row action is this?

HW7 $\begin{pmatrix} 1 & 0 & 0 \\ -k & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{pmatrix}$ What row action is this?

HW8 Find a 3x3 matrix that does $\rho_2 \to 5\rho_2$.

HW9 Find a 3x3 matrix that does $\rho_3 \leftrightarrow \rho_2$

HW10 Find a 3x3 matrix that does $\rho_3 \to \rho_3 - 4\rho_2$

Also for extra credit you may wish to learn Cube Actions and the matrices that make these actions.