Whiteboard: 1/60, f4.5, iso 640, EV+1.3, Auto Focus, H4n **INPUT @ 95%** gain.

BBB 1/60, f6.7, iso 640, **WB Use Grey Card**, Manual Audio at level 11, focal length ½ way to ∞. H4n **INPUT @ 95%** gain.

BBB Check Horizontal Level & Billy's Lights!! Middle line at bottom 1/3 of desk & remember Billy and Bo chair locations!

Universal Gravitational Potential Energy Derivation **Switch audio to 96kHz!!**

Mr.p: (60 seconds of listening for this group of videos)

Mr.p: Good morning. We have already learned about Universal Gravitational Potential Energy; today we are going to derive the equation for it. … Billy, please read the Universal Gravitational Potential Energy Equation.

$$U_g = -\frac{Gm_1m_2}{r}$$

- Billy: [♪ Flipping Physics ♪] The universal gravitational potential energy which exists between two masses equals the negative of the universal gravitational constant times mass one times mass two all divided by the distance between the centers of mass of the two masses.
- Bo: Isn't the distance r … squared in this equation?

$$F_g = \frac{Gm_1m_2}{r^2}$$

- Bobby: You're confusing this with Newton's Universal Law of Gravitation.
- Bo: Duh.
- Bobby: You're welcome.

Mr.p: We have already shown that the force of gravity is a conservative force. Bobby, please tell me what that means and what equation you should remember whenever you know a force is conservative.

- Bobby: Well, the work done by a conservative force on an object is always independent of the path taken by the object. That is what it means to be a conservative force. As far as the equation we should remember …

$$F_{\text{conservative}} = -\frac{dU}{dx} \Rightarrow F_g = -\frac{dU_g}{dr}$$

- Bo: … A conservative force equals the negative of the derivative, with respect to position, of the potential energy associated with that force.
- Bobby: (yeah) Thanks. Then the force of gravity equals the negative of the derivative of gravitational potential energy with respect to position … however, the position symbol should be r because the force of gravity always acts along the radius toward the other object.

Mr.p: Thank you. … Now we can multiply both sides by dr …

$F_g = -\frac{dU_g}{dr} \Rightarrow dU_g = -F_g dr \Rightarrow \int_{U_i}^{U_f} dU_g = -\int_{r_i}^{r_f} F_g dr$ and take the definite integral of both sides. … The integral with respect to gravitational potential energy is just gravitational potential energy. … We can substitute in the limits. … And final gravitational potential energy minus initial gravitational potential energy equals the change in potential energy. That is the left-hand side of the equation. … What about the right-hand side? Can any one of you tell me what the definite integral of the force of gravity with respect to position is equal to?

- Bo: … Is it impulse? $\text{Impulse} = \vec{J} = \Delta\vec{p} = \int_{t_i}^{t_f} \vec{F} \mathrm{d}t$
- **Mr.p: (No, Impulse is the definite integral of force with respect to *time*.)**
- Bobby: … It's work. The definite integral of force with respect to time equals work.
- Billy: Oh yeah!

$$\Rightarrow \left[U_g\right]_{U_i}^{U_f} = U_{g_f} - U_{g_i} = \Delta U_g = -\int_{r_i}^{r_f} F_g \mathrm{d}r = -W_{F_g} \Rightarrow W_{\text{conservative force}} = -\Delta U$$

- Bo: Yep.
- Billy: … Oh oh, we just showed that the work done by a conservative force equals the change in potential energy associated with that force. That's cool!

- Bobby and Bo: Yeah. Yep.

{switch perspective}

- Mr.p: Very nice. … Now, because the force of gravity is an attractive force, the vector universal gravitational force equation is that the vector of the force of gravity from object 1 on object 2 equals the negative of the quantity … universal gravitational constant times mass 1 times mass 2 all divided by the square of the distance between centers of mass of the two masses all times unit vector r from object 1 to object 2. … $\vec{F}_{12} = -\frac{Gm_1m_2}{r^2}\hat{r}_{12}$ The negative is in this vector equation because the force of gravity is an attractive force, in other words, the force from object 1 on object 2 is opposite in direction from the unit vector from object 1 to object 2. … That means, when we substitute the force of gravity into this equation, we need to substitute in the negative of the universal gravitational constant times mass 1 times mass 2 all divided by the square of the distance between centers of mass.

- **BBB: (Okay. Sure. Yeah.)** $\Rightarrow \Delta U_g = -\int_{r_i}^{r_f} \left(-\frac{Gm_1m_2}{r^2}\right) \mathrm{d}r = Gm_1m_2 \int_{r_i}^{r_f} \frac{1}{r^2} \mathrm{d}r$
- **Bobby: (mr.p?)**
- Mr.p: Yes Bobby?

{switch perspective}

- Bobby: We could understand that the negative needs to be in front of the force of gravity in the integral using the work as the integral of the dot product of force with respect to position, right?
- **Mr.p: (Yes Bobby. But please explain a bit more.)**
- Bobby: Well, those two vectors, force of gravity and dr, are opposite in direction and that gives an angle of 180°, and the cosine of 180° is negative 1. That means the work

done by the force of gravity equals the integral of the negative of force of gravity with respect to position. That's why that negative is there inside the integral.

$$W = \int \vec{F} \cdot \mathrm{d}\vec{r} \Rightarrow W_{F_g} = \int F_g \cos(180^\circ)\,\mathrm{d}r = \int \left(-F_g\right) \mathrm{d}r$$

Mr.p: Very nice Bobby. Thanks. … Mr.p: Bo, please evaluate that integral.

Bo: Uh … Well, a negative times a negative makes the right-hand side positive and the universal gravitation constant, mass 1, and mass 2 are all constant with respect to position, so they can come out from under the integral. … One over r squared is the same thing as r raised to the negative 2 power. … $\Rightarrow \Delta U_g = Gm_1m_2 \int_{r_i}^{r_f} r^{-2}\mathrm{d}r = Gm_1m_2 \left[\frac{r^{-1}}{-1}\right]_{r_i}^{r_f} = -Gm_1m_2 \left[\frac{1}{r}\right]_{r_i}^{r_f}$ The integral of that is r to the negative 1 power over negative 1. … Bring the negative out front and r to the negative 1 is the inverse of r. … And we can substitute in the limits to get the change in gravitational potential energy equals the negative of the universal gravitational constant times mass 1 times mass 2 times the quantity the inverse of r final minus the inverse of r initial. … And, I think … I'm done.

$\Rightarrow \Delta U_g = -Gm_1m_2\left[\frac{1}{r_f} - \frac{1}{r_i}\right]$ and let $U_{g_i} = 0$ where $F_g = 0 \rightarrow r_i = \infty$

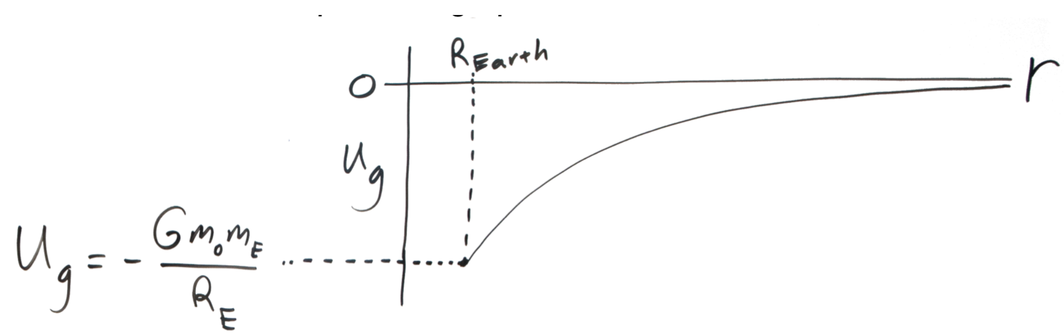
Mr.p: Thank you Bo. … Now, with gravitational potential energy, we need to know where gravitational potential energy equals zero. With universal gravitational potential energy, that location is where the force of gravitational attraction between the two objects equals zero. Billy, where is that location?

- Billy: Uh … I don't get it.
- Bobby: … He is asking how far apart the two objects need to be for the force of gravitational attraction to equal zero.

- Billy: But … It will never equal zero … unless the two objects are infinitely far apart.
- Bobby: I think that is what he is looking for. If r is infinitely large, then Newton's universal law of gravitation equals zero.
- Billy: So, we set the location of zero universal gravitational potential energy where r equals infinity, and the two objects are infinitely far apart?
- Bo: Yep, we did this last time. The point of zero gravitational potential energy is where the force of gravity also equals zero. That happens when r is infinitely large.
- Billy: (Oh.) … Yeah.

{switch perspective}

- Mr.p: Thanks. … Setting the initial distance between the two objects to be



infinitely large means the initial gravitational potential energy is defined as zero at that location. … One over infinity also equals zero. … $\Rightarrow U_{g_f} - U_{g_i} = -Gm_1m_2\left[\frac{1}{r_f} - \frac{1}{\infty}\right] \Rightarrow U_{g_f} - 0 = -Gm_1m_2\left[\frac{1}{r_f} - 0\right] \Rightarrow U_g = -\frac{Gm_1m_2}{r}$ That means universal gravitational potential energy equals the negative of the quantity Newton's universal gravitational constant times mass 1 times mass 2 all divided by the distance between the centers of mass of the two objects. We have just derived the equation for universal gravitational potential energy.

- **BBB: (Great! Okay. Nice.)**
- Mr.p: Previously we showed that the universal gravitational potential energy between an object and the Earth, or any planet for that matter, looks like this, however, we never determined the universal gravitational potential energy if

the object moves inside the planet.

- **Billy: (Inside the planet?)**
- Mr.p: Yes. Inside the planet. If you are interested in taking a detailed look at the force of gravity and gravitational potential energy which exist between an object and a planet all the way from the center of the planet to infinitely far away, we will go over that in a future lesson. The link should have just appeared and is also in the video description.
- **Billy: (What video description? What is going on here?)**
- Mr.p: Thank you very much for learning with me, I enjoy learning with you.