Linear Algebra MAT313 Fall 2022
Professor Sormani
Lesson 1 Linear Systems and Vectors

Please be sure to mark down the date and time that you start this lesson. Carefully take notes on pencil and paper while watching the lesson videos. Pause the lesson to try classwork before watching the video going over that classwork. If you work with any classmates, be sure to write their names on the problems you completed together. Please wear masks when meeting with classmates even if you meet off campus.

You will cut and paste the photos of your notes and completed classwork and a selfie taken holding up the first page of your work in a googledoc entitled:

MAT313F22-lesson1-lastname-firstname

and share editing of that document with me sormanic@gmail.com. You will also include your homework and any corrections to your homework in this doc.

If you have a question, type QUESTION in your googledoc next to the point in your notes that has a question and email me with the subject MAT313 QUESTION. I will answer your question by inserting a photo into your googledoc or making an extra video.

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Linear Algebra Welcome Video
(if you have not watched it yet, please watch it now)

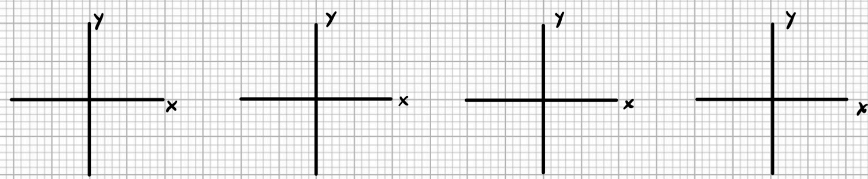
The videos and classwork today will take more than two hours but the homework is shorter than usual.

Lesson 1 Playlist 313F21-1 with 12 videos and 20 classwork problems. If you wish you may first watch the videos about linear systems and take a break before watching the videos about vectors.

Class notes including solutions to classwork followed by homework:

Examples of Linear Equations

① $x + y = 4$ ② $x - y = 4$ ③ $x + y = 8$ ④ $2x + 2y = 8$



one equation
two variables : x and y
graph the solution to each of these

① $x + y = 4$

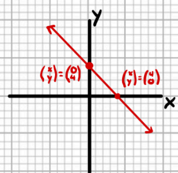
$y = 4 - x$ (basic algebra)
$y = -x + 4$

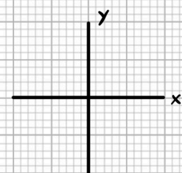
slope $= -1$
y intercept $= 4$

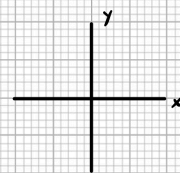
$y = mx + b$ ← will not use this in this course because it does not work with more variables

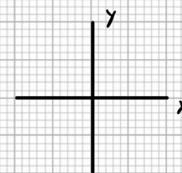
Examples of Linear Equations

(1) $x+y=4$ (2) $x-y=4$ (3) $x+y=8$ (4) $2x+2y=8$









one equation $\boxed{ax+by=z}$ The solution is a line in the plane.
two variables : x and y

graph the solution to each of these

(1) $x+y=4$ Solve graphically (know it is a line)

x	y
0	4
4	0

$0+y=4$
$x+0=4$

Classwork
Use this method to graph (2), (3), + (4)
Pause and do now.

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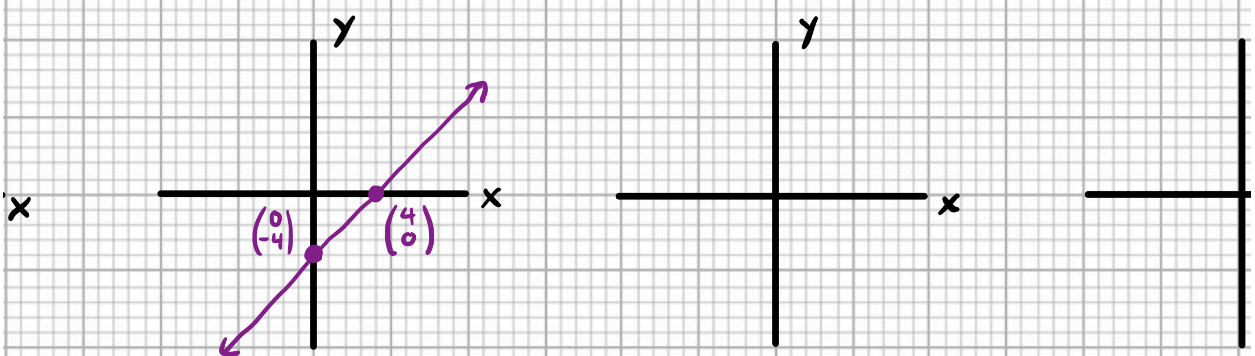
Linear Algebra 0

Scratchwork | Linear Algebra 0

② $x - y = 4$ ③ $x + y = 8$ ④ $2x + 2$



② $x - y = 4$

$0 - y = 4$
$y = -4$

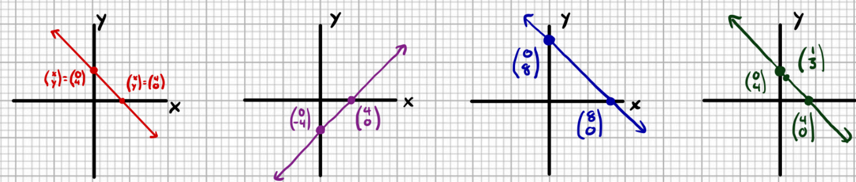
x	y
0	-4
4	0

$\binom{0}{-4}$

$x - 0 = 4$
$x = 4$

Examples of Linear Equations

① $x+y=4$ ② $x-y=4$ ③ $x+y=8$ ④ $2x+2y=8$



All points here solve the equation

③ $x+y=8$

$0+y=8 \rightarrow$ | $x+0=8 \rightarrow$

x	y
0	8
8	0

④ $2x+2y=8$

$2\cdot 1+2y=8$
$2y=6$
$y=3$

$2x+2\cdot 0=8$
$2x=8$
$x=4$

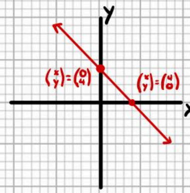
x	y
1	3
4	0

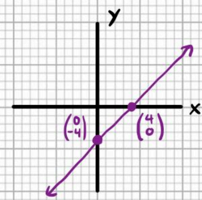
Just need any two points

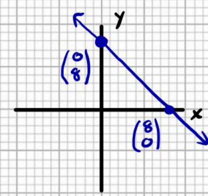
Next Solve systems of linear equations.

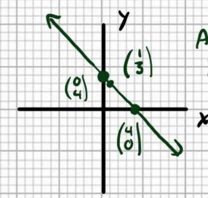
Examples of Linear Equations

① $x + y = 4$ ② $x - y = 4$ ③ $x + y = 8$ ④ $2x + 2y = 8$









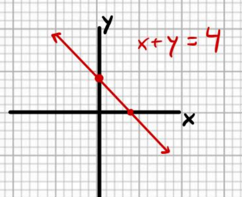
All points here solve the equation

Examples of Systems of Linear Equations

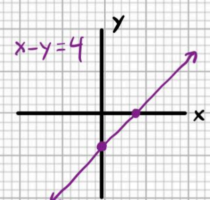
⑤ $x + y = 4$ AND $x - y = 4$ $\boxed{\begin{matrix} x + y = 4 \\ x - y = 4 \end{matrix}}$

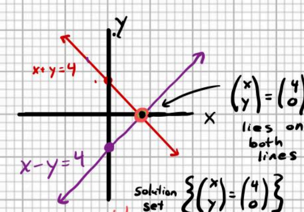
solutions to both equations

Find points that lie on both lines.



and

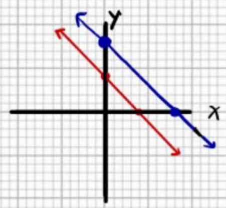




Check $x=4$ $y=0$ solves the system

$4+0=4$ ✓ $4-0=4$ ✓

⑥ $x+y=4$ AND $x+y=8$ $\boxed{\begin{matrix} x+y=4 \\ x+y=8 \end{matrix}}$ Pause + Solve graphically



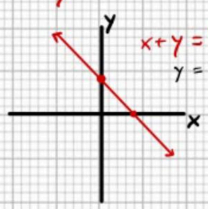
$x+y=4$
$x+y=8$
are parallel
never meet

} no solution

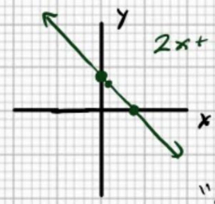
$\varnothing$
empty set

Pause + solve!

⑦ $x+y=4$ AND $2x+2y=8$



$x+y=4$
$y=4-x$



$2x+2y=8$
$2y=8-2x$
$y=4-x$

x can have any value in $\mathbb{R}$
"x is free"

The same line! Every point on the line is a solution!

$$\left\{ \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ 4-x \end{pmatrix} \,\middle|\, x \in \mathbb{R} \right\}$$

real line

Notice $2x+2y=8$
divide both sides by 2
$x+y=4$
multiply both sides by 2
$2x+2y=8$

$x+y=4 \iff 2x+2y=8$
iff symbol
"if and only if"

(8) $x+2y=8$ AND $x-2y=0$

(9) $x+2y=8$ AND $2x+4y=16$

(10) $x+2y=8$ AND $2x+4y=0$

pause + try solutions are in the classnotes.

Solutions

(8) $x+2y=8$ AND $x-2y=0$

$$\boxed{\begin{array}{l} x+2y=8 \\ x-2y=0 \end{array}}$$

x	y
0	4
8	0

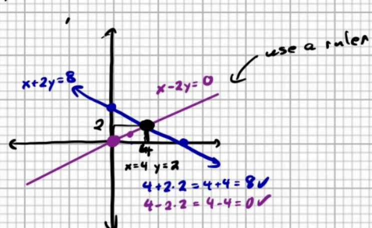
$0+2y=8$, $2y=8$, $y=4$

$x+2\cdot 0=8$, $x=8$

x	y
0	0
2	1

$0-2y=0$, $y=0$

$2-2y=0$, $y=1$



Solution Set $= \left\{ \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4 \\ 2 \end{pmatrix} \right\}$

only one point

(9) $x+2y=8$ AND $2x+4y=16$

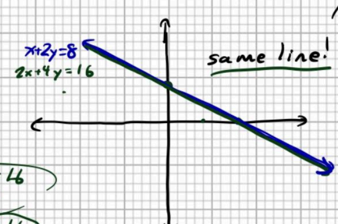
$$\boxed{\begin{array}{l} x+2y=8 \\ 2x+4y=16 \end{array}}$$

$2x+4y=16$

x	y
0	4
8	0

$2\cdot 0+4y=16$, $y=4$

$2x+4\cdot 0=16$, $x=8$



a line

Solution Set $= \left\{ \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ 4-\frac{1}{2}x \end{pmatrix} \middle| \; x \in \mathbb{R} \right\}$

$x+2y=8$

$2y=8-x$

$y=4-\frac{1}{2}x$

(10) $x+2y=8$ AND $2x+4y=0$

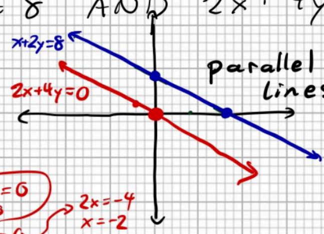
$$\boxed{\begin{array}{l} x+2y=8 \\ 2x+4y=0 \end{array}}$$

$2x+4y=0$

x	y
0	0
-2	1

$2\cdot 0+4y=0$, $y=0$

$2x+4\cdot 1=0$, $2x=-4$, $x=-2$



$2y=8-x$

$y=-\frac{1}{2}x+4$

$4y=0-2x$

$y=-\frac{2}{4}x=-\frac{1}{2}x$

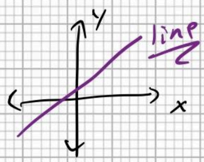
same slope $=\frac{-1}{2}$

Solution Set $=\emptyset$

no solution

Definition of a Linear Equation

a linear equation with two variables:



coefficients

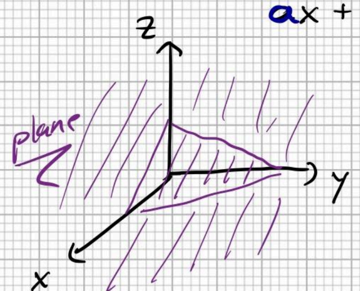
$ax + by = c$ where a, b, c are given real numbers and x, y are unknown variables

the solution is always a line.

with three variables:

coefficients

$ax + by + cz = d$ where a, b, c, d are given and x, y, z are unknown



Solution set is a plane (will discuss more later)

more generally we will have m variables with m very large.

Defn a linear equation with m variables

$x_1, x_2, x_3 \dots x_m$ are unknown variables

each has a coefficient a_i in $\mathbb{R}$

$$a_1x_1 + a_2x_2 + a_3x_3 + \dots + a_mx_m = d$$

we are given $a_1, a_2 \dots a_m \in \mathbb{R}$ and $d \in \mathbb{R}$

find $x_1 \dots x_m$ in $\mathbb{R}$

Sum notation:

$$\sum_{j=1}^{m} a_j x_j = d$$

$$\begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{pmatrix} \in \mathbb{R}^m \quad m\text{-vector}$$

(11) $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 5 \\ 8 \\ 9 \end{pmatrix}$ $a_1 = 2$ $a_2 = 4$ $a_3 = 6$

$d = 10$

Rewrite $\sum_{j=1}^{m} a_j x_j = d$ using these values: check if true or false

Solution:

(11) $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 5 \\ 8 \\ 9 \end{pmatrix}$ $a_1 = 2$ $a_2 = 4$ $a_3 = 6$

$d = 10$

Rewrite $\sum_{j=1}^{3} a_j x_j = d$ using these values and check if true or false

Solution: $a_1 x_1 + a_2 x_2 + a_3 x_3 = d$ by sum notation

stop at $\underline{\underline{3}}$ top of the sum

$2 \cdot 5 + 4 \cdot 8 + 6 \cdot 9 = 10$ false

(12) $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 2 \\ 4 \\ 6 \\ 8 \end{pmatrix}$ $a_1 = 1$ $a_2 = 3$ $a_3 = 5$ $a_4 = 7$

Find the value of d

$d = \sum_{j=1}^{4} a_j x_j$ solutions in notes

SOLUTION

(12) $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 2 \\ 4 \\ 6 \\ 8 \end{pmatrix}$ $a_1 = 1$ $a_2 = 3$ $a_3 = 5$ $a_4 = 7$

Find the value of d

$d = \sum_{j=1}^{4} a_j x_j = \sum_{j=1}^{4} a_j x_j = a_1 x_1 + a_2 x_2 + a_3 x_3 + a_4 x_4$

$= 1 \cdot 2 + 3 \cdot 4 + 5 \cdot 6 + 7 \cdot 8$ stop at 4

$= 2 + 12 + 30 + 56 = 44 + 56 = 100$ $\boxed{d = 100}$

Defn: A system of $\underline{n}$ linear equations with m unknown variables

$$
\begin{aligned}
&a_{1,1}x_1 + a_{1,2}x_2 + \cdots + a_{1,m}x_m = d_1 && \leftarrow \text{equation 1}\\
&a_{2,1}x_1 + a_{2,2}x_2 + \cdots + a_{2,m}x_m = d_2 && \leftarrow \text{equation 2}\\
&\cdots\\
&a_{n,1}x_1 + a_{n,2}x_2 + \cdots + a_{n,m}x_m = d_n && \leftarrow \text{equation } n
\end{aligned}
$$

Can also use sum notation

$$\boxed{\sum_{j=1}^{m} a_{i,j}x_j = d_i \quad \text{for } i = 1 \text{ to } n}$$

(13) $a_{1,1} = 1$ $a_{1,2} = 2$ $a_{1,3} = 3$ $d_1 = 10$

$a_{2,1} = 4$ $a_{2,2} = 5$ $a_{2,3} = 6$ $d_2 = 11$

Rewrite the system $\sum_{j=1}^{3} a_{ij} x_j = d_i$ for $i = 1, 2$

with $x_1 = x$
$x_2 = y$
$x_3 = z$

pause + try

(13) $a_{1,1} = 1$ $a_{1,2} = 2$ $a_{1,3} = 3$ $d_1 = 10$

$a_{2,1} = 4$ $a_{2,2} = 5$ $a_{2,3} = 6$ $d_2 = 11$

Rewrite the system $\boxed{\sum_{j=1}^{3} a_{ij} x_j = d_i \text{ for } i=1,2}$

with $x_1 = x$
$x_2 = y$
$x_3 = z$

pause + try

2 equations:

$i=1$ $\sum_{j=1}^{3} a_{1j} x_j = d_1$ $a_{1,1} x_1 + a_{1,2} x_2 + a_{1,3} x_3 = d_1$

$1x + 2y + 3z = 10$

$i=2$ $\sum_{j=1}^{3} a_{2j} x_j = d_2$ $a_{2,1} x_1 + a_{2,2} x_2 + a_{2,3} x_3 = d_2$

$4x + 5y + 6z = 11$

$$\boxed{\begin{aligned} 1x + 2y + 3z &= 10 \\ 4x + 5y + 6z &= 11 \end{aligned}}$$

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Linear Algebra 0

Scratchwork | Linear Algebra 0

(14) Rewrite the system

$$\boxed{\begin{aligned} 5x+2y &= 10 \\ 2x-4y &= 0 \\ x+y &= 8 \end{aligned}}$$

using sum notation

and tell us the value of each a_{ij}, d_i, x_j

use sum notation

$$\boxed{\sum_{j=1}^{m} a_{i,j} x_j = d_i \quad \text{for } i=1 \text{ to } n}$$

coefficients

$a_{i,j}$

i^{th} equation

j^{th} variable x_j

pause + solve

(14) Rewrite the system
$$\boxed{\begin{aligned} 5x+2y &= 10 \\ 2x-4y &= 0 \\ 1x+1y &= 8 \end{aligned}}$$
✓ ✓ ✓

using sum notation

and tell us the value of each a_{ij}, d_i, x_j

use sum notation

$$\boxed{\sum_{j=1}^{m} a_{i,j}\, x_j = d_i \quad \text{for } i=1 \text{ to } n}$$

coefficients
$a_{i,j}$
i^{th} equation
j^{th} variable x_j

$$\sum_{j=1}^{2} a_{ij}\, x_j = d_i \quad \text{for } i=1 \text{ to } n=3$$

$\leftarrow$ because we have $m=2$ variables $x_1=x$ $x_2=y$

because we have 3 equations

equation $i=1$ $a_{1,1}=5$ $a_{1,2}=2$ $d_1=10$

equation $i=2$ $a_{2,1}=2$ $a_{2,2}=-4$ $d_2=0$

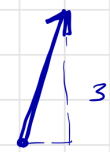
equation $i=3$ $a_{3,1}=1$ $a_{3,2}=1$ $d_3=8$

because $x=1\cdot x$

Vectors in $\mathbb{R}^m$

$m=2$ vector $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \in \mathbb{R}^2$

Example $\begin{pmatrix} 1 \\ 3 \end{pmatrix}$



$m=3$ vector $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \in \mathbb{R}^3$

seen in vector calculus

vector $\begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{pmatrix} \in \mathbb{R}^m$

$\mathbb{R}$ reals

$(-\infty, \infty)$

Example

$\begin{pmatrix} 1 \\ 2 \\ 7 \end{pmatrix} \in \mathbb{R}^3$

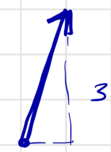
$\begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \\ 1 \\ 0 \\ 0 \\ 1 \end{pmatrix} \in \mathbb{R}^8$

we will have $m = 10{,}000$ or higher

Vectors in $\mathbb{R}^m$

$m=2$ vector $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \in \mathbb{R}^2$

Example $\begin{pmatrix} 1 \\ 3 \end{pmatrix}$



$m=3$ vector $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \in \mathbb{R}^3$ seen in vector calculus

vector $\begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{pmatrix} \in \mathbb{R}^m$

$\mathbb{R}$ $\underbrace{\text{reals}}$ $(-\infty, \infty)$

$\longleftrightarrow$

Example

$\begin{pmatrix} 1 \\ 2 \\ 7 \end{pmatrix} \in \mathbb{R}^3$

$\begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \\ 1 \\ 0 \\ 0 \\ 1 \end{pmatrix} \in \mathbb{R}^8$

we will have $m = 10{,}000$ or higher

$\vec{x} \in \mathbb{R}^4$ $\quad \vec{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix}$ $\quad \vec{z} \in \mathbb{R}^5$ $\quad \vec{z} = \begin{pmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \\ z_5 \end{pmatrix}$

$\vec{y} \in \mathbb{R}^4$ $\quad \vec{y} = \begin{pmatrix} y_1 \\ y_2 \\ x_3 \\ y_4 \end{pmatrix}$

y_i is the i^{th} "entry" "component" of vector $\vec{y}$

Addition of Vectors:

<u>Defn</u>: Given $\vec{x}$ and $\vec{y} \in \mathbb{R}^m$ then

$$\vec{x} + \vec{y} = \begin{pmatrix} x_1 + y_1 \\ x_2 + y_2 \\ \vdots \\ x_m + y_m \end{pmatrix} \in \mathbb{R}^m$$

Addition of Vectors:

Defn: Given $\vec{x}$ and $\vec{y} \in \mathbb{R}^m$ then

$$\vec{x}+\vec{y} = \begin{pmatrix} x_1+y_1 \\ x_2+y_2 \\ \vdots \\ x_m+y_m \end{pmatrix} \in \mathbb{R}^m$$

$\in$ in symbol

(15) $$\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + \begin{pmatrix} 4 \\ 5 \\ 6 \end{pmatrix} = \begin{pmatrix} 1+4 \\ 2+5 \\ 3+6 \end{pmatrix} = \begin{pmatrix} 5 \\ 7 \\ 9 \end{pmatrix} \in \mathbb{R}^3$$

(16) $$\begin{pmatrix} 1 \\ 3 \\ 5 \\ 7 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1+0 \\ 3+0 \\ 5+0 \\ 7+0 \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \\ 5 \\ 7 \end{pmatrix} \in \mathbb{R}^4$$

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Linear Algebra 0

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Defn Scalar Multiplication of Vectors

Given a vector $\vec{v} \in \mathbb{R}^m$ and a scalar $t \in \mathbb{R}$

we define the scalar product

$$t\vec{v} = t\begin{pmatrix} v_1 \\ v_2 \\ \vdots \\ v_m \end{pmatrix} = \begin{pmatrix} tv_1 \\ tv_2 \\ \vdots \\ tv_m \end{pmatrix}$$

Rescales the vector by t

(17) $2\begin{pmatrix} 1 \\ 4 \end{pmatrix} = ?$ graph

(18) $-4\begin{pmatrix} 3 \\ 2 \end{pmatrix} = ?$ graph

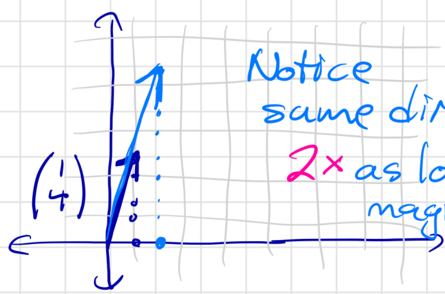
pause + try

we define the scalar product

$$t\vec{v} = t\begin{pmatrix} v_1 \\ v_2 \\ \vdots \\ v_m \end{pmatrix} = \begin{pmatrix} tv_1 \\ tv_2 \\ \vdots \\ tv_m \end{pmatrix}$$

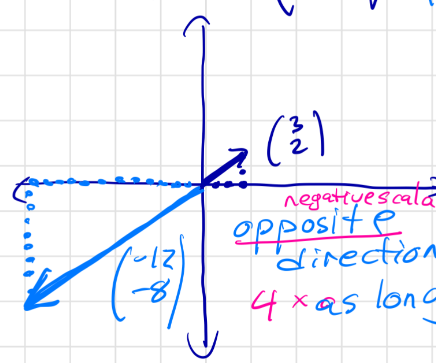
Rescales the vector by t

(17) $2\begin{pmatrix} 1 \\ 4 \end{pmatrix} = \begin{pmatrix} 2\cdot 1 \\ 2\cdot 4 \end{pmatrix} = \begin{pmatrix} 2 \\ 8 \end{pmatrix}$



Notice same direction
2× as long a magnitude

(18) $-4\begin{pmatrix} 3 \\ 2 \end{pmatrix} = \begin{pmatrix} -4\cdot 3 \\ -4\cdot 2 \end{pmatrix} = \begin{pmatrix} -12 \\ -8 \end{pmatrix}$



negative scalar
opposite direction
4× as long

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Linear Algebra 0

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Subtraction of vectors: Given $v, w \in \mathbb{R}^m$

$$\vec{v} - \vec{w} = \vec{v} + \underbrace{(-1)}_{\substack{\text{scalar} \\ \text{product}}}\vec{w} \in \mathbb{R}^m$$

(reverses direction)

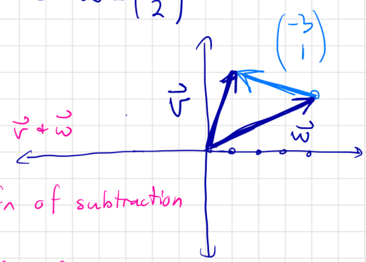
(19) Graph $\vec{v} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$ and $\vec{w} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$

and find $\vec{v} - \vec{w}$

pause & try

(19) Graph $\vec{v} = \begin{pmatrix}1\\3\end{pmatrix}$ and $\vec{w} = \begin{pmatrix}4\\2\end{pmatrix}$

and find $\vec{v} - \vec{w}$



$\vec{v} - \vec{w} = \begin{pmatrix}1\\3\end{pmatrix} - \begin{pmatrix}4\\2\end{pmatrix}$ by our $\vec{v} + \vec{w}$

$= \begin{pmatrix}1\\3\end{pmatrix} + (-1)\begin{pmatrix}4\\2\end{pmatrix}$ by defn of subtraction

$= \begin{pmatrix}1\\3\end{pmatrix} + \begin{pmatrix}(-1)4\\(-1)2\end{pmatrix}$ by defn of scalar mult.

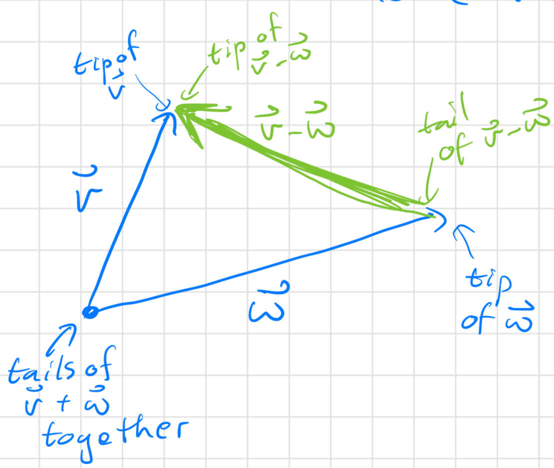
$= \begin{pmatrix}1\\3\end{pmatrix} + \begin{pmatrix}-4\\-2\end{pmatrix}$ by arithmetic $(-1)4 = -4$, $(-1)2 = -2$

$= \begin{pmatrix}1+(-4)\\3+(-2)\end{pmatrix}$ by defn of vector addition

$= \begin{pmatrix}-3\\1\end{pmatrix}$ by arithmetic $1-4=-3$, $3-2=1$

Justified my steps

In fact $\vec{v}-\vec{w}$ is always a vector whose tip is at the tip of $\vec{v}$ and whose tail is at the tip of $\vec{w}$

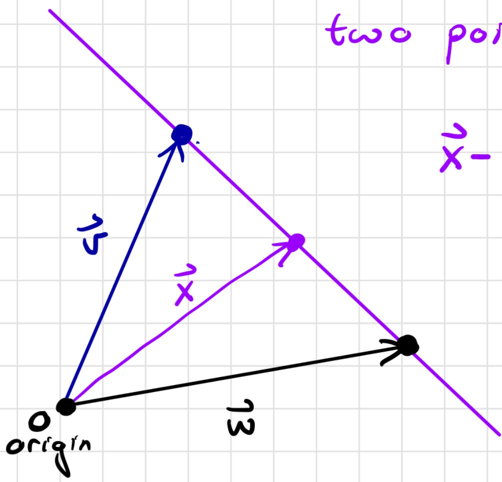


For more about vector addition and scalar multiplicat on, see Additional Resources at the end of this.lesson

Lines written in Vector Notation

Find a formula for a line through two points $\vec{v} = \begin{pmatrix} v_1 \\ \vdots \\ v_m \end{pmatrix}$ and $\vec{w} = \begin{pmatrix} w_1 \\ \vdots \\ w_m \end{pmatrix}$

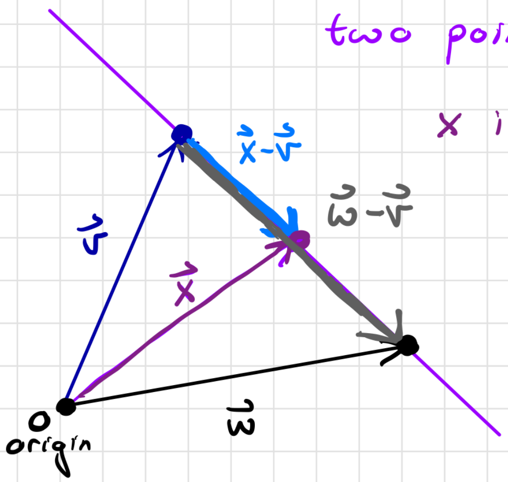
$\vec{x} - \vec{v}$



Lines written in Vector Notation

Find a formula for a line through two points $\vec{v} = \begin{pmatrix} v_1 \\ \vdots \\ v_m \end{pmatrix}$ and $\vec{w} = \begin{pmatrix} w_1 \\ \vdots \\ w_m \end{pmatrix}$

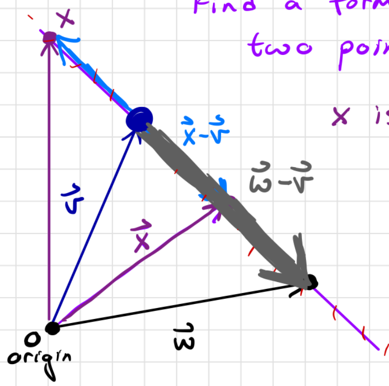
x is a typical point on the line



$$\vec{x}-\vec{v} = t\,(\vec{w}-\vec{v})$$

Lines written in Vector Notation

Find a formula for a line through two points $\vec{v} = \begin{pmatrix} v_1 \\ \vdots \\ v_m \end{pmatrix}$ and $\vec{w} = \begin{pmatrix} w_1 \\ \vdots \\ w_m \end{pmatrix}$



x is a typical point on the line

$$\vec{x} - \vec{v} = t\,(\vec{w} - \vec{v})$$

$$\left\{ \vec{x} = \vec{v} + t\,\underbrace{(\vec{w} - \vec{v})} \;\middle|\; t \in \mathbb{R} \right\}$$

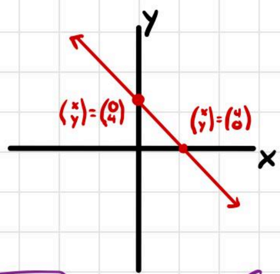
position vector $\vec{v}$ on the line

direction vector $(\vec{w} - \vec{v})$ where $\vec{w}$ is also on the line

as we change the value of t we get different points $\vec{x}$ lying on the line

(20)(a) Write this line in vector notation

$x+y=4$ using $\vec{v}=\begin{pmatrix}0\\4\end{pmatrix}$ and $\vec{w}=\begin{pmatrix}4\\0\end{pmatrix}$



$$\left\{\vec{x}=\underbrace{\vec{v}}_{\text{position}}+t\,\underbrace{(\vec{w}-\vec{v})}_{\text{direction}}\;\middle|\;t\in\mathbb{R}\right\}$$

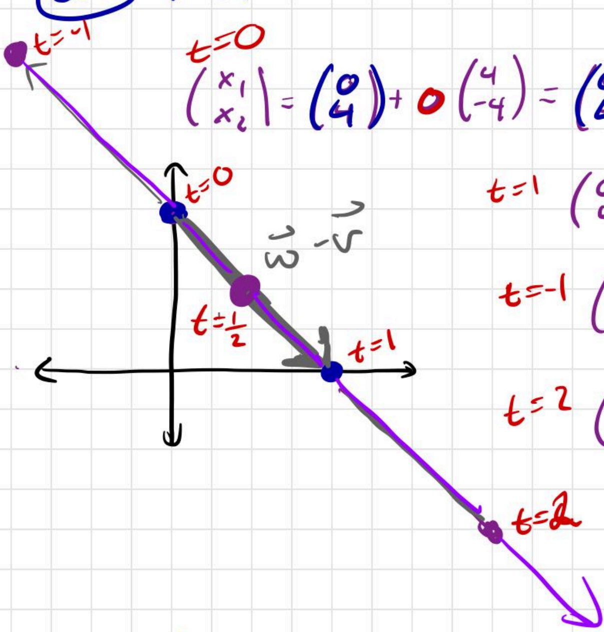
position $\begin{pmatrix}0\\4\end{pmatrix}$

direction $\begin{pmatrix}4\\0\end{pmatrix}-\begin{pmatrix}0\\4\end{pmatrix}=\begin{pmatrix}4-0\\0-4\end{pmatrix}=\begin{pmatrix}4\\-4\end{pmatrix}$

$\begin{pmatrix}X\\Y\end{pmatrix}=\begin{pmatrix}x_1\\x_2\end{pmatrix}$

$$\left\{\begin{pmatrix}x_1\\x_2\end{pmatrix}=\begin{pmatrix}0\\4\end{pmatrix}+t\begin{pmatrix}4\\-4\end{pmatrix}\;\middle|\;t\in\mathbb{R}\right\}$$

(20b) Plot the vectors for $t=0, t=1, t=-1, t=2, t=4$ and $t=\frac{1}{2}$



$t=0$

$\begin{pmatrix}x_1\\x_2\end{pmatrix}=\begin{pmatrix}0\\4\end{pmatrix}+0\begin{pmatrix}4\\-4\end{pmatrix}=\begin{pmatrix}0\\4\end{pmatrix}+\begin{pmatrix}0\\0\end{pmatrix}=\begin{pmatrix}0\\4\end{pmatrix}$ position $\vec{v}$

$t=1$ $\begin{pmatrix}0\\4\end{pmatrix}+1\begin{pmatrix}4\\-4\end{pmatrix}=\begin{pmatrix}4\\0\end{pmatrix}\leftarrow\vec{w}$

$t=-1$ $\begin{pmatrix}0\\4\end{pmatrix}+(-1)\begin{pmatrix}4\\-4\end{pmatrix}=\begin{pmatrix}-4\\8\end{pmatrix}$

$t=2$ $\begin{pmatrix}0\\4\end{pmatrix}+2\begin{pmatrix}4\\-4\end{pmatrix}=\begin{pmatrix}8\\-4\end{pmatrix}$

$t=\frac{1}{2}$ $\begin{pmatrix}0\\4\end{pmatrix}+\left(\frac{1}{2}\right)\begin{pmatrix}4\\-4\end{pmatrix}=\begin{pmatrix}2\\2\end{pmatrix}$

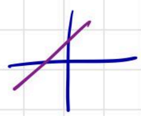
$$\left\{\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}0\\4\end{pmatrix}+t\begin{pmatrix}4\\-4\end{pmatrix}\;\middle|\;t\in\mathbb{R}\right\}$$

Lines in $\mathbb{R}^3$, $\mathbb{R}^4$ and even $\mathbb{R}^m$

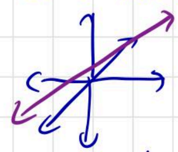
$$\Big\{ \vec{x} = \underset{\text{position}}{\vec{v}} + t\,\underbrace{(\vec{w} - \vec{v})}_{\text{direction}} \;\Big|\; t \in \mathbb{R} \Big\}$$

works in all dimensions.

$m=2$ in $\mathbb{R}^2$



$m=3$ in $\mathbb{R}^3$



$m=4$ and higher cannot be drawn but we can still describe the set.

Example in $\mathbb{R}^4$

Find the line through $\vec{v} = \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix}$ and $\vec{w} = \begin{pmatrix} 0 \\ 1 \\ 5 \\ 8 \end{pmatrix}$

Solution: try first if you wish

Direction $\vec{w} - \vec{v} = \begin{pmatrix} 0 \\ 1 \\ 5 \\ 8 \end{pmatrix} - \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix} = \begin{pmatrix} 0-1 \\ 1-2 \\ 5-3 \\ 8-4 \end{pmatrix} = \begin{pmatrix} -1 \\ -1 \\ 2 \\ 4 \end{pmatrix}$

$$\left\{ \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix} + t \begin{pmatrix} -1 \\ -1 \\ 2 \\ 4 \end{pmatrix} \;\middle|\; t \in \mathbb{R} \right\}$$

Check $\vec{v}$ and $\vec{w}$ are on the line

at $t=0$ $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix} + 0 \begin{pmatrix} -1 \\ -1 \\ 2 \\ 4 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix}$ ✓ $\vec{v} \in$ line

at $t=1$
$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix} + 1\begin{pmatrix} -1 \\ -1 \\ 2 \\ 4 \end{pmatrix} = \begin{pmatrix} 1-1 \\ 2-1 \\ 3+2 \\ 4+4 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 5 \\ 8 \end{pmatrix} \checkmark \in \text{line}$$

at $t=2$
$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix} + 2\begin{pmatrix} -1 \\ -1 \\ 2 \\ 4 \end{pmatrix} = \begin{pmatrix} 1-2 \\ 2-2 \\ 3+4 \\ 4+8 \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \\ 7 \\ 12 \end{pmatrix} \text{ on the line.}$$

See similar homework.

[Linear Algebra Lesson 1 Playlist](#) (12 videos)

[How to check you have watched all the videos](#)

Additional Resources at the end of this lesson

Homework:

At the top of your lesson 1 googledoc, write your full name as on the Lehman registration and also any preferred names or pronouns if you wish. Let me know a little about your career and education goals. It is fine to mention a few possible directions of interest. It is always great to have options and to double major or have a minor.

Include photos of your notes and then do the ten homework problems below showing all work:

Lesson 1 Homework

① Rewrite the system

$$\boxed{\begin{aligned} 2x+0y+5z&=4\\ 2x+4y+8z&=3\\ x-y+z&=8 \end{aligned}}$$

as a sum

$$\sum_{j=1}^{m} a_{ij}x_j = d_i \quad \text{for } i=1 \text{ to } n$$

with $x_1 = x$ $x_2 = y$ $x_3 = z$

What is m? What is n?

What are the $a_{i,j}$ and d_i?

② Write the system $\sum_{j=1}^{4} a_{ij}x_j = d_i$ for $i=1$ to 3

with $x_1 = x$ $x_2 = y$ $x_3 = z$ $x_4 = w$

$d_i = i^2$ (that is $d_1 = 1$ $d_2 = 2^2 = 4$ $d_3 = 3^2 = 9 \ldots$)

and $a_{i,j} = (i+j)$ ($a_{1,1} = 1+1 = 2$ $a_{1,2} = 1+2 = 3 \ldots$)

(3) Let $\vec{v} = \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix}$ and $\vec{w} = \begin{pmatrix} 5 \\ 5 \\ 5 \\ 5 \end{pmatrix}$

find $\vec{w} - \vec{v}$ slowly justifying each step. (see final example)

(4) (a) Find the formula for the line through $\vec{v} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ and $\vec{w} = \begin{pmatrix} 5 \\ 2 \end{pmatrix}$

(see classwork 20)

(b) Plot the points for $t=0$, $t=1$, $t=-1$, $t=2$, and $t=\frac{1}{2}$ and verify they are on the line.

(5) Consider $y = mx + b$

Check that $\vec{v} = \begin{pmatrix} 0 \\ b \end{pmatrix}$ and $\vec{w} = \begin{pmatrix} 1 \\ m+b \end{pmatrix}$ are on the line.

Find the vector formula for this line.

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Linear Algebra 0

Scratchwork | Linear Algebra 0

(6) Find the vector formula for the line through $\begin{pmatrix}1\\0\\1\\0\end{pmatrix}$ and $\begin{pmatrix}0\\2\\0\\2\end{pmatrix}$

Before doing (7)-(10)

Read Beezer Preliminaries on Complex numbers and then practice at IXL

Links are below.

(7) $(2+4i)+(5-3i)=\,?$

(8) $5(2+4i)=\,?$

(9) $(a+bi)(a-bi)=\,?$

(10) $\dfrac{4+3i}{5+6i}=\,?$ (Hint multiply top and bottom by $(5-6i)$)

Read Beezer Preliminaries on Complex numbers and then practice at IXL before doing (7)-(10). If you do not have time for this right now, you may do this after Exam 1.

Additional Resources:

Rootmath Linear Algebra Section 1.1: about vector addition and scalar multiplication:
https://www.youtube.com/playlist?list=PLA738885C1D6E75A4

Lines for Linear Algebra videos and notes

To submit homework follow the directions at the top of this document.