BC Calculus Assignment 8.3

L'Hôspital's rule and relative rates

1. To do L'Hôspital's Rule, you find a _____________ by taking the _____________________ of the top and bottom of a fraction.

2. L'Hôspital's Rule only applies if you're evaluating a limit that gives you — or — .

3. When $f(x)$ grows faster than $g(x)$, $\lim\limits_{x \to \infty} \frac{f(x)}{g(x)} =$ ____________.

4. When $g(x)$ grows faster than $f(x)$, $\lim\limits_{x \to \infty} \frac{f(x)}{g(x)} =$ ____________.

5. When $f(x)$ and $g(x)$ grow at the same rate, $\lim\limits_{x \to \infty} \frac{f(x)}{g(x)} =$ a _________________.

Circle the function that grows faster, or if they grow at the same rate, write "same rate".

6. x^2 and $5x^2$

7. $5x^2$ and x^3

8. x^3 and 3^x

9. $\ln x$ and $\sqrt{x}$

10. Put the following functions in order of growth, from slowest to fastest, given $a > 1$:

$$x^a,\ x^{-a},\ x^{1/a},\ a^x,\ x!,\ \log_a x,\ ax$$

11. $\lim\limits_{x \to 0} \frac{4e^x - 4}{x}$ is

12. $\lim\limits_{h \to 0} \dfrac{2e^{h} - 2}{5h}$ is

13. $\lim\limits_{x \to 2} \dfrac{\sin(x-2)}{2e^{x-2} - x}$ is

14. $\lim\limits_{x \to 0} \dfrac{e^{x} - \sin x - 3x - 1}{4x^{2} - 6x}$ is

15. $\lim\limits_{x \to 0} \dfrac{x^{2}}{x - \sin x}$ is

16. If *a* and *b* are positive constants, then $\lim\limits_{x \to \infty} \dfrac{\ln(bx+1)}{\ln(ax^{4}+4)} =$

17. Which of the following limits are greater than 0 ? *Circle all that apply*

I. $\lim\limits_{x \to 0^{-}} \dfrac{|x|}{x}$

II. $\lim\limits_{x \to 3} \dfrac{x^{2} - 7x + 12}{3 - x}$

III. $\lim\limits_{x \to \infty} \dfrac{1 - x}{1 + x}$

x	$f(x)$	$f'(x)$	$f''(x)$	$f'''(x)$
2	0	0	5	7

18. The third derivative of the function f is continuous on the interval (0, 4). Values for f and its first three derivatives at $x = 2$ are given in the table above. What is $\lim_{x \to 2} \frac{(x-2)^2}{f(x)}$?

19. $\lim_{x \to 3} \frac{\int_3^x e^{3t}\, dt}{x^2 - 9}$ is

20. Let g be a continuously differentiable function with $g(-2) = 4$ and $g'(-2) = 6$. What is $\lim_{x \to -2} \frac{\int_{-2}^x g(t)\, dt}{g(x) - 4}$?

21. For which of the following does $\lim_{x \to \infty} f(x) = \infty$? *Circle all that apply*

I. $f(x) = \frac{\ln x}{x^{99}}$

II. $f(x) = \frac{e^x}{\ln x}$

III. $f(x) = \frac{x^{99}}{e^x}$

1. Consider the differential equation $\frac{dy}{dx} = y^2(2x + 2)$. Let $y = f(x)$ be the particular solution to the differential equation with initial condition $f(0) = -1$.

(a) Find $\lim_{x \to 0} \frac{f(x) + 1}{\sin x}$. Show the work that leads to your answer.

1: L'Hospital's Rule
1: answer

(b) Use Euler's method, starting at $x = 0$ with two steps of equal size, to approximate $f(\frac{1}{2})$.

1: Euler's method
1: answer

(c) Find $y = f(x)$, the particular solution to the differential equation with initial condition $f(0) = -1$.

1: separation of variables
1: antiderivatives
1: constant of integration
1: uses initial condition
1: solves for y

2. Let f be the function given by $f(x) = 2xe^{2x}$.

(a) Find $\lim_{x \to -\infty} f(x)$ and $\lim_{x \to \infty} f(x)$.

1: $\lim_{x \to \infty} f(x)$

1: $\lim_{x \to -\infty} f(x)$

(b) Find the absolute minimum value of f. Justify that your answer is an absolute minimum.

1: solve f'(x) for appropriate value

1: evaluate f at critical point

1: justify absolute minimum value

(c) What is the range of f?

1: answer

(d) Consider the family of functions defined by $y = bxe^{bx}$, where b is a nonzero constant. Show that the absolute minimum value of bxe^{bx} is the same for all nonzero values of b.

1: set y' equal to appropriate value

1: solve y'

1: evaluate y at critical number and get value independent of b