Name:

APPLICATION: Insuring Multiple Events with Compound Probability

Level 1

Redundancy and Reliability
A common practice in engineering is redundancy. Redundancy is having backups of a component to improve the reliability of a product. Simply put: If a part fails, it has a backup to take over so the object still works. Let's look at an example of redundancy in computer servers.

Scenario: An internet web hosting company, World Wide Hosting, boasts high reliability in their advertisements. To reduce any risk of servers going down, each main server has backups so that if the main server crashes, the backup can take over without any websites going offline. The probability of any main server crashing is 10%.

1. **World Wide Hosting wants to increase the reliability of their main server, so they add a backup server that has a probability of crashing, called the failure rate, of 12%.**
    a. Assuming the servers are completely independent, use set notation to describe the probability of both servers crashing.
    b. What is the probability of a network outage on any given day?

2. **World Wide Hosting wants to increase the reliability even more, so they add a third backup to their main server. The third backup has a failure rate of 15%. What is the probability of a network outage with three independent servers running?**

3. **After some testing, World Wide Hosting finds that their servers are not completely independent. If a server crashes due to a power surge, all backups after the failed server have a 20% increased chance of failing. Note: The 20% increased chance of failing will only be applied to each server one time.**
    a. During a storm, the main server fails due to an electrical surge. What is the new probability of two backup servers given that an electrical surge has occurred?
    b. What is the probability of a network outage if the main server fails due to an electrical surge?

Level 2

Auto Insurance
An auto insurance company, InsurAuto, is setting its monthly premiums for 2023 and has created the following table based off of policyholder data.

Probability of motor vehicle accident per 30,000 miles driven

	16 to 24	25 to 40	40 to 65	65+
Male	0.17	0.11	0.06	0.13
Female	0.14	0.09	0.04	0.12

- Subtract 2% for individuals that have taken a defensive driving course
- Subtract 5% for individuals that own a car with crash avoidance features
- Add 3% for drivers in cities with a population more than 100,000

1. **What is the expected value of the following individual?**
    - 67 year old man that pays a $210 per month premium
    - Has not taken a defensive driving course
    - Does not have a car with crash avoidance features
    - Lives in Collingswood, NJ (population 14,000)?
    - Receives an average payout of $1100 if they file a claim

InsurAuto hopes to earn $25 profit per policyholder per month and pays about $2000 on average per auto claim.

2. **InsurAuto sets its premiums on an individual basis. For each person described below, calculate the premium that InsurAuto must set in order to earn an expected value of $25 per policyholder per month.**
    a. Alonso is a 22 year old male who lives in New York City (population 8.4 million). He has never taken a defensive driving course but does have a car with crash avoidance features.

    b. Aaliyah is a 43 year old woman who lives in Kirkland, WA (population 91,951). She took a defensive driving course recently but does not own a car with crash avoidance features.

3. **Explain why Alonso's premium is so much higher than Aaliyah's.**

4. **Carrie is a 26 year old woman living in Melrose, MA (population 28,254). She has not taken a defensive driving course nor does she have a car with crash avoidance features. The probabilities listed in the table are per 30,000 miles driven. If Carrie drives 30,000 miles each year for 5 years:**
    a. If C = a crash, how would you write the probability of not crashing using set notation?

    b. What is the probability of her getting into exactly 2 motor vehicle accidents? Note: It does not matter which year contains the crash, just that there are exactly two.

Level 3

The Law of Large Numbers and Insurance

Insurance companies rely on insuring a large pool of individuals in order to take advantage of the Law of Large Numbers. First, watch this video on the Law of Large Numbers. Then we will perform an experiment to see this mathematical concept in action.

1. **In your own words, what does the law of large numbers tell us?**

Now let's set up our own insurance company and see how the law of large numbers affects our bottom line. The insurance company charges a monthly premium of $100. We are going to use a standard, six sided die to simulate the likelihood that someone will file an insurance claim each month.

- On a 1, the policyholder will file a claim and be paid $300
- On a 2, 3, 4, 5, or 6, no claim will be made

2. **Are the dice rolls being used to simulate a claim independent or dependent?**

3. **Define a claim using a variable, then write a statement that represents NOT a claim.**

4. **What is the theoretical probability that someone will file a claim 3 months in a row?**

5. **Let's calculate the expected value of an insurance policy using theoretical probabilities. Complete the table to calculate the expected value of an insurance policies**

Dice Outcome	Theoretical Probability	Profit/Loss	x*P(x)
1		-200	
2		+100	
3		+100	
4		+100	
5		+100	
6		+100	
		Expected Value	

6. **Why is the profit/loss of a die roll of 1 only -200 if the claim pays $300?**

7. **Now let's see what our actual outcomes are. Use [Roll a Die] to simulate 10 rolls and calculate the expected value after the 10 rolls.**
    a. Set number of rolls to 10
    b. Click Go
    c. Click Show Stats and record your results in the table below.

10 Trials

Dice Outcome	Experimental Probability	Profit/Loss	x*P(x)
1		-200	
2		+100	
3		+100	
4		+100	
5		+100	
6		+100	
		Expected Value	

8. **Perform the same calculations but let's increase the number of policyholders to 1,000. Use [Roll a Die] again but update the number of rolls to 1,000, then record your results in the table below.**

1,000 Trials

Dice Outcome	Experimental Probability	Profit/Loss	x*P(x)
1		-200	
2		+100	
3		+100	
4		+100	
5		+100	
6		+100	
		Expected Value	

9. **Let's increase the number of policyholders one more time, this time to 1,000,000. Use [Roll a Die] again but update the number of rolls to 1,000,000, then record your results in the table below. Note that there will be a little lag on the website due to the high number of simulated rolls.**

1,000,000 Trials

Dice Outcome	Experimental Probability	Profit/Loss	x*P(x)
1		-200	
2		+100	
3		+100	
4		+100	
5		+100	
6		+100	
		Expected Value	

10. **Look back at your theoretical and experimental expected values. Write a summary statement about how the experimental expected values compared to the theoretical value as the number of trials increased.**

11. **How do you think an insurance company uses the law of large numbers to its advantage?**