Suppose we want to solve a problem such as finding the orbital velocity of a satellite in orbit around a body. How do we do that? It seems so freaking complicated! It’s actually not that difficult. When you break the formula down one piece at a time and understand what each piece means, then solving the equation becomes as easy as multiplication, addition, subtraction, and division.

The formula for finding the orbital velocity of an object in orbit is as follows:

$$V = \sqrt{\frac{GM}{R}}$$

> For the sake of entering calculations into a calculator, you could also express the equation this way: V = sqrt(GM/R)

But what does all that mean? What is V, what is GM? What is R?

In the following few paragraphs, we will look at what each of these letters mean and then show how to plug them into the formula.

Scientific Notation

I want to start by briefly talking about Scientific Notation. If you haven't brushed up on Scientific Notation in a while, you might not understand what things like 6.67259E-11 and 3.301880e23 actually mean.

Scientific notation is just a way to show very long numbers using a smaller amount of space.

3.301880e23 means that we are going to move the decimal 23 places to the right, and fill in the gaps with zeros. So 3.301880e23 becomes 330188000000000000000000.

Let's look at it this way:

3.301880 is our starting number.
33.01880 is 1 decimal to the right.
330.1880 is 2 decimals to the right.
3301.880 is 3 decimals to the right.
33018.80 is 4 decimals to the right.
330188.0 is 5 decimals to the right.
3301880. is 6 decimals to the right.

But now we are out of numbers, so as we continue to move the decimal to the right, we fill in the gaps with zeros.

33018800. is 7 decimals to the right, plus we added 1 zero to fill in the gap.
330188000. is 8 decimals to the right, plus we added 2 zeros to fill in the gaps.
3301880000. is 9 decimals to the right, plus we added 3 zeros to fill in the gaps.

And so on until we finally get to:

330188000000000000000000 is 23 decimals to the right.

You will sometimes see this kind of notation with a + sign in it.

3.301880e+23

3.301880e23 and 3.301880e+23 mean exactly the same thing.

Scientific Notation

On the other hand, 6.67259E-11 means we are going to move the decimal point 11 places to the left. Note that the notation is E-11 instead of E11 (or E+11). The negative sign means we have to move the decimal to the left instead of to the right.

Again, let's look at it this way:

6.67259 is our starting number.
.667259 is 1 decimal point to the left.

But here we have already ran out of numbers. So how do we keep moving the decimal to the left if there aren't any numbers? We just fill in the gaps with zeros.

.0667259 is 2 decimal points, plus we added 1 zero to fill in the gap.
.00667259 is 3 decimal points, plus we added 2 zeros to fill in the gaps.
.000667259 is 4 decimal points, plus we added 3 zeros to fill in the gaps.
.0000667259 is 5 decimal points, plus we added 3 zeros to fill in the gaps.

And so on until you get to:

0.0000000000667259

We have moved the decimal point 11 places to the left, and filled in the 10 gaps with zeros.

6.67259E-11 = 0.0000000000667259

Another way of expressing 6.67259E-11 is to say 6.67259 * 10^-11. That's 6.67259 multiplied by 10 raised to the power of -11.

If you want to *solve* 6.67259 * 10^-11 using a calculator like Windows calculator, put in the exponent first. So type -11 and hit equals. Then press the 10x button (which is to the left of the number 0), then * 6.67259 and press equals. That will give you 0.0000000000667259

G (Gravitational Constant)

The letter G comes up in most formulas. G (also known as "big G") is the gravitational constant. (Read about the gravitational constant on wikipedia if you care to know more about it.) But for our purpose:

G = 6.67259E-11

There are slightly different definitions for the gravitational constant depending on which source you check, but Orbiter uses 6.67259E-11 (according to Orbiter.pdf page 128 [Orbiter 2010] or page 136 [Orbiter 2016].) Since we are trying to solve problems for Orbiter, we must use the same gravitational constant that Orbiter uses.

In a spreadsheet like on google docs, the gravitational constant can be expressed in two ways:

=6.67259*pow(10,-11)

or the more simplified format:

=6.67259E-11

Either way works.

M (Mass)

M is the mass of the body you are orbiting. If we are orbiting Mercury, then the Mass is 3.301880e+23. (This comes from Orbiter\Config\Mercury.cfg)

When typing the mass into a calculator like windows calculator, just type it as 3.301880e23 (leave out the + sign).

3.301880e+23 = 330188000000000000000000 (330,188,000,000,000,000,000,000)

That's three hundred thirty sextillion, one hundred eighty-eight quintillion.

GM (Gravitational Constant multiplied by Mass)

Multiplying the gravitational constant by the mass is very common when solving orbit equations. GM is known as the standard gravitational parameter. (Read about the standard gravitational parameter on wikipedia if you care to know more about it.) But it's worth knowing that GM is sometimes represented as *μ* in a formula. So the following two equations mean exactly the same thing:

$$V = \sqrt{\frac{\mu}{R}}$$ is the same as $$V = \sqrt{\frac{GM}{R}}$$

Calculator form: V = sqrt(*μ*/R) is the same as V = sqrt(GM/R)

μ is just a short form of GM.

So if we needed to solve GM for Mercury, it would look like this:

0.0000000000667259 * 330188000000000000000000

Obviously with that many numbers, it's easy to make mistakes, so using scientific notation makes more sense:

6.67259E-11 * 3.301880e23 = 220320914692 (220,320,914,692)

That's two hundred twenty billion, three hundred twenty million, nine hundred fourteen thousand, six hundred ninety-two.

2GM (2 * Gravitational Constant multiplied by Mass)

This is the same as GM, but you multiply it by 2.

So the equation would be:

2 * 6.67259E-11 * 3.301880e23 = 440641829384 (440,641,829,384)

That's four hundred forty billion, six hundred forty-one million, eight hundred twenty-nine thousand, three hundred eighty-four.

R (Radius)

The letter R is the radius of the orbit. If we are orbiting Mercury, then the radius of the orbit is 2.440e6 + our orbital altitude measured in meters. (The 2.440e6 figure comes from Orbiter\Config\Mercury.cfg)

So if our orbital altitude at Mercury is 100x100km, then:

R = 2.440e6 + 100000 = 2540000 (2,540,000)

That’s two million, five hundred forty thousand.

Rp / Ra

In a circular orbit, all points of the orbit have the same altitude, so Rp and Ra are the same as R + orbital altitude.

In a non circular orbit, Rp is radius + periapsis altitude, and Ra is radius + apoapsis altitude.

So if our orbit around Mercury is 50x100km, then:

Rp is 2.440e6 + 50000 = 2490000
Ra is 2.440e6 + 100000 = 2540000

<u>a (semi-major axis) SMa</u>

The semi-major axis is the radius of an orbit at its two most distant points. In the case of a circular orbit, the semi-major axis is simply the radius of the orbit since all points are equally distant.

The semi-major axis is required if you need to calculate the orbital period of an elliptical orbit. Solving for a is simple once you have Rp and Ra. You just add Rp and Ra together, and then divide by 2.

$$a = \frac{Rp + Ra}{2}$$

Calculator form: a = (Rp+Ra)/2

If our orbit around Mercury is 50x100km, then:

Rp is 2.440e6 + 50000   = 2490000
Ra is 2.440e6 + 100000 = 2540000

$$a = \frac{2,490,000 + 2,540,000}{2}$$

$$a = \frac{5,030,000}{2}$$

$$a = 2,515,000$$

Calculator form:

a = (2490000 + 2540000) / 2
a = 5030000 / 2
a = 2515000

The semi-major axis is 2,515,000 meters, or 2.5150 M (million meters.)

Velocity of a Circular Orbit

Now that we know what all the stuff means, we can finally calculate something useful. Once again, the equation to calculate orbital velocity is:

$$V = \sqrt{\frac{GM}{R}}$$

V is the velocity we are trying to find. We don't know what V is until we solve the equation.

When we plug in the actual numbers for a 100km circular orbit around Mercury, we get:

$$V = \sqrt{\frac{6.67259E-11 \cdot 3.301880e23}{2.440e6 + 100000}}$$

$$V = 2945.1743173414$$

Calculator form: sqrt(6.67259E-11 * 3.301880e23 / (2.440e6+100000)) = 2945.1743173414

That means the velocity is = 2945.1743173414. We have now solved V. We don't need this many decimal points though, so we'll round to the 100th decimal.

V = 2945.17 m/s

So if you were in orbit 100 kilometers above the surface of Mercury, you would be traveling 2945.17 meters per second. (6,588 miles per hour)

How fast would you be traveling if you were in a circular orbit 200 kilometers above Mercury?

Velocity of an Elliptical Orbit

V is velocity. So Vp is the periapsis velocity, and Va is the apoapsis velocity. In this case, we don't know what Vp or Va are. We need to solve the equation to find Vp and Va.

$$Vp = \sqrt{\frac{2GM \cdot Ra}{Rp(Rp + Ra)}}$$

$$Va = \sqrt{\frac{2GM \cdot Rp}{Ra(Rp + Ra)}}$$

Calculator form:

Vp = sqrt(2GM*Ra/(Rp*(Rp+Ra)))

Va = sqrt(2GM*Rp/(Ra*(Rp+Ra)))

Calculating the orbital period of a circular orbit (example 1)

Suppose we want to know how long it takes to orbit Mercury in a 100km circular orbit. The orbital period will be represented by the letter T because we want to know how much time it takes. T is the unknown. We are solving the equation to find T.

$$T = \sqrt{\frac{4 \cdot \pi^2 \cdot R^3}{GM}}$$

Calculator form: T = sqrt((4 * pi^2 * R^3)/ GM)

pi^2 = 9.86960

Referring back to the section on Radius, we know that R = 2.440e6 + 100000 = 2540000

So R^3 = 2540000^3 = 16387064000000000000

Now we have everything we need to plug in the numbers for the formula.

T = sqrt((4 * 9.86960 * 16387064000000000000) / (6.67259E-11 * 3.301880e23))

T = 5418.7920314102

It takes 5,418.8 seconds to orbit Mercury 1 time if we have a 100km circular orbit.

Another way to express this without first solving for pi squared and R cubed is:

T = sqrt((4 * pow(PI,2) * pow(2.440e6+100000,3)) / (6.67259E-11 * 3.301880e23))

A scientific calculator like the one at the link below can accept equations like that which makes it easier to plug in different numbers and then copy/paste the formula into the calculator.

http://ostermiller.org/calc/calculator.html

Calculating the orbital period of a circular orbit (example 2)

If we wanted to know the orbital period for a 100km circular orbit around the moon, we would just replace the radius and mass, R and M, with the proper radius and mass for the moon. If we replace 2.440e6 with 1.74E+06, then we have the radius of the moon instead of the radius of Mercury. So we then need to replace 3.301880e23 with 7.35E+22 so that we have the mass of the moon instead of the mass of Mercury.

```
T = sqrt((4 * pow(PI,2) * pow(1.74E+06+100000,3)) / (6.67259E-11 * 7.35E+22))
```

Now we can solve for T and find the orbital period for a 100km circular orbit around the moon.

Of course, if we want to know the orbital period for a different altitude, we can substitute 100000 for any altitude we want. If we want to know the orbital period at just 10 meters above the surface:

```
T = sqrt((4 * pow(PI,2) * pow(1.74E+06+10,3)) / (6.67259E-11 * 7.35E+22))
```

Of course a 10 meter orbit isn’t possible due to the fact that you would crash into various hillsides and mountains on the moon, but you can still solve for T and find the hypothetical orbital period.

Calculating the orbital period of an elliptical orbit

Most of the time we do not have the luxury of having circular orbits. Therefore knowing how to solve the orbital period for elliptical orbits is essential.

T = (2 * pi * sqrt(a^3 / GM))

If our orbit around Mercury is 50x100km, then:

Rp is 2.440e6 + 50000 = 2490000
Ra is 2.440e6 + 100000 = 2540000

a = (Rp + Ra) / 2
a = 2515000

T = 2 * PI * sqrt(pow(a,3) / GM)

T = 2 * PI * sqrt(pow(2515000,3) / (6.67259E-11 * 3.301880e23))

T = 5338.98 seconds

Our elliptical orbit has an orbital period of 5,339 seconds.

Calculating the eccentricity of an orbit

It can be useful to know how circular the orbit is. A perfectly circular orbit has an eccentricity of 0.0000. A hyperbolic trajectory has an eccentricity of at least 1.0 or greater. A hyperbolic trajectory means that our velocity is high enough to escape the gravitational pull of the body we are orbiting.

We calculate the eccentricity using the following equation:

$$e = (Ra - Rp) / (Ra + Rp)$$

If our orbit around Mercury is 50x100km, then:

Rp is 2.440e6 + 50000 = 2490000
Ra is 2.440e6 + 100000 = 2540000

$$e = (2540000 - 2490000) / (2540000 + 2490000)$$

$$e = 0.0099$$

Calculating the dV required to raise/lower an orbit

V=2993
G=6.67259E-11 (Gravitational Constant)
M=3.301880e23 (Mercury's Mass)
R=2.440e6 (Mercury's Radius)
Sa=20000 (starting altitude = 20km)
Ta=5000 (target altitude = 5km)

dV=V-(sqrt(2*G*M*((R+(Ta)))/((R+(Sa))*(R+(Sa)+R+(Ta)))))

In this example, we need 4.58 m/s to lower our PeA from 20km to 5km.

* To derive the orbital velocity with an equation:

V=sqrt(G*M/(R+(Sa)))

Calculating the acceleration due to gravity for a given body

Acceleration due to gravity is denoted by a lower case g.

G=6.67259E-11 (Gravitational Constant)
M=3.301880e23 (Mercury's Mass)
R=2.440e6 (Mercury's Radius)
μ=G*M

$g = (GM) / R^2$

This formula can also be stated as: $g = \mu / R^2$

$g = 3.70063347708949\ m/s^2$

Let’s try it for Earth:

G=6.67259E-11 (Gravitational Constant)
M=5.973698968e24 (Earth's Mass)
R=6.37101e6 (Earth's Radius)
μ=G*M

$g = \mu / R^2$

$g = 9.82021961405143\ m/s^2$

Calculating escape velocity for a given body

G=6.67259E-11 (Gravitational Constant)
M=3.301880e23 (Mercury's Mass)
R=2.440e6 (Mercury's Radius)
μ=G*M

$V^{esc} = \sqrt{2} * (G*M) / (R+\text{Orbital altitude in meters})$

If our orbital altitude around Mercury is 100km, then:

$V^{esc} = \sqrt{2} * \mu / (R+100,000)$

Calculating the cost of a plane change maneuver

dV = Sin(RInc/2) * 2 * Orbital Velocity

If we are in a 1000km x 1000km orbit around Venus and are out of plane with the target object by 34.46 degrees, then:

dV = Sin(34.46 / 2) * 2 * 6787

Note: When we take the RInc, divide by 2, and then get the sin of that value, we need our answer in degrees, not radians.

Sin(34.46 / 2) = 0.29620819206693534163546562206883

If you get -0.99881148646225619207573516946914, then you are getting radians, not degrees.

When we solve the entire equation, we get: 4,021 m/s dV. (That’s a very expensive plane change.)

http://www.orbiter-forum.com/showthread.php?t=26682