

MTH603
Assignment#2

Question No. 1: Consider the following table of values

x	2	4	6	8	10	12	14
F(x)	23	93	259	559	1031	1713	2643

Use Newton's backwards difference formula to compute $f(11)$ with and $p = 0.5$

Solution

x	Y=f _x	∇y_n	$\nabla^2 y_n$	$\nabla^3 y_n$	$\nabla^4 y_n$	$\nabla^5 y_n$	$\nabla^6 y_n$
2	23						
4	93	70					
6	259	166	96				
8	559	300	134	38			
10	1031	472	172	38	0		
12	1713	682	210	38	0	0	
14	2643	930	248	38	0	0	0

Newton's backwards Formula

$$y_x = y_n + p \nabla y_n + \frac{p(p+1)}{2!} \nabla^2 y_n + \frac{p(p+1)(p+2)}{3!} \nabla^3 y_n + \frac{p(p+1)(p+2)(p+3)}{4!} \nabla^4 y_n \\ + \frac{p(p+1)(p+2)(p+3)(p+4)}{5!} \nabla^5 y_n + \frac{p(p+1)(p+2)(p+3)(p+4)(p+5)}{6!} \nabla^6 y_n$$

$$y_x = 2643 + 0.5(930) + \frac{0.5(0.5+1)}{2} (248) + \frac{0.5(0.5+1)(0.5+2)}{6} (38)$$

$$y_x = 2643 + 465 + 93 + 11.875$$

$$= 3212.875$$

Question No. 2: Find Newton's forward difference table from the following data.

x	0.0	0.1	0.2	0.3	0.4
$f(x)$	1	0.9048	0.8187	0.6408	0.4703

Solution:

x	$Y=f_x$	∇y_n	$\nabla^2 y_n$	$\nabla^3 y_n$	$\nabla^4 y_n$
0.0	1				
0.1	0.9048	-0.0952			
0.2	0.8187	-0.0861	-0.0091		
0.3	0.6408	-0.1779	-0.0918	-0.0827	
0.4	0.4703	-0.1705	-0.0074	-0.0844	-0.0017

$$y_x = y_0 + p \nabla y_0 + \frac{p(p-1)}{2!} \nabla^2 y_0 + \frac{p(p-1)(p-2)}{3!} \nabla^3 y_0 + \frac{p(p-1)(p-2)(p-3)}{4!} \nabla^4 y_0$$

$$p = \frac{(x - x_0)}{h}$$

$$= \frac{(x - 0.0)}{0.1} = (10x - 0)$$

$$= 1 + 10x - (0.0952) + \frac{10x(10x-1)}{2!}(-0.0091) + \frac{10x(10x-1)(10x+2)}{3!}(-0.0827)$$

$$+ \frac{10x(10x-1)(10x+2)(10x+3)}{4!}(-0.0017)$$

$$= 1 + 0.952x + \frac{-0.182x^2 + 0.091x}{2} + \frac{-16.54x^3 + 24.81x^2 + 0.091x}{6} +$$

$$\frac{-0.068x^4 + 4.08x^3 + 1.19x^2 + 0.102x}{24}$$

$$= 1 + 0.952x + \frac{2.184x^2 + 1.092x - 66.16x^3 + 99.24x^2 - 6.616x - 0.068x^4 - 4.08x^3 + 1.19x^2 + 0.102x}{24}$$

$$= \frac{1}{24} - 0.068x - 70.24x + 100.43x + 8.762x$$