FREEHOLD REGIONAL HIGH SCHOOL DISTRICT
OFFICE OF CURRICULUM AND INSTRUCTION
INTERNATIONAL BACCALAUREATE PROGRAM

IB Mathematics: Analysis & Approaches, SL & HL Year 1

Grade Level: 11

Credits: 5

BOARD OF EDUCATION ADOPTION DATE: August 26, 2021

FREEHOLD REGIONAL HIGH SCHOOL DISTRICT

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IB Mathematics: Analysis & Approaches, SL & HL
Year 1

Course Description

From the International Baccalaureate Organization: "Mathematics: analysis and approaches is for students who enjoy developing their mathematics to become fluent in the construction of mathematical arguments and develop strong skills in mathematical thinking. They will also be fascinated by exploring real and abstract applications of these ideas, with and without technology. Students who take Mathematics: analysis and approaches will be those who enjoy the thrill of mathematical problem solving and generalization.

This course recognizes the need for analytical expertise in a world where innovation is increasingly dependent on a deep understanding of mathematics. This course includes topics that are both traditionally part of a pre-university mathematics course (for example, functions, trigonometry, calculus) as well as topics that are amenable to investigation, conjecture and proof, for instance the study of sequences and series at both SL and HL, and proof by induction at HL.

The course allows the use of technology, as fluency in relevant mathematical software and hand-held technology is important regardless of choice of course. However, Mathematics: analysis and approaches has a strong emphasis on the ability to construct, communicate and justify correct mathematical arguments.

Students who choose Mathematics: analysis and approaches at SL or HL should be comfortable in the manipulation of algebraic expressions and enjoy the recognition of patterns and understand the mathematical generalization of these patterns. Students who wish to take Mathematics: analysis and approaches at a higher level will have strong algebraic skills and the ability to understand simple proof. They will be students who enjoy spending time with problems and get pleasure and satisfaction from solving challenging problems."

Course Sequence and Pacing

Unit Title	**HL Pacing**	**SL Pacing**
1: Foundational Knowledge	6 sessions	18 sessions
2: Further Trigonometry	13 sessions	28 sessions
3: Further Functions	22 sessions	40 sessions
4: Complex Numbers (HL only)	18 sessions	n/a
5: Combinatorics	10 sessions	20 sessions
6: Reasoning and Proof (HL only)	18 sessions	n/a
7: Overview: Linear Algebra (HL only)	8 sessions	n/a
8: Vectors (HL Only)	25 sessions	n/a

IB Mathematics: Analysis & Approaches, SL & HL, Year 1
Unit 1 Overview: Foundational Knowledge **Duration: 6 HL, 18 SL sessions**

IB Topic Essential Understandings:
Number and algebra allow us to represent patterns, show equivalencies and make generalizations which enable us to model real-world situations. Algebra is an abstraction of numerical concepts and employs variables which allow us to solve mathematical problems.

Models are depictions of real-life events using expressions, equations or graphs while a function is defined as a relation or expression involving one or more variables. Creating different representations of functions to model the relationships between variables, visually and symbolically as graphs, equations and tables represent different ways to communicate mathematical ideas.

Geometry and trigonometry allows us to quantify the physical world, enhancing our spatial awareness in two and three dimensions. This topic provides us with the tools for analysis, measurement and transformation of quantities, movements and relationships.

IB Content-Specific Conceptual Understandings

Modelling real-life situations with the structure of arithmetic and geometric sequences and series allows for prediction, analysis and interpretation.

Formulae are a generalization made on the basis of specific examples, which can then be extended to new examples.

Different representations of functions, symbolically and visually as graphs, equations and tables provide different ways to communicate mathematical relationships.

Moving between different forms to represent functions allows for deeper understanding and provides different approaches to problem solving.

The relationships between the length of the sides and the size of the angles in a triangle can be used to solve many problems involving position, distance, angles and area.

Equivalent measurement systems, such as degrees and radians, can be used for angles to facilitate ease of calculation.

AHL
Patterns can be identified in behaviours which can give us insight into appropriate strategies to model or solve them.

Section and IB Sub-Topics	**HL Duration (sessions)**	**SL Duration (sessions)**
A: Sequences and Series SL 1.2, SL 1.3, SL 1.4	2	5
B: Introduction to Trigonometry SL 3.1, SL 3.2, SL 3.3, SL 3.4, SL 3.6, AHL 3.9	2	6
C: Function Analysis SL 2.1, SL 2.2, SL 2.3, SL 2.4, SL 2.5, SL 2.6, SL 2.7, SL 2.10, SL 2.11	2	7

IB Mathematics: Analysis & Approaches, SL & HL, Year 1
Unit 1: Foundational Knowledge
Section 1A: Sequences and Series **Duration: 2 HL, 5 SL sessions**

IB Topic Essential Understandings:
Number and algebra allow us to represent patterns, show equivalencies and make generalizations which enable us to model real-world situations. Algebra is an abstraction of numerical concepts and employs variables which allow us to solve mathematical problems.

Models are depictions of real-life events using expressions, equations or graphs while a function is defined as a relation or expression involving one or more variables. Creating different representations of functions to model the relationships between variables, visually and symbolically as graphs, equations and tables represent different ways to communicate mathematical ideas.

Geometry and trigonometry allows us to quantify the physical world, enhancing our spatial awareness in two and three dimensions. This topic provides us with the tools for analysis, measurement and transformation of quantities, movements and relationships.

IB Content-Specific Conceptual Understandings

Modelling real-life situations with the structure of arithmetic and geometric sequences and series allows for prediction, analysis and interpretation.

Formulae are a generalization made on the basis of specific examples, which can then be extended to new examples.

IB Content		**Standards-Aligned Objectives.** Instruction and assessment will align to the following objectives, with IB command terms in bold
SL 1.2	Arithmetic sequences and series. Use of the formulae for the n^{th} term and the sum of the first n terms of the sequence. Use of sigma notation for sums of arithmetic sequences. Applications. Analysis, interpretation and prediction where a model is not perfectly arithmetic in real life.	**Distinguish** between arithmetic and geometric patterns in given sequences. **Deduce** into the explicit formula, given an arithmetic sequence in recursive form. **Describe** arithmetic sequences from recursive and explicit formulas. **Apply** arithmetic sequences to real-world scenarios using approximation where appropriate.
SL 1.3	Geometric sequences and series. Use of the formulae for the n^{th} term and the sum of the first n terms of the sequence. Use of sigma notation for the sums of geometric sequences. Applications.	**Deduce** into the explicit formula, given a geometric sequence in recursive form. **Describe** geometric sequences from recursive and explicit formulas. **Apply** geometric sequences to real-world scenarios using approximation where appropriate.
SL 1.4	Financial applications of geometric sequences and series: • compound interest	**Apply** geometric sequences to problems involving financial situations.

	• annual depreciation.	

TOK Connections	International Mindedness Connections
SL 1.2: Is all knowledge concerned with identification and use of patterns? Consider Fibonacci numbers and connections with the golden ratio. **SL 1.3:** How do mathematicians reconcile the fact that some conclusions seem to conflict with our intuitions? Consider for instance that a finite area can be bounded by an infinite perimeter **SL 1.4:** How have technological advances affected the nature and practice of mathematics? Consider the use of financial packages for instance.	**SL 1.4:** Do all societies view investment and interest in the same way?

IB Mathematics: Analysis & Approaches, SL & HL, Year 1 **Unit 1: Foundational Knowledge** **Section 1.B: Introduction to Trigonometry** **Duration: 2 HL, 6 SL sessions**		
IB Topic Essential Understandings: Number and algebra allow us to represent patterns, show equivalencies and make generalizations which enable us to model real-world situations. Algebra is an abstraction of numerical concepts and employs variables which allow us to solve mathematical problems. Models are depictions of real-life events using expressions, equations or graphs while a function is defined as a relation or expression involving one or more variables. Creating different representations of functions to model the relationships between variables, visually and symbolically as graphs, equations and tables represent different ways to communicate mathematical ideas. Geometry and trigonometry allows us to quantify the physical world, enhancing our spatial awareness in two and three dimensions. This topic provides us with the tools for analysis, measurement and transformation of quantities, movements and relationships.		
IB Content-Specific Conceptual Understandings		
Different representations of functions, symbolically and visually as graphs, equations and tables provide different ways to communicate mathematical relationships. Moving between different forms to represent functions allows for deeper understanding and provides different approaches to problem solving. The relationships between the length of the sides and the size of the angles in a triangle can be used to solve many problems involving position, distance, angles and area. Equivalent measurement systems, such as degrees and radians, can be used for angles to facilitate ease of calculation. **AHL** Patterns can be identified in behaviours which can give us insight into appropriate strategies to model or solve them.		
IB Content		**Standards-Aligned Objectives.** Instruction and assessment will align to the following objectives, with IB command terms in bold
SL 3.1	The distance between two points in three-dimensional space, and their midpoint. Volume and surface area of three-dimensional solids including right-pyramid, right cone, sphere, hemisphere and combinations of these solids. The size of an angle between two intersecting lines or between a line and a plane.	**Calculate** the distance between two points in a three-dimensional space, and their midpoint using a variety of methods. **Calculate** the volume and surface area of three-dimensional solids. **Hence** determine the parameters for the three-dimensional solid.
SL 3.2	Use of sine, cosine and tangent ratios to find the sides and angles of right-angled triangles. The sine rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$. The cosine rule: $c^2 = a^2 + b^2 - 2ab\cos C$; $\cos C = \frac{a^2+b^2-c^2}{2ab}$.	**Solve** right-angled triangles using sine, cosine and tangent ratios. **Determine** the appropriate method for solving oblique triangles, and **hence** find all unknown sides and angles. **Calculate** the area of an non-right angled triangle.

	Area of a triangle as $\frac{1}{2}$absinC.	
SL 3.3	Applications of right and non-right angled trigonometry, including Pythagoras's theorem. Angles of elevation and depression. Construction of labelled diagrams from written statements.	**Apply** properties of right and non-right angled triangle trigonometry to various real-world scenarios. **Sketch** right and non-right angled triangles to determine the appropriate method for solving a real-world problem.
SL 3.4	The circle: radian measure of angles; length of an arc; area of a sector.	**Express** angle measures in different units and **compare and contrast** the value of using each measure. **Deduce** the relationship between the area of a circle and the area of a sector; circumference of a circle and arc length.
SL 3.6	The Pythagorean identity $\cos^2 \theta + \sin^2 \theta = 1$. Double angle identities for sine and cosine. The relationship between trigonometric ratios.	**Explore** trigonometric ratios to **deduce** various identities. **Calculate** trigonometric ratios using appropriate known formulae. **Simplify** trigonometric expressions and **solve** trig equations using appropriate known formulae.
AHL 3.9	Definition of the reciprocal trigonometric ratios sec θ, cosec θ and cot θ. Pythagorean identities: $1 + \tan^2 \theta = \sec^2 \theta$ and $1 + \cot^2 \theta = \text{cosec}^2 \theta$.	**Explore** trigonometric ratios to **deduce** various identities. **Calculate** trigonometric ratios using appropriate known formulae. **Simplify** trigonometric expressions and **solve** trig equations using appropriate known formulae.

TOK Connections	International Mindedness Connections
SL 3.1: What is an axiomatic system? Are axioms self-evident to everybody? **SL 3.2:** Is it ethical that Pythagoras gave his name to a theorem that may not have been his own creation? What criteria might we use to make such a judgment? **SL 3.3:** If the angles of a triangle can add up to less than 180°, 180° or more than 180°, what does this tell us about the nature of mathematical knowledge? **SL 3.4**: Which is a better measure of angle: radian or degree? What criteria can/do/should mathematicians use to make such decisions? **HL 3.9**: What is the relationship between concepts and facts? To what extent do the concepts that we use shape the conclusions that we reach?	**SL 3.2:** Diagrams of Pythagoras' theorem occur in early Chinese and Indian manuscripts. The earliest references to trigonometry are in Indian mathematics; the use of triangulation to find the curvature of the Earth in order to settle a dispute between England and France over Newton's gravity. **SL 3.3:** The use of triangulation to find the curvature of the Earth in order to settle a dispute between England and France over Newton's gravity.

IB Mathematics: Analysis & Approaches, SL & HL, Year 1 **Unit 1: Foundational Knowledge** **Section 1.C: Function Analysis** **Duration: 2 HL, 7 SL sessions**		
IB Topic Essential Understandings: Number and algebra allow us to represent patterns, show equivalencies and make generalizations which enable us to model real-world situations. Algebra is an abstraction of numerical concepts and employs variables which allow us to solve mathematical problems. Models are depictions of real-life events using expressions, equations or graphs while a function is defined as a relation or expression involving one or more variables. Creating different representations of functions to model the relationships between variables, visually and symbolically as graphs, equations and tables represent different ways to communicate mathematical ideas. Geometry and trigonometry allows us to quantify the physical world, enhancing our spatial awareness in two and three dimensions. This topic provides us with the tools for analysis, measurement and transformation of quantities, movements and relationships.		
IB Content-Specific Conceptual Understandings		
Formulae are a generalization made on the basis of specific examples, which can then be extended to new examples. Different representations of functions, symbolically and visually as graphs, equations and tables provide different ways to communicate mathematical relationships. Moving between different forms to represent functions allows for deeper understanding and provides different approaches to problem solving.		
IB Content		**Standards-Aligned Objectives.** Instruction and assessment will align to the following objectives, with IB command terms in bold
SL 2.1	Different forms of the equation of a straight line. Gradient; intercepts. Lines with gradients m_1 and m_2 ● Parallel lines $m_1 = m_2$. ● Perpendicular lines $m_1 \times m_2 = -1$.	**Compare and contrast** different forms of linear functions in order to identify when to use each. **Find** and **interpret** gradients and intercepts for linear functions. **Determine** the relationship between different types of lines (including parallel and perpendicular).
SL 2.2	Concept of a function, domain, range and graph. Function notation, for example f(x), v(t), C(n). The concept of a function as a mathematical model. Informal concept that an inverse function reverses or undoes the effect of a function. Inverse function as a reflection in the line y = x, and the notation $f^{-1}(x)$.	**Explore** the properties of functions, including domain, range and graph. **Represent** functions in various forms and convert between them. **Explore** the relationship between a function and its inverse.

SL 2.3	The graph of a function; its equation $y = f(x)$. Creating a sketch from information given or a context, including transferring a graph from screen to paper. Using technology to graph functions including their sums and differences.	**Represent** functions in various forms and convert between them. **Sketch** the graph of a function using various approaches. **Compose** functions using mathematical operations.
SL 2.4	Key features of graphs. Point of intersection of two curves or lines using technology.	**Determine** key features of graphs. **Find** the point of intersection of two curves using technology.
SL 2.5	Composite functions. Identity function. Finding the inverse function $f^{-1}(x)$	**Compose** functions using mathematical operations **Verify** inverse functions using composition
SL 2.6	The quadratic function $f(x) = ax^2 + bx + c$: its graph, y-intercept $(0, c)$. Axis of symmetry. The form $f(x) = a(x - p)(x - q)$, x-intercepts $(p, 0)$ and $(q,0)$. The form $f(x) = a(x - h)^2 + k$, vertex (h, k).	**Compare and contrast** different forms of quadratic functions in order to identify various features of the function. **Sketch** the graphs of quadratic functions.
SL 2.7	Solution of quadratic equations and inequalities. The quadratic formula. The discriminant $\Delta = b^2 - 4ac$ and the nature of the roots, that is, two distinct real roots, two equal real roots, no real roots.	**Solve** quadratic functions using various methods. **Determine** the nature of the roots of a quadratic based on the discriminant.
SL 2.10	Solutions of equations, both graphically and analytically. Use of technology to solve a variety of equations, including those where there is no appropriate analytic approach. Applications of graphing skills and solving equations that relate to real-life situations.	**Solve** equations, both graphically and analytically, using a variety of methods for a variety of real-world scenarios.
SL 2.11	Transformations of graphs. • Translations: $y = f(x) + b$; $y = f(x - a)$. • Reflections (in both axes): $y = -f(x)$; $y = f(-x)$. • Vertical stretch with scale factor p: $y = p\,f(x)$. • Horizontal stretch with scale factor $\frac{1}{q}$: $y = f(qx)$. • Composite transformations.	**Investigate** various transformations of functions. **Demonstrate** an understanding of various transformations from a variety of approaches, including analytic and graphical.

TOK Connections	International Mindedness Connections
SL 2.1: Descartes showed that geometric problems could be solved algebraically and vice versa. What does this tell us about mathematical representation and mathematical knowledge?	**SL 2.2:** The development of functions by Rene Descartes (France), Gottfried Wilhelm Leibnitz (Germany) and Leonhard Euler (Switzerland); the notation for functions was developed by a number of different mathematicians in the 17th and 18th

SL 2.2: Do you think mathematics or logic should be classified as a language? **SL 2.3:** Does studying the graph of a function contain the same level of mathematical rigour as studying the function algebraically? What are the advantages and disadvantages of having different forms and symbolic language in mathematics? **SL 2.5:** Do you think mathematics or logic should be classified as a language? **SL 2.6:** Are there fundamental differences between mathematics and other areas of knowledge? If so, are these differences more than just methodological differences? **SL 2.10:** What assumptions do mathematicians make when they apply mathematics to real-life situations?	centuries–how did the notation we use today become internationally accepted? **SL 2.4:** Bourbaki group analytical approach versus the Mandlebrot visual approach

IB Mathematics: Analysis & Approaches, SL & HL, Year 1
Unit 2 Overview: Further Trigonometry **Duration: 13 HL, 28 SL sessions**

IB Topic Essential Understandings:
Geometry and trigonometry allows us to quantify the physical world, enhancing our spatial awareness in two and three dimensions. This topic provides us with the tools for analysis, measurement and transformation of quantities, movements and relationships.

IB Content-Specific Conceptual Understandings

The properties of shapes depend on the dimension they occupy in space.

Volume and surface area of shapes are determined by formulae, or general mathematical relationships or rules expressed using symbols or variables.

The relationships between the length of the sides and the size of the angles in a triangle can be used to solve many problems involving position, distance, angles and area.

Equivalent measurement systems, such as degrees and radians, can be used for angles to facilitate ease of calculation.

Different representations of the values of trigonometric relationships, such as exact or approximate, may not be equivalent to one another.

The trigonometric functions of angles may be defined on the unit circle, which can visually and algebraically represent the periodic or symmetric nature of their values.

AHL
Position and movement can be modelled in three-dimensional space using vectors.

The relationships between algebraic, geometric and vector methods can help us to solve problems and quantify those positions and movements.

Section and IB Sub-Topics	HL Duration	SL Duration
A: Spatial Geometry SL 3.1	2 sessions	3 sessions
B: Triangle Trigonometry SL 3.2, SL 3.3	2 sessions	4 sessions
C: Circle Trigonometry SL 3.4	2 sessions	4 sessions
D: Trigonometry Ratios SL 3.5, SL 3.6	2 sessions	5 sessions
E: Trigonometric Graphs and Applications SL 3.7, AHL 3.11	3 sessions	7 sessions
F: Algebra and Trigonometry SL 3.8, AHL 3.9, AHL 3.10	2 sessions	5 sessions

IB Mathematics: Analysis & Approaches, SL & HL, Year 1 Unit 2: Further Trigonometry Section 2.A: Spatial Geometry		Duration: 2 HL, 3 SL sessions
IB Topic Essential Understandings: Geometry and trigonometry allows us to quantify the physical world, enhancing our spatial awareness in two and three dimensions. This topic provides us with the tools for analysis, measurement and transformation of quantities, movements and relationships.		
IB Content-Specific Conceptual Understandings		
The properties of shapes depend on the dimension they occupy in space. Volume and surface area of shapes are determined by formulae, or general mathematical relationships or rules expressed using symbols or variables.		
IB Content		**Standards-Aligned Objectives.** Instruction and assessment will align to the following objectives, with IB command terms in bold
SL 3.1	The distance between two points in three-dimensional space, and their midpoint. Volume and surface area of three-dimensional solids including right-pyramid, right cone, sphere, hemisphere and combinations of these solids. The size of an angle between two intersecting lines or between a line and a plane.	**Calculate** the distance between two points in a three-dimensional space, and their midpoint using a variety of methods. **Calculate** the volume and surface area of three-dimensional solids. **Hence** determine the parameters for the three-dimensional solid.

TOK Connections	International Mindedness Connections
SL 3.1: What is an axiomatic system? Are axioms self-evident to everybody? **SL 3.1**: Demonstrate Cavalieri's principle using a variety of materials.Explore how would we calculate the volume if the cross-sectional areas are not the same? Revisit when learning integral calculus.	**SL 3.1:** History of the distance formula. **SL 3.1:** How were the Pyramids built and how long would it take for 1000 people to construct with modern technology? What would it cost?

IB Mathematics: Analysis & Approaches, SL & HL, Year 1
Unit 2: Further Trigonometry
Section 2.B: Triangle Trigonometry **Duration: 2 HL, 4 SL sessions**

IB Topic Essential Understandings:
Geometry and trigonometry allows us to quantify the physical world, enhancing our spatial awareness in two and three dimensions. This topic provides us with the tools for analysis, measurement and transformation of quantities, movements and relationships.

IB Content-Specific Conceptual Understandings

The relationships between the length of the sides and the size of the angles in a triangle can be used to solve many problems involving position, distance, angles and area.

IB Content		**Standards-Aligned Objectives.** Instruction and assessment will align to the following objectives, with IB command terms in bold
SL 3.2	Use of sine, cosine and tangent ratios to find the sides and angles of right-angled triangles. The sine rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$. The cosine rule: $c^2 = a^2 + b^2 - 2ab\cos C$; $\cos C = \frac{a^2 + b^2 - c^2}{2ab}$. Area of a triangle as $\frac{1}{2}ab\sin C$.	**Determine** when to use which trig ratio based on given information **Apply** the sine rule and cosine rule to **solve** non-right triangles **Calculate** the area of a non-right triangle
SL 3.3	Applications of right and non-right angled trigonometry, including Pythagoras's theorem. Angles of elevation and depression. Construction of labelled diagrams from written statements.	**Apply** rules of right and non-right triangles to **solve** real world problems including problems involving angles of elevation and depression **Construct** diagrams that represent written statements

TOK Connections	International Mindedness Connections
SL 3.2: Is it ethical that Pythagoras gave his name to a theorem that may not have been his own creation? What criteria might we use to make such a judgment? **SL 3.3:** Is it ethical that Pythagoras gave his name to a theorem that may not have been his own creation? What criteria might we use to make such a judgement?	**SL 3.3**: Diagrams of Pythagoras' theorem occur in early Chinese and Indian manuscripts. The earliest references to trigonometry are in Indian mathematics; the use of triangulation to find the curvature of the Earth in order to settle a dispute between England and France over Newton's gravity.

IB Mathematics: Analysis & Approaches, SL & HL, Year 1 **Unit 2: Further Trigonometry** **Section 2.C: Circle Trigonometry**		**Duration: 2 HL, 4 SL sessions**
IB Topic Essential Understandings: Geometry and trigonometry allows us to quantify the physical world, enhancing our spatial awareness in two and three dimensions. This topic provides us with the tools for analysis, measurement and transformation of quantities, movements and relationships.		
IB Content-Specific Conceptual Understandings		
Equivalent measurement systems, such as degrees and radians, can be used for angles to facilitate ease of calculation. Different representations of the values of trigonometric relationships, such as exact or approximate, may not be equivalent to one another. The trigonometric functions of angles may be defined on the unit circle, which can visually and algebraically represent the periodic or symmetric nature of their values.		
IB Content		**Standards-Aligned Objectives.** Instruction and assessment will align to the following objectives, with IB command terms in bold
SL 3.4	The circle: radian measure of angles; length of an arc; area of a sector.	**Investigate** the relationship between central angle, arc length and sector area. **Solve** for missing values of a circle using known information about arc length and/or sector area.

TOK Connections	International Mindedness Connections
SL 3.4: Which is a better measure of angle: radian or degree? What criteria can/do/should mathematicians use to make such decisions?	**SL 3.4:** Seki Takakazu calculating π to ten decimal places; Hipparchus, Menelaus and Ptolemy; Why are there 360 degrees in a complete turn? Links to Babylonian mathematics

IB Mathematics: Analysis & Approaches, SL & HL, Year 1 **Unit 2: Further Trigonometry** **Section 2.D: Trigonometry Ratios**			**Duration: 2 HL, 5 SL sessions**
IB Topic Essential Understandings: Geometry and trigonometry allows us to quantify the physical world, enhancing our spatial awareness in two and three dimensions. This topic provides us with the tools for analysis, measurement and transformation of quantities, movements and relationships.			
IB Content-Specific Conceptual Understandings			
Different representations of the values of trigonometric relationships, such as exact or approximate, may not be equivalent to one another. The trigonometric functions of angles may be defined on the unit circle, which can visually and algebraically represent the periodic or symmetric nature of their values.			
IB Content			**Standards-Aligned Objectives.** Instruction and assessment will align to the following objectives, with IB command terms in bold
SL 3.5	Definition of cosθ, sinθ in terms of the unit circle Definition of tanθ as sinθ/cosθ Exact values of trigonometric ratios of 0, π/6 , π/4 , π/3 , π/2 and their multiples. Extension of the sine rule to the ambiguous case		**Define** trigonometric ratios from known relationships on the unit circle. **Hence find** trig ratios for special angles. **Extend** understanding of sine rule to ambiguous case
SL 3.6	The Pythagorean identity $\cos^2 \theta + \sin^2 \theta = 1$. Double angle identities for sine and cosine. The relationship between trigonometric ratios		**Use** known relationships between trig ratios to **simplify** trigonometric expressions, **solve** trigonometric equations and **show that** trig expressions are equivalent.

TOK Connections	**International Mindedness Connections**
SL 3.5: Trigonometry was developed by successive civilizations and cultures. To what extent is mathematical knowledge embedded in particular traditions or bound to particular cultures? How have key events in the history of mathematics shaped its current form and methods?	**SL 3.5:** The first work to refer explicitly to the sine as a function of an angle is the Aryabhatiya of Aryabhata (ca 510).

IB Mathematics: Analysis & Approaches, SL & HL, Year 1 **Unit 2 : Further Trigonometry** **Section 2.E: Trigonometric Graphs and Applications**		**Duration: 3 HL, 7 SL sessions**
IB Topic Essential Understandings: Geometry and trigonometry allows us to quantify the physical world, enhancing our spatial awareness in two and three dimensions. This topic provides us with the tools for analysis, measurement and transformation of quantities, movements and relationships.		
IB Content-Specific Conceptual Understandings		
The relationships between the length of the sides and the size of the angles in a triangle can be used to solve many problems involving position, distance, angles and area. The trigonometric functions of angles may be defined on the unit circle, which can visually and algebraically represent the periodic or symmetric nature of their values.		

IB Content		**Standards-Aligned Objectives.** Instruction and assessment will align to the following objectives, with IB command terms in bold
SL 3.7	The circular functions sinx, cosx, and tanx; amplitude, their periodic nature, and their graphs Composite functions of the form f(x) = asin(b(x + c)) + d. Transformations. Real-life contexts.	**Graph** trigonometric functions for given domains. **Transform** trigonometric functions analytically and graphically **Model** real-world applications using trigonometric functions
AHL 3.11	Relationships between trigonometric functions and the symmetry properties of their graphs.	**Explore** relationship between trigonometric functions and symmetry

TOK Connections	International Mindedness Connections
SL 3.7: Music can be expressed using mathematics. What does this tell us about the relationship between music and mathematics? **AHL 3.11:** Mathematics and knowledge claims: how can there be an infinite number of discrete solutions to an equation?	None applicable

IB Mathematics: Analysis & Approaches, SL & HL, Year 1 **Unit 2: Further Trigonometry** **Section 2.F: Algebra and Trigonometry** **Duration: 2 HL, 5 SL sessions**

IB Topic Essential Understandings:
Geometry and trigonometry allows us to quantify the physical world, enhancing our spatial awareness in two and three dimensions. This topic provides us with the tools for analysis, measurement and transformation of quantities, movements and relationships.

IB Content-Specific Conceptual Understandings

The relationships between the length of the sides and the size of the angles in a triangle can be used to solve many problems involving position, distance, angles and area.

Equivalent measurement systems, such as degrees and radians, can be used for angles to facilitate ease of calculation.

Different representations of the values of trigonometric relationships, such as exact or approximate, may not be equivalent to one another.

The trigonometric functions of angles may be defined on the unit circle, which can visually and algebraically represent the periodic or symmetric nature of their values.

IB Content		**Standards-Aligned Objectives.** Instruction and assessment will align to the following objectives, with IB command terms in bold
SL 3.8	Solving trigonometric equations in a finite interval, both graphically and analytically Equations leading to quadratic equations in sinx, cosx or tanx.	**Solve** trigonometric equations graphically and analytically **Represent** trigonometric equations in the form of quadratic equations; **hence use** quadratic properties to **solve** trigonometric equations.
AHL 3.9	Definition of the reciprocal trigonometric ratios secθ, cosecθ and cotθ. Pythagorean identities: $1 + \tan^2 \theta = \sec^2 \theta$ and $1 + \cot^2 \theta = \csc^2 \theta$ The inverse functions f(x) = arcsinx, f(x) = arccosx, f(x) = arctanx; their domains and ranges; their graphs	**Sketch** inverse trig functions and **compare and contrast** their domains with their inverses. **Use** known relationships between trig ratios to **simplify** trigonometric expressions, **solve** trigonometric equations and **show that** trig expressions are equivalent.
AHL 3.10	Compound angle identities. Double angle identity for tan.	**Investigate** relationship between angle sums and differences. **Use** the double angle identity to **find** trigonometric ratios, **simplify** expressions, **solve** equations and **verify** equations.

TOK Connections	**International Mindedness Connections**
AHL 3.9: What is the relationship between concepts and facts? To what extent do the concepts that we use shape the conclusions that we reach?	**AHL 3.9:** The origin of degrees in the mathematics of Mesopotamia and why we use minutes and seconds for time; the origin of the word sine.

IB Mathematics: Analysis & Approaches, SL & HL, Year 1 Unit 3 Overview: Further Functions	Duration: 22 HL, 40 SL sessions

IB Topic Essential Understandings:
Models are depictions of real-life events using expressions, equations or graphs while a function is defined as a relation or expression involving one or more variables. Creating different representations of functions to model the relationships between variables, visually and symbolically as graphs, equations and tables represent different ways to communicate mathematical ideas.

IB Content-Specific Conceptual Understandings

Logarithm laws provide the means to find inverses of exponential functions which model real-life situations.

Different representations of functions, symbolically and visually as graphs, equations and tables provide different ways to communicate mathematical relationships.

The parameters in a function or equation correspond to geometrical features of a graph and can represent physical quantities in spatial dimensions.

Moving between different forms to represent functions allows for deeper understanding and provides different approaches to problem solving.

Our spatial frame of reference affects the visible part of a function and by changing this “window” can show more or less of the function to best suit our needs.

Equivalent representations of quadratic functions can reveal different characteristics of the same relationship.

Functions represent mappings that assign to each value of the independent variable (input) one and only one dependent variable (output).

AHL
Extending results from a specific case to a general form can allow us to apply them to a larger system.

Patterns can be identified in behaviours which can give us insight into appropriate strategies to model or solve them.

The intersection of a system of equations may be represented graphically and algebraically and represents the solution that satisfies the equations.

Section and IB Sub-Topics	HL Duration	SL Duration
A: Function Analysis SL 2.2, SL 2.5, AHL 2.14, AHL 2.15, AHL 2.16	5 sessions	13 sessions
B: Linear Functions SL 2.1, SL 2.3, SL 2.4, SL 2.10, SL 2.11	1 sessions	5 sessions
C: Quadratic and Other Polynomial Functions SL 2.3, SL 2.4, SL 2.6, SL 2.7, SL 2.10, SL 2.11, AHL 2.12	4 sessions	9 sessions
D: Logarithms and Exponential Functions SL 2.3, SL 2.4, SL 2.9, SL 2.10, SL 2.11	8 sessions	13 sessions
E: Rational Functions (HL only) SL 2.3, SL 2.4, SL 2.8, SL 2.10, SL 2.11, AHL 2.13	4 sessions	n/a

IB Mathematics: Analysis & Approaches, SL & HL, Year 1
Unit 3: Further Functions
Section 3.A: Function Analysis — **Duration: 5 HL, 13 SL sessions**

IB Topic Essential Understandings:
Models are depictions of real-life events using expressions, equations or graphs while a function is defined as a relation or expression involving one or more variables. Creating different representations of functions to model the relationships between variables, visually and symbolically as graphs, equations and tables represent different ways to communicate mathematical ideas.

IB Content-Specific Conceptual Understandings

Different representations of functions, symbolically and visually as graphs, equations and tables provide different ways to communicate mathematical relationships.

The parameters in a function or equation correspond to geometrical features of a graph and can represent physical quantities in spatial dimensions.

Moving between different forms to represent functions allows for deeper understanding and provides different approaches to problem solving.

Our spatial frame of reference affects the visible part of a function and by changing this “window” can show more or less of the function to best suit our needs.

Functions represent mappings that assign to each value of the independent variable (input) one and only one dependent variable (output).

AHL
Extending results from a specific case to a general form can allow us to apply them to a larger system.

Patterns can be identified in behaviours which can give us insight into appropriate strategies to model or solve them.

IB Content		**Standards-Aligned Objectives.** Instruction and assessment will align to the following objectives, with IB command terms in bold
SL 2.2	Concept of a function, domain, range and graph. Function notation, for example f(x), v(t), C(n). The concept of a function as a mathematical model. Informal concept that an inverse function reverses or undoes the effect of a function. Inverse function as a reflection in the line y = x, and the notation $f^{-1}(x)$	**Understand** the concept of a function and its inverse that reverses the effect of the function. **Model** functions & inverse functions as a reflection in the line y=x.
SL 2.5	Composite functions Identity function. Finding the inverse function $f^{-1}(x)$.	**Compose** functions algebraically. **Find** inverse functions analytically. **Verify** inverse functions using compositions.

AHL 2.14	Odd and even functions Finding the inverse function, $f^{-1}(x)$, including domain restriction. Self-inverse functions	**Explore** rotations and reflections of functions and **hence** classify them as odd, even or neither.
AHL 2.15	Solutions of $g(x) \geq f(x)$, both graphically and analytically	**Determine** solutions of polynomial inequalities graphically and analytically.
AHL 2.16	The graphs of the functions, $y = \|f(x)\|$ and $y = f(\|x\|)$, $y = 1/f(x)$, $y = f(ax + b)$, $y = [f(x)]^2$. Solution of modulus equations and inequalities	**Graph** functions and **Interpret** key features of their graphs. **Find** the solutions of modulus equations and inequalities.

TOK Connections	International Mindedness Connections
SL 2.2 Do you think mathematics or logic should be classified as a language? **AHL 2.14** If systems of notation and measurement are culturally and historically situated, does this mean mathematics cannot be seen as independent of culture? **AHL 2.15** Are there differences in terms of value that different cultures ascribe to mathematics, or to the relative value that they ascribe to different areas of knowledge?	**SL 2.2** The development of functions by Rene Descartes (France), Gottfried Wilhelm Leibnitz (Germany) and Leonhard Euler (Switzerland); the notation for functions was developed by a number of different mathematicians in the 17th and 18th centuries–how did the notation we use today become internationally accepted? **AHL 2.14** The notation for functions was developed by a number of different mathematicians in the 17th and 18th centuries. How did the notation we use today become internationally accepted? **AHL 2.16** The Bourbaki group analytic approach versus Mandlebrot visual approach.

IB Mathematics: Analysis & Approaches, SL & HL, Year 1
Unit 3: Further Functions
Section 3.B: Linear Functions **Duration: 1 HL, 5 SL sessions**

IB Topic Essential Understandings:
Models are depictions of real-life events using expressions, equations or graphs while a function is defined as a relation or expression involving one or more variables. Creating different representations of functions to model the relationships between variables, visually and symbolically as graphs, equations and tables represent different ways to communicate mathematical ideas.

IB Content-Specific Conceptual Understandings

Different representations of functions, symbolically and visually as graphs, equations and tables provide different ways to communicate mathematical relationships.

The parameters in a function or equation correspond to geometrical features of a graph and can represent physical quantities in spatial dimensions.

Moving between different forms to represent functions allows for deeper understanding and provides different approaches to problem solving.

Our spatial frame of reference affects the visible part of a function and by changing this “window” can show more or less of the function to best suit our needs.

Functions represent mappings that assign to each value of the independent variable (input) one and only one dependent variable (output).

AHL
Extending results from a specific case to a general form can allow us to apply them to a larger system.

Patterns can be identified in behaviours which can give us insight into appropriate strategies to model or solve them.

The intersection of a system of equations may be represented graphically and algebraically and represents the solution that satisfies the equations.

IB Content		**Standards-Aligned Objectives.** Instruction and assessment will align to the following objectives, with IB command terms in bold
SL 2.1	Different forms of the equation of a straight line. Gradient; intercepts. Lines with gradients m_1 and m_2. Parallel lines $m_1 = m_2$. Perpendicular lines $m_1 \times m_2 = -1$.	**Distinguish** between the different forms of the equation of a straight line. **Identify** the key features of linear equations, including gradient and intercepts. **Understand** the relationship between parallel and perpendicular lines.
SL 2.3	The graph of a function; its equation $y = f(x)$. Creating a sketch from information given or a context, including transferring a graph from screen to paper. Using technology to graph functions including their sums and differences	**Graph** linear equations by hand and through the use of technology.

SL 2.4	Determine key features of graphs. Finding the point of intersection of two curves or lines using technology.	**Determine** key features of graphs. **Find** the points of intersection of two curves or lines using technology.
SL 2.10	Solving equations, both graphically and analytically. Use of technology to solve a variety of equations, including those where there is no appropriate analytic approach. Applications of graphing skills and solving equations that relate to real-life situations.	**Solve** equations, both graphically and analytically and through the use of technology. **Model** and **analyze** real life applications.
SL 2.11	Transformations of graphs. Translations: $y = f(x) + b$; $y = f(x - a)$. Reflections (in both axes): $y = -f(x)$; $y = f(-x)$. Vertical stretch with scale factor p: $y = p\, f(x)$. Horizontal stretch with scale factor 1 q : $y = f(qx)$. Composite transformations.	**Demonstrate** transformations of graphs. **Describe** composite transformations.

TOK Connections	International Mindedness Connections
SL 2.1 Descartes showed that geometric problems could be solved algebraically and vice versa. What does this tell us about mathematical representation and mathematical knowledge? **SL 2.3** Does studying the graph of a function contain the same level of mathematical rigour as studying the function algebraically? What are the advantages and disadvantages of having different forms and symbolic language in mathematics? **SL 2.10** What assumptions do mathematicians make when they apply mathematics to real-life situations?	**SL 2.4** Bourbaki group analytical approach versus the Mandlebrot visual approach.

IB Mathematics: Analysis & Approaches, SL & HL, Year 1
Unit 3: Further Functions
Section 3.C: Quadratic and Other Polynomial Functions **Duration: 4 HL, 9 SL sessions**

IB Topic Essential Understandings:
Models are depictions of real-life events using expressions, equations or graphs while a function is defined as a relation or expression involving one or more variables. Creating different representations of functions to model the relationships between variables, visually and symbolically as graphs, equations and tables represent different ways to communicate mathematical ideas.

IB Content-Specific Conceptual Understandings

Different representations of functions, symbolically and visually as graphs, equations and tables provide different ways to communicate mathematical relationships.

The parameters in a function or equation correspond to geometrical features of a graph and can represent physical quantities in spatial dimensions.

Moving between different forms to represent functions allows for deeper understanding and provides different approaches to problem solving.

Our spatial frame of reference affects the visible part of a function and by changing this "window" can show more or less of the function to best suit our needs.

Equivalent representations of quadratic functions can reveal different characteristics of the same relationship.

Functions represent mappings that assign to each value of the independent variable (input) one and only one dependent variable (output).

AHL
Extending results from a specific case to a general form can allow us to apply them to a larger system.

Patterns can be identified in behaviours which can give us insight into appropriate strategies to model or solve them.

The intersection of a system of equations may be represented graphically and algebraically and represents the solution that satisfies the equations.

IB Content		**Standards-Aligned Objectives.** Instruction and assessment will align to the following objectives, with IB command terms in bold
SL 2.3	The graph of a function; its equation y = f(x). Creating a sketch from information given or a context, including transferring a graph from screen to paper. Using technology to graph functions including their sums and differences	**Graph** polynomial equations by hand and through the use of technology.
SL 2.4	Determine key features of graphs. Finding the point of intersection of two curves or lines using technology.	**Determine** key features of graphs. **Find** the point of intersection of two curves or lines using technology.

SL 2.6	The quadratic function $f(x) = ax^2 + bx + c$: its graph, y -intercept (0, c). Axis of symmetry. The form $f(x) = a(x - p)(x - q)$, x-intercepts (p, 0) and (q, 0). The form $f(x) = a (x - h)^2 + k$, vertex (h, k)	**Convert** between forms of quadratic functions. **Identify** the key features of the graphs of quadratic functions..
SL 2.7	Solution of quadratic equations and inequalities. The quadratic formula. The discriminant $\Delta = b^2 - 4ac$ and the nature of the roots, that is, two distinct real roots, two equal real roots, no real roots.	**Solve** quadratic equations using the Quadratic Formula. **Evaluate** the discriminant of a quadratic in order to analyze the roots of a function.
SL 2.10	Solving equations, both graphically and analytically. Use of technology to solve a variety of equations, including those where there is no appropriate analytic approach. Applications of graphing skills and solving equations that relate to real-life situations.	**Solve** equations, both graphically, analytically and using technology where there is an absence of an analytic approach. **Model** and **analyze** real life applications.
SL 2.11	Transformations of graphs. Translations: $y = f(x) + b$; $y = f(x - a)$. Reflections (in both axes): $y = - f(x)$; $y = f(- x)$. Vertical stretch with scale factor p: $y = p f(x)$. Horizontal stretch with scale factor 1 q : $y = f(qx)$. Composite transformations.	**Demonstrate** transformations of graphs. **Describe** composite transformations.
AHL 2.12	Polynomial functions, their graphs and equations; zeros, roots and factors. The factor and remainder theorems. Sum and product of the roots of polynomial equations.	**Graph** polynomial functions. **Identify** the roots and zeros of polynomial functions using the remainder and factor theorem. **Find** the sum and product of the roots of polynomial equations.

TOK Connections	International Mindedness Connections
SL 2.3 Does studying the graph of a function contain the same level of mathematical rigour as studying the function algebraically? What are the advantages and disadvantages of having different forms and symbolic language in mathematics? **SL 2.6** Are there fundamental differences between mathematics and other areas of knowledge? If so, are these differences more than just methodological differences? **SL 2.10** What assumptions do mathematicians make when they apply mathematics to real-life situations?	**SL 2.4** Bourbaki group analytical approach versus the Mandlebrot visual approach. **SL 2.7** The Babylonian method of multiplication: $ab = (a + b^{2} - a^{2} - {}^{2}/2$. Sulba Sutras in ancient India and the Bakhshali Manuscript contained an algebraic formula for solving quadratic equations.

AHL 2.12 Is it an oversimplification to say that some areas of knowledge give us facts whereas other areas of knowledge give us interpretations?	

IB Mathematics: Analysis & Approaches, SL & HL, Year 1
Unit 3: Further Functions
Section 3.D: Logarithms and Exponential Functions **Duration: 8 HL, 13 SL sessions**

IB Topic Essential Understandings:
Models are depictions of real-life events using expressions, equations or graphs while a function is defined as a relation or expression involving one or more variables. Creating different representations of functions to model the relationships between variables, visually and symbolically as graphs, equations and tables represent different ways to communicate mathematical ideas.

IB Content-Specific Conceptual Understandings

Logarithm laws provide the means to find inverses of exponential functions which model real-life situations.

Different representations of functions, symbolically and visually as graphs, equations and tables provide different ways to communicate mathematical relationships.

The parameters in a function or equation correspond to geometrical features of a graph and can represent physical quantities in spatial dimensions.

Moving between different forms to represent functions allows for deeper understanding and provides different approaches to problem solving.

Our spatial frame of reference affects the visible part of a function and by changing this "window" can show more or less of the function to best suit our needs.

Equivalent representations of quadratic functions can reveal different characteristics of the same relationship.

Functions represent mappings that assign to each value of the independent variable (input) one and only one dependent variable (output).

AHL
Extending results from a specific case to a general form can allow us to apply them to a larger system.

Patterns can be identified in behaviours which can give us insight into appropriate strategies to model or solve them.

The intersection of a system of equations may be represented graphically and algebraically and represents the solution that satisfies the equations.

IB Content		**Standards-Aligned Objectives.** Instruction and assessment will align to the following objectives, with IB command terms in bold
SL 2.3	The graph of a function; its equation $y = f(x)$. Creating a sketch from information given or a context, including transferring a graph from screen to paper. Using technology to graph functions including their sums and differences	**Graph** logarithmic and exponential equations by hand and through the use of technology.
SL 2.4	Determine key features of graphs.	**Determine** key features of graphs.

	Finding the point of intersection of two curves or lines using technology.	**Find** the point of intersection of two curves or lines using technology.
SL 2.9	Exponential functions and their graphs: $f(x) = a^x$, $a > 0$, $f(x) = e^x$ Logarithmic functions and their graphs: $f(x) = \log_a x$, $x > 0$, $f(x) = \ln x$, $x > 0$.	**Analyze** exponential and logarithmic functions.
SL 2.10	Solving equations, both graphically and analytically. Use of technology to solve a variety of equations, including those where there is no appropriate analytic approach. Applications of graphing skills and solving equations that relate to real-life situations.	**Solve** equations, both graphically, analytically and using technology where there is an absence of an analytic approach. **Model** and **analyze** real life applications.
SL 2.11	Transformations of graphs. Translations: $y = f(x) + b$; $y = f(x - a)$. Reflections (in both axes): $y = - f(x)$; $y = f(- x)$. Vertical stretch with scale factor p: $y = p\, f(x)$. Horizontal stretch with scale factor 1 q : $y = f(qx)$. Composite transformations.	**Demonstrate** transformations of graphs. **Describe** composite transformations.

TOK Connections	International Mindedness Connections
SL 2.3 Does studying the graph of a function contain the same level of mathematical rigour as studying the function algebraically? What are the advantages and disadvantages of having different forms and symbolic language in mathematics? **SL 2.9** What role do “models” play in mathematics? Do they play a different role in mathematics compared to their role in other areas of knowledge? **SL 2.10** What assumptions do mathematicians make when they apply mathematics to real-life situations?	**SL 2.4** Bourbaki group analytical approach versus the Mandlebrot visual approach.

IB Mathematics: Analysis & Approaches, SL & HL, Year 1 **Unit 3: Further Functions** **Section 3.E: Rational Functions (HL only)** **Duration: 4 HL, n/a SL sessions**
IB Topic Essential Understandings: Models are depictions of real-life events using expressions, equations or graphs while a function is defined as a relation or expression involving one or more variables. Creating different representations of functions to model the relationships between variables, visually and symbolically as graphs, equations and tables represent different ways to communicate mathematical ideas.
IB Content-Specific Conceptual Understandings
Different representations of functions, symbolically and visually as graphs, equations and tables provide different ways to communicate mathematical relationships. The parameters in a function or equation correspond to geometrical features of a graph and can represent physical quantities in spatial dimensions. Moving between different forms to represent functions allows for deeper understanding and provides different approaches to problem solving. Our spatial frame of reference affects the visible part of a function and by changing this “window” can show more or less of the function to best suit our needs. Equivalent representations of quadratic functions can reveal different characteristics of the same relationship. Functions represent mappings that assign to each value of the independent variable (input) one and only one dependent variable (output). **AHL** Extending results from a specific case to a general form can allow us to apply them to a larger system. Patterns can be identified in behaviours which can give us insight into appropriate strategies to model or solve them. The intersection of a system of equations may be represented graphically and algebraically and represents the solution that satisfies the equations.

IB Content		**Standards-Aligned Objectives.** Instruction and assessment will align to the following objectives, with IB command terms in bold
SL 2.3	The graph of a function; its equation y = f(x). Creating a sketch from information given or a context, including transferring a graph from screen to paper. Using technology to graph functions including their sums and differences	**Graph** rational equations by hand and through the use of technology.
SL 2.4	Determine key features of graphs. Finding the point of intersection of two curves or lines using technology.	**Determine** key features of graphs. **Find** the point of intersection of two curves or lines using technology.

SL 2.8	The reciprocal function f(x) = $\frac{1}{x}$, x ≠ 0: its graph and self-inverse nature. Rational functions of the form f(x) = $\frac{ax+b}{cx+d}$ and their graphs. Equations of vertical and horizontal asymptotes	**Analyze** the reciprocal function. **Identify** the equations of the vertical and horizontal asymptotes.
SL 2.10	Solving equations, both graphically and analytically. Use of technology to solve a variety of equations, including those where there is no appropriate analytic approach. Applications of graphing skills and solving equations that relate to real-life situations.	**Solve** equations, both graphically, analytically and using technology where there is an absence of an analytic approach. **Model** and **analyze** real life applications.
SL 2.11	Transformations of graphs. Translations: y = f(x) + b; y = f(x − a). Reflections (in both axes): y = − f(x); y = f(− x). Vertical stretch with scale factor p: y = p f(x). Horizontal stretch with scale factor 1 q : y = f(qx). Composite transformations.	**Demonstrate** transformations of graphs. **Describe** composite transformations.
AHL 2.13	Rational functions of the form f(x) = $\frac{ax+b}{cx^2+dx+e}$, and f(x) = $\frac{ax^2+bx+c}{dx+e}$	**Analyze** rational functions.

TOK Connections	International Mindedness Connections
SL 2.3 Does studying the graph of a function contain the same level of mathematical rigour as studying the function algebraically? What are the advantages and disadvantages of having different forms and symbolic language in mathematics? **SL 2.8** What are the implications of accepting that mathematical knowledge changes over time? **SL 2.10** What assumptions do mathematicians make when they apply mathematics to real-life situations? **AHL 2.13** Does studying the graph of a function contain the same level of mathematical rigour as studying the function algebraically? What are the advantages and disadvantages of having different forms and symbolic language in mathematics?	**SL 2.4** Bourbaki group analytical approach versus the Mandlebrot visual approach.

IB Mathematics: Analysis & Approaches, SL & HL, Year 1 Unit 4 Overview: Complex Numbers (HL Only)	Duration: 18 HL, 0 SL sessions	
IB Topic Essential Understandings: Number and algebra allow us to represent patterns, show equivalencies and make generalizations which enable us to model real-world situations. Algebra is an abstraction of numerical concepts and employs variables which allow us to solve mathematical problems.		
IB Content-Specific Conceptual Understandings		
AHL Representing complex numbers in different forms allows us to easily carry out seemingly difficult calculations.		
Section and IB Sub-Topics	**HL Duration**	**SL Duration**
A: Properties of & Operations with Complex Numbers AHL 1.12, AHL 1.13	5 sessions	N/A
B: Graphs of Complex Numbers AHL 1.12	7 sessions	N/A
C: Roots of Complex Numbers AHL 1.14	6 sessions	N/A

IB Mathematics: Analysis & Approaches, SL & HL, Year 1 **Unit 4 : Complex Numbers (HL only)** **Section 4.A: Properties of & Operations with Complex Numbers** **Duration: 5 HL, 0 SL sessions**
IB Topic Essential Understandings: Number and algebra allow us to represent patterns, show equivalencies and make generalizations which enable us to model real-world situations. Algebra is an abstraction of numerical concepts and employs variables which allow us to solve mathematical problems.
IB Content-Specific Conceptual Understandings
AHL Representing complex numbers in different forms allows us to easily carry out seemingly difficult calculations.

IB Content		**Standards-Aligned Objectives.** Instruction and assessment will align to the following objectives, with IB command terms in bold
AHL 1.12	Complex numbers: the number i, where $i^2 = -1$. Cartesian form $z = a + bi$; the terms real part, imaginary part, conjugate, modulus and argument.	**Understand** complex numbers as a distinct set of numbers. **Represent** complex numbers in cartesian form and **distinguish** the real parts from the imaginary parts.
AHL 1.13	Modulus–argument (polar) form: $z = r(\cos\theta + i\sin\theta) = r\text{cis}\theta$ Euler form: $z = re^{i\theta}$ Sums, products and quotients in Cartesian, polar or Euler forms and their geometric interpretation.	**Represent** complex numbers in cartesian, polar and Euler forms and transform between each. **Interpret** geometrically the sums, products and quotients of complex numbers in Cartesian, polar or Euler forms.

TOK Connections	**International Mindedness Connections**
AHL 1.12 How does language shape knowledge? For example, do the words “imaginary” and “complex” make the concepts more difficult than if they had different names? **AHL 1.13** Why might it be said that $e^{i\pi} + 1 = 0$ is beautiful? What is the place of beauty and elegance in mathematics? What about the place of creativity?	n/a

IB Mathematics: Analysis & Approaches, SL & HL, Year 1
Unit 4 : Complex Numbers (HL only)
Section 4.B: Graphs of Complex Numbers **Duration: 7 HL, 0 SL sessions**

IB Topic Essential Understandings:
Number and algebra allow us to represent patterns, show equivalencies and make generalizations which enable us to model real-world situations. Algebra is an abstraction of numerical concepts and employs variables which allow us to solve mathematical problems.

IB Content-Specific Conceptual Understandings

AHL
Representing complex numbers in different forms allows us to easily carry out seemingly difficult calculations.

IB Content		**Standards-Aligned Objectives.** Instruction and assessment will align to the following objectives, with IB command terms in bold
AHL 1.12	The complex plane.	**Compare and contrast** the complex plane with the coordinate plane. **Represent** complex numbers on a coordinate plane.

TOK Connections	**International Mindedness Connections**
AHL 1.12 How does language shape knowledge? For example, do the words “imaginary” and “complex” make the concepts more difficult than if they had different names?	n/a

IB Mathematics: Analysis & Approaches, SL & HL, Year 1
Unit 4: Complex Numbers (HL only)
Section 4.C: Roots of Complex Numbers — **Duration: 6 HL, 0 SL sessions**

IB Topic Essential Understandings:
Number and algebra allow us to represent patterns, show equivalencies and make generalizations which enable us to model real-world situations. Algebra is an abstraction of numerical concepts and employs variables which allow us to solve mathematical problems.

IB Content-Specific Conceptual Understandings

AHL
Representing complex numbers in different forms allows us to easily carry out seemingly difficult calculations.

IB Content		**Standards-Aligned Objectives.** Instruction and assessment will align to the following objectives, with IB command terms in bold
AHL 1.14	Complex conjugate roots of quadratic and polynomial equations with real coefficients. Complex roots occur in conjugate pairs. De Moivre's theorem and its extension to rational exponents. Powers and roots of complex numbers.	**Solve** polynomials with complex roots **Explain** why complex roots of polynomials occur in conjugate pairs. **Apply** De Moivre's theorem to simplify rational expressions **Simplify** complex numbers with rational exponents

TOK Connections	**International Mindedness Connections**
AHL 1.14 Could we ever reach a point where everything important in a mathematical sense is known? Reflect on the creation of complex numbers before their applications were known.	n/a

IB Mathematics: Analysis & Approaches, SL & HL, Year 1 **Unit 5 Overview: Combinatorics** **Duration: 10 HL, 20 SL sessions**		
IB Topic Essential Understandings: Number and algebra allow us to represent patterns, show equivalencies and make generalizations which enable us to model real-world situations. Algebra is an abstraction of numerical concepts and employs variables which allow us to solve mathematical problems		
IB Content-Specific Conceptual Understandings		
Patterns in numbers inform the development of algebraic tools that can be applied to find unknowns. The binomial theorem is a generalization which provides an efficient method for expanding binomial expressions. **AHL** Proof serves to validate mathematical formulae and the equivalence of identities.		
Section and IB Sub-Topics	**HL Duration**	**SL Duration**
A: Counting Principles, including Permutations & Combinations AHL 1.10	5 sessions	9 sessions
B: Pascal's Triangle SL 1.9	2 sessions	3 sessions
C: Binomial Theorem SL 1.9, AHL 1.10	3 sessions	8 sessions

IB Mathematics: Analysis & Approaches, SL & HL, Year 1 **Unit 5: Combinatorics** **Section 5.A: Counting Principles, including Permutations & Combinations**		**Duration: 5 HL, 9 SL sessions**
IB Topic Essential Understandings: Number and algebra allow us to represent patterns, show equivalencies and make generalizations which enable us to model real-world situations. Algebra is an abstraction of numerical concepts and employs variables which allow us to solve mathematical problems		
IB Content-Specific Conceptual Understandings		
Patterns in numbers inform the development of algebraic tools that can be applied to find unknowns. **AHL** Proof serves to validate mathematical formulae and the equivalence of identities.		
IB Content		**Standards-Aligned Objectives.** Instruction and assessment will align to the following objectives, with IB command terms in bold
AHL 1.10	Counting principles, including permutations and combinations.	**Apply** counting principles. **Distinguish** between real world situations that can be represented using combinations and permutations.

TOK Connections	**International Mindedness Connections**
AHL 1.10 What counts as understanding in mathematics? Is it more than just getting the right answer?	**AHL 1.10** The properties of “Pascal’s triangle” have been known in a number of different cultures long before Pascal (for example the Chinese mathematician Yang Hui).

IB Mathematics: Analysis & Approaches, SL & HL, Year 1 **Unit 5 : Combinatorics** **Section 5.B: Pascal's Triangle** **Duration: 2 HL, 3 SL sessions**

IB Topic Essential Understandings:
Number and algebra allow us to represent patterns, show equivalencies and make generalizations which enable us to model real-world situations. Algebra is an abstraction of numerical concepts and employs variables which allow us to solve mathematical problems

IB Content-Specific Conceptual Understandings

Patterns in numbers inform the development of algebraic tools that can be applied to find unknowns.

The binomial theorem is a generalization which provides an efficient method for expanding binomial expressions.

IB Content		**Standards-Aligned Objectives.** Instruction and assessment will align to the following objectives, with IB command terms in bold
SL 1.9	Use of Pascal's triangle and nCr	**Generate** Pascal's triangle and **apply** in order to **identify** binomial coefficients.

TOK Connections	**International Mindedness Connections**
SL 1.9 How have notable individuals shaped the development of mathematics as an area of knowledge? Consider Pascal and "his" triangle.	**SL 1.9** The properties of "Pascal's triangle" have been known in a number of different cultures long before Pascal. (for example the Chinese mathematician Yang Hui).

IB Mathematics: Analysis & Approaches, SL & HL, Year 1
Unit 5 : Combinatorics
Section 5.C: Binomial Theorem **Duration: 3 HL, 8 SL sessions**

IB Topic Essential Understandings:
Number and algebra allow us to represent patterns, show equivalencies and make generalizations which enable us to model real-world situations. Algebra is an abstraction of numerical concepts and employs variables which allow us to solve mathematical problems

IB Content-Specific Conceptual Understandings

Patterns in numbers inform the development of algebraic tools that can be applied to find unknowns.

The binomial theorem is a generalization which provides an efficient method for expanding binomial expressions.

AHL
Proof serves to validate mathematical formulae and the equivalence of identities.

IB Content		**Standards-Aligned Objectives.** Instruction and assessment will align to the following objectives, with IB command terms in bold
SL 1.9	The binomial theorem: expansion of $(a + b)^n$, $n \in \mathbb{N}$.	**Apply** Pascal's triangle and the Binomial Theorem to expand binomials.
AHL 1.10	Extension of the binomial theorem to fractional & negative indices, ie $(a + b)^n$, $n \in \mathbb{Q}$.	**Extend** the Binomial Theorem to expand binomials with fractional and negative indices.

TOK Connections	International Mindedness Connections
SL 1.9 How have notable individuals shaped the development of mathematics as an area of knowledge? Consider Pascal and "his" triangle. **AHL 1.10** What counts as understanding in mathematics? Is it more than just getting the right answer?	**AHL 1.10** The properties of "Pascal's triangle" have been known in a number of different cultures long before Pascal (for example the Chinese mathematician Yang Hui).

IB Mathematics: Analysis & Approaches, SL & HL, Year 1 **Unit 6 Overview: Reasoning and Proof (HL only)**	**Duration: 18 HL, 0 SL sessions**

IB Topic Essential Understandings:
Number and algebra allow us to represent patterns, show equivalencies and make generalizations which enable us to model real-world situations. Algebra is an abstraction of numerical concepts and employs variables which allow us to solve mathematical problems.

IB Content-Specific Conceptual Understandings

Numbers and formulae can appear in different, but equivalent, forms, or representations, which can help us to establish identities.

AHL
Proof serves to validate mathematical formulae and the equivalence of identities.

Section and IB Sub-Topics	HL Duration	SL Duration
A: Proof by Deduction SL 1.6	5 sessions	N/A
B: Proof by Contradiction & Counterexample AHL 1.15	6 sessions	N/A
C: Proof by Mathematical Induction AHL 1.15	7 sessions	N/A

IB Mathematics: Analysis & Approaches, SL & HL, Year 1 **Unit 6 : Reasoning and Proof** **Section 6.A: Proof by Deduction** **Duration: 5 HL, 0 SL sessions**		
IB Topic Essential Understandings: Number and algebra allow us to represent patterns, show equivalencies and make generalizations which enable us to model real-world situations. Algebra is an abstraction of numerical concepts and employs variables which allow us to solve mathematical problems.		
IB Content-Specific Conceptual Understandings		
Numbers and formulae can appear in different, but equivalent, forms, or representations, which can help us to establish identities **AHL** Proof serves to validate mathematical formulae and the equivalence of identities.		
IB Content		**Standards-Aligned Objectives.** Instruction and assessment will align to the following objectives, with IB command terms in bold
SL 1.6	Simple deductive proof, numerical and algebraic; how to lay out a left-hand side to right-hand side (LHS to RHS) proof. The symbols and notation for equality and identity.	**Perform** simple deductive proofs. **Understand** symbols and notations used in equations and identities.

TOK Connections	**International Mindedness Connections**
SL 1.6 Is mathematical reasoning different from scientific reasoning, or reasoning in other Areas of Knowledge?	n/a

IB Mathematics: Analysis & Approaches, SL & HL, Year 1 **Unit 6 : Reasoning and Proof** **Section 6.B: Proof by Contradiction & Counterexample**	**Duration: 6 HL, 0 SL sessions**

IB Topic Essential Understandings:
Number and algebra allow us to represent patterns, show equivalencies and make generalizations which enable us to model real-world situations. Algebra is an abstraction of numerical concepts and employs variables which allow us to solve mathematical problems.

IB Content-Specific Conceptual Understandings

AHL
Proof serves to validate mathematical formulae and the equivalence of identities.

IB Content		**Standards-Aligned Objectives.** Instruction and assessment will align to the following objectives, with IB command terms in bold
AHL 1.15	Proof by contradiction. Use of a counterexample to show that a statement is not always true.	**Prove** that a statement is true using contradiction or a counterexample

TOK Connections	**International Mindedness Connections**
AHL 1.15 What is the role of the mathematical community in determining the validity of a mathematical proof? Do proofs provide us with completely certain knowledge? What is the difference between the inductive method in science and proof by induction in mathematics?	**AHL 1.15** How did the Pythagoreans find out that 2 is irrational?

IB Mathematics: Analysis & Approaches, SL & HL, Year 1 Unit 6 : Reasoning and Proof Section 6.C: Proof by Mathematical Induction Duration: 7 HL, 0 SL sessions		
IB Topic Essential Understandings: Number and algebra allow us to represent patterns, show equivalencies and make generalizations which enable us to model real-world situations. Algebra is an abstraction of numerical concepts and employs variables which allow us to solve mathematical problems		
IB Content-Specific Conceptual Understandings		
AHL Proof serves to validate mathematical formulae and the equivalence of identities.		
IB Content		**Standards-Aligned Objectives.** Instruction and assessment will align to the following objectives, with IB command terms in bold
AHL 1.15	Proof by mathematical induction.	**Prove** statements to be true using mathematical induction.

TOK Connections	International Mindedness Connections
AHL 1.15 What is the role of the mathematical community in determining the validity of a mathematical proof? Do proofs provide us with completely certain knowledge? What is the difference between the inductive method in science and proof by induction in mathematics?	**AHL 1.15** How did the Pythagoreans find out that 2 is irrational?

IB Mathematics: Analysis & Approaches, SL & HL, Year 1 **Unit 7 Overview: Linear Algebra (HL only)**		**Duration: 8 HL, 0 SL sessions**
IB Topic Essential Understandings: Number and algebra allow us to represent patterns, show equivalencies and make generalizations which enable us to model real-world situations. Algebra is an abstraction of numerical concepts and employs variables which allow us to solve mathematical problems.		
IB Content-Specific Conceptual Understandings		
AHL Representing partial fractions and complex numbers in different forms allows us to easily carry out seemingly difficult calculations. The solution for systems of equations can be carried out by a variety of equivalent algebraic and graphical methods.		
Section and IB Sub-Topics	**HL Duration**	**SL Duration**
A: Solving Systems using Matrices AHL 1.16	5 sessions	N/A
B: Partial Fractions AHL 1.11	3 sessions	N/A

IB Mathematics: Analysis & Approaches, SL & HL, Year 1 Unit 7: Linear Algebra (HL only) Section 7.A: Solving Systems using Matrices			Duration: 5 HL, 0 SL sessions
IB Topic Essential Understandings: Number and algebra allow us to represent patterns, show equivalencies and make generalizations which enable us to model real-world situations. Algebra is an abstraction of numerical concepts and employs variables which allow us to solve mathematical problems.			
IB Content-Specific Conceptual Understandings			
AHL The solution for systems of equations can be carried out by a variety of equivalent algebraic and graphical methods.			
IB Content		**Standards-Aligned Objectives.** Instruction and assessment will align to the following objectives, with IB command terms in bold	
AHL 1.16	Solutions of systems of linear equations (a maximum of three equations in three unknowns), including cases where there is a unique solution, an infinite number of solutions or no solution.	**Calculate** solutions of systems of linear equations (a maximum of three equations in three unknowns), including cases where there is a unique solution, an infinite number of solutions or no solution.	

TOK Connections	International Mindedness Connections
AHL 1.16 Mathematics, Sense, Perception and Reason: If we can find solutions in higher dimensions can we reason that these spaces exist beyond our sense perception?	n/a

IB Mathematics: Analysis & Approaches, SL & HL, Year 1 **Unit 7: Linear Algebra** **Section 7.B: Partial Fractions** **Duration: 3 HL, 0 SL sessions**		
IB Topic Essential Understandings: Number and algebra allow us to represent patterns, show equivalencies and make generalizations which enable us to model real-world situations. Algebra is an abstraction of numerical concepts and employs variables which allow us to solve mathematical problems.		
IB Content-Specific Conceptual Understandings		
AHL Representing partial fractions and complex numbers in different forms allows us to easily carry out seemingly difficult calculations.		
IB Content		**Standards-Aligned Objectives.** Instruction and assessment will align to the following objectives, with IB command terms in bold
AHL 1.11	Partial fractions.	**Convert** rational expressions into partial fractions.

TOK Connections	**International Mindedness Connections**
n/a	n/a

IB Mathematics: Analysis & Approaches, SL & HL, Year 1 Unit 8 Overview: Vectors (HL Only) Duration: 25 HL, 0 SL sessions		
IB Topic Essential Understandings: Geometry and vectors allow us to quantify the physical world, enhancing our spatial awareness in two and three dimensions. This topic provides us with the tools for analysis, measurement and transformation of quantities, movements and relationships.		
IB Content-Specific Conceptual Understandings		
AHL Position and movement can be modelled in three-dimensional space using vectors. The relationships between algebraic, geometric and vector methods can help us to solve problems and quantify those positions and movements.		
Section and IB Sub-Topics	**HL Duration**	**SL Duration**
A: Vector Properties AHL 3.12	6 sessions	N/A
B: Vector Operations AHL 3.12, AHL 3.13, AHL 3.16	6 sessions	N/A
C: Coordinate Geometry and Vectors AHL 3.14, AHL 3.15, AHL 3.17	7 sessions	N/A
D: Application of Vectors AHL 3.17,AHL 3.18	6 sessions	N/A

IB Mathematics: Analysis & Approaches, SL & HL, Year 1
Unit 8: Vectors (HL Only)
Section 8.A: Vector Properties **Duration: 6 HL, 0 SL sessions**

IB Topic Essential Understandings:
Geometry and vectors allow us to quantify the physical world, enhancing our spatial awareness in two and three dimensions. This topic provides us with the tools for analysis, measurement and transformation of quantities, movements and relationships.

IB Content-Specific Conceptual Understandings

AHL
Position and movement can be modelled in three-dimensional space using vectors.

The relationships between algebraic, geometric and vector methods can help us to solve problems and quantify those positions and movements.

IB Content		**Standards-Aligned Objectives.** Instruction and assessment will align to the following objectives, with IB command terms in bold
AHL 3.12	Concept of a vector; position vectors; displacement vectors. Representation of vectors using directed line segments. Base vectors i, j, k. Components of a vector: $\boldsymbol{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = v_1\boldsymbol{i} + v_2\boldsymbol{j} + v_3\boldsymbol{k}.$	**Understand** the concept of a vector; position vectors; displacement vectors. **Represent** vectors using directed line segments. **Identify** the components of a vector.

TOK Connections	International Mindedness Connections
AHL 3.12 Vectors are used to solve many problems in position location. This can be used to save a lost sailor or destroy a building with a laser-guided bomb. To what extent does possession of knowledge carry with it an ethical obligation?	n/a

IB Mathematics: Analysis & Approaches, SL & HL, Year 1
Unit 8 : Vectors (HL Only)
Section 8.B: Vector Operations **Duration: 6 HL, 0 SL sessions**

IB Topic Essential Understandings:
Geometry and vectors allow us to quantify the physical world, enhancing our spatial awareness in two and three dimensions. This topic provides us with the tools for analysis, measurement and transformation of quantities, movements and relationships.

IB Content-Specific Conceptual Understandings

AHL
Position and movement can be modelled in three-dimensional space using vectors.

The relationships between algebraic, geometric and vector methods can help us to solve problems and quantify those positions and movements.

IB Content		**Standards-Aligned Objectives.** Instruction and assessment will align to the following objectives, with IB command terms in bold
AHL 3.12	Algebraic and geometric approaches to the following: ● the sum and difference of two vectors ● the zero vector 0, the vector −v ● multiplication by a scalar, kv, parallel vectors ● magnitude of a vector $\|v\|$; unit vectors $\frac{v}{\|v\|}$ ● position vectors $\vec{OA} = \boldsymbol{a}, \quad \vec{OB} = \boldsymbol{b}$ ● Displacement vector $\vec{AB} = \boldsymbol{b} - \boldsymbol{a}$ Proofs of geometrical properties using vectors	**Apply** properties of vectors to **perform** operations on vectors. **Prove** vector properties using a variety of methods.
AHL 3.13	The definition of the scalar product of two vectors. The angle between two vectors. Perpendicular vectors; parallel vectors.	**Understand** the definition of the scalar product of two vectors. **Calculate** the scalar product and angle between two vectors. **Classify** vectors as parallel, perpendicular, or skew.
AHL 3.16	The definition of a vector product of two vectors Properties of the vector product Geometrical interpretation of $\|v \ x \ w\|$	**Understand** the definition of a vector product of two vectors. **Apply** the properties of the vector product.

TOK Connections	**International Mindedness Connections**
AHL 3.12 Vectors are used to solve many problems in position location. This can be used to save a lost sailor or destroy a building with a laser-guided bomb. To what extent does possession of knowledge carry with it an ethical obligation? **AHL 3.13** The nature of mathematics: why this definition of scalar product?	n/a

AHL 3.16 To what extent is certainty attainable in mathematics? Is certainty attainable, or desirable, in other areas of knowledge?	

IB Mathematics: Analysis & Approaches, SL & HL, Year 1 **Unit 8 : Vectors (HL Only)** **Section 8.C: Coordinate Geometry and Vectors**		**Duration: 7 HL, 0 SL sessions**
IB Topic Essential Understandings: Geometry and vectors allow us to quantify the physical world, enhancing our spatial awareness in two and three dimensions. This topic provides us with the tools for analysis, measurement and transformation of quantities, movements and relationships.		
IB Content-Specific Conceptual Understandings		
AHL Position and movement can be modelled in three-dimensional space using vectors. The relationships between algebraic, geometric and vector methods can help us to solve problems and quantify those positions and movements.		
IB Content		**Standards-Aligned Objectives.** Instruction and assessment will align to the following objectives, with IB command terms in bold
AHL 3.14	Vector equation of a line in two and three dimensions: **r** = **a** + λ**b**. The angle between two lines. Simple applications to kinematics	**Write** equations of vectors in various forms. **Calculate** the angle between two lines. **Apply** properties of kinematics to solve real world problems.
AHL 3.15	Coincident, parallel, intersecting and skew lines, distinguishing between these cases. Points of intersection.	**Classify** lines as coincident, parallel, intersecting and skew. **Find** the points of intersection of systems of equations.
AHL 3.17	Vector equations of a plane: **r** = **a** + λ**b** + μ**c**, where **b** and **c** are non-parallel vectors within the plane. **r** · **n** = **a** · **n**, where **n** is a normal to the plane and **a** is the position vector of a point on the plane. Cartesian equation of a plane ax + by + cz = d.	**Write** equations of planes in various forms. **Calculate** and **interpret** dot products of vector equations.

TOK Connections	International Mindedness Connections
AHL 3.14 Why might it be argued that one form of representation is superior to another? What criteria might a mathematician use in making such an argument? **AHL 3.15** How can there be an infinite number of discrete solutions to an equation? What does this suggest about the nature of mathematical knowledge and how it compares to knowledge in other disciplines?	n/a

IB Mathematics: Analysis & Approaches, SL & HL, Year 1 **Unit 8 : Vectors (HL Only)** **Section 8.D: Application of Vectors** **Duration: 6 HL, 0 SL sessions**		
IB Topic Essential Understandings: Geometry and vectors allow us to quantify the physical world, enhancing our spatial awareness in two and three dimensions. This topic provides us with the tools for analysis, measurement and transformation of quantities, movements and relationships.		
IB Content-Specific Conceptual Understandings		
AHL Position and movement can be modelled in three-dimensional space using vectors. The relationships between algebraic, geometric and vector methods can help us to solve problems and quantify those positions and movements.		
IB Content		**Standards-Aligned Objectives.** Instruction and assessment will align to the following objectives, with IB command terms in bold
AHL 3.17	Vector equations of a plane: $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b} + \mu\mathbf{c}$, where **b** and **c** are non-parallel vectors within the plane. $\mathbf{r} \cdot \mathbf{n} = \mathbf{a} \cdot \mathbf{n}$, where **n** is a normal to the plane and **a** is the position vector of a point on the plane. Cartesian equation of a plane $ax + by + cz = d$.	**Write** equations of planes in various forms. **Calculate** and **interpret** dot products of vector equations.
AHL 3.18	Intersections of: a line with a plane; two planes; three planes. Angle between: a line and a plane; two planes.	**Find** the intersection of lines and planes, two planes and three planes. **Find** the angle between a line and plane and between two planes.

TOK Connections	International Mindedness Connections
AHL 3.18 Mathematics and the knower: are symbolic representations of three-dimensional objects easier to deal with than visual representations? What does this tell us about our knowledge of mathematics in other dimensions?	n/a