

FREEHOLD REGIONAL HIGH SCHOOL DISTRICT OFFICE OF CURRICULUM AND INSTRUCTION INTERNATIONAL BACCALAUREATE PROGRAM

IB Mathematics: Analysis & Approaches, SL & HL Year 1

Grade Level: 11

Credits: 5

BOARD OF EDUCATION ADOPTION DATE: August 26, 2021

FREEHOLD REGIONAL HIGH SCHOOL DISTRICT

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Course Description

From the International Baccalaureate Organization: "Mathematics: analysis and approaches is for students who enjoy developing their mathematics to become fluent in the construction of mathematical arguments and develop strong skills in mathematical thinking. They will also be fascinated by exploring real and abstract applications of these ideas, with and without technology. Students who take Mathematics: analysis and approaches will be those who enjoy the thrill of mathematical problem solving and generalization.

This course recognizes the need for analytical expertise in a world where innovation is increasingly dependent on a deep understanding of mathematics. This course includes topics that are both traditionally part of a pre-university mathematics course (for example, functions, trigonometry, calculus) as well as topics that are amenable to investigation, conjecture and proof, for instance the study of sequences and series at both SL and HL, and proof by induction at HL.

The course allows the use of technology, as fluency in relevant mathematical software and hand-held technology is important regardless of choice of course. However, Mathematics: analysis and approaches has a strong emphasis on the ability to construct, communicate and justify correct mathematical arguments.

Students who choose Mathematics: analysis and approaches at SL or HL should be comfortable in the manipulation of algebraic expressions and enjoy the recognition of patterns and understand the mathematical generalization of these patterns. Students who wish to take Mathematics: analysis and approaches at a higher level will have strong algebraic skills and the ability to understand simple proof. They will be students who enjoy spending time with problems and get pleasure and satisfaction from solving challenging problems."

Course Sequence and Pacing

Unit Title	HL Pacing	SL Pacing		
1: Foundational Knowledge	6 sessions	18 sessions		
2: Further Trigonometry	13 sessions	28 sessions		
3: Further Functions	22 sessions	40 sessions		
4: Complex Numbers (HL only)	18 sessions	n/a		
5: Combinatorics	10 sessions	20 sessions		
6: Reasoning and Proof (HL only)	18 sessions	n/a		
7: Overview: Linear Algebra (HL only)	8 sessions	n/a		
8: Vectors (HL Only)	25 sessions	n/a		

Unit 1 Overview: Foundational Knowledge Duration: 6 HL, 18 SL sessions

IB Topic Essential Understandings:

Number and algebra allow us to represent patterns, show equivalencies and make generalizations which enable us to model real-world situations. Algebra is an abstraction of numerical concepts and employs variables which allow us to solve mathematical problems.

Models are depictions of real-life events using expressions, equations or graphs while a function is defined as a relation or expression involving one or more variables. Creating different representations of functions to model the relationships between variables, visually and symbolically as graphs, equations and tables represent different ways to communicate mathematical ideas.

Geometry and trigonometry allows us to quantify the physical world, enhancing our spatial awareness in two and three dimensions. This topic provides us with the tools for analysis, measurement and transformation of quantities, movements and relationships.

IB Content-Specific Conceptual Understandings

Modelling real-life situations with the structure of arithmetic and geometric sequences and series allows for prediction, analysis and interpretation.

Formulae are a generalization made on the basis of specific examples, which can then be extended to new examples.

Different representations of functions, symbolically and visually as graphs, equations and tables provide different ways to communicate mathematical relationships.

Moving between different forms to represent functions allows for deeper understanding and provides different approaches to problem solving.

The relationships between the length of the sides and the size of the angles in a triangle can be used to solve many problems involving position, distance, angles and area.

Equivalent measurement systems, such as degrees and radians, can be used for angles to facilitate ease of calculation.

AHL

Patterns can be identified in behaviours which can give us insight into appropriate strategies to model or solve them.

Section and IB Sub-Topics	HL Duration (sessions)	SL Duration (sessions)
A: Sequences and Series SL 1.2, SL 1.3, SL 1.4	2	5
B: Introduction to Trigonometry SL 3.1, SL 3.2, SL 3.3, SL 3.4, SL 3.6, AHL 3.9	2	6
C: Function Analysis SL 2.1, SL 2.2, SL 2.3, SL 2.4, SL 2.5, SL 2.6, SL 2.7, SL 2.10, SL 2.11	2	7

Unit 1: Foundational Knowledge Section 1A: Sequences and Series

IB Topic Essential Understandings:

Number and algebra allow us to represent patterns, show equivalencies and make generalizations which enable us to model real-world situations. Algebra is an abstraction of numerical concepts and employs variables which allow us to solve mathematical problems.

Duration: 2 HL, 5 SL sessions

Models are depictions of real-life events using expressions, equations or graphs while a function is defined as a relation or expression involving one or more variables. Creating different representations of functions to model the relationships between variables, visually and symbolically as graphs, equations and tables represent different ways to communicate mathematical ideas.

Geometry and trigonometry allows us to quantify the physical world, enhancing our spatial awareness in two and three dimensions. This topic provides us with the tools for analysis, measurement and transformation of quantities, movements and relationships.

IB Content-Specific Conceptual Understandings

Modelling real-life situations with the structure of arithmetic and geometric sequences and series allows for prediction, analysis and interpretation.

Formulae are a generalization made on the basis of specific examples, which can then be extended to new examples.

IB Content		Standards-Aligned Objectives. Instruction and assessment will align to the following objectives, with IB command terms in bold	
SL 1.2	Arithmetic sequences and series. Use of the formulae for the n th term and the sum of the first n terms of the sequence. Use of sigma notation for sums of arithmetic sequences. Applications. Analysis, interpretation and prediction where a model is not perfectly arithmetic in real life.	Distinguish between arithmetic and geometric patterns in given sequences. Deduce into the explicit formula, given an arithmetic sequence in recursive form. Describe arithmetic sequences from recursive and explicit formulas. Apply arithmetic sequences to real-world scenarios using approximation where appropriate.	
SL 1.3	Geometric sequences and series. Use of the formulae for the n th term and the sum of the first n terms of the sequence. Use of sigma notation for the sums of geometric sequences. Applications.	Deduce into the explicit formula, given a geometric sequence in recursive form. Describe geometric sequences from recursive and explicit formulas. Apply geometric sequences to real-world scenarios using approximation where appropriate.	
SL 1.4	Financial applications of geometric sequences and series: • compound interest	Apply geometric sequences to problems involving financial situations.	

annual depreciation.	
TOK Connections	International Mindedness Connections
SL 1.2: Is all knowledge concerned with identification and use of patterns? Consider Fibonacci numbers and connections with the golden ratio.	SL 1.4: Do all societies view investment and interest in the same way?
SL 1.3: How do mathematicians reconcile the fact that some conclusions seem to conflict with our intuitions? Consider for instance that a finite area can be bounded by an infinite perimeter	
SL 1.4: How have technological advances affected the nature and practice of mathematics? Consider the use of financial packages for instance.	

Unit 1: Foundational Knowledge

Section 1.B: Introduction to Trigonometry

IB Topic Essential Understandings:

Number and algebra allow us to represent patterns, show equivalencies and make generalizations which enable us to model real-world situations. Algebra is an abstraction of numerical concepts and employs variables which allow us to solve mathematical problems.

Duration: 2 HL, 6 SL sessions

Models are depictions of real-life events using expressions, equations or graphs while a function is defined as a relation or expression involving one or more variables. Creating different representations of functions to model the relationships between variables, visually and symbolically as graphs, equations and tables represent different ways to communicate mathematical ideas.

Geometry and trigonometry allows us to quantify the physical world, enhancing our spatial awareness in two and three dimensions. This topic provides us with the tools for analysis, measurement and transformation of quantities, movements and relationships.

IB Content-Specific Conceptual Understandings

Different representations of functions, symbolically and visually as graphs, equations and tables provide different ways to communicate mathematical relationships.

Moving between different forms to represent functions allows for deeper understanding and provides different approaches to problem solving.

The relationships between the length of the sides and the size of the angles in a triangle can be used to solve many problems involving position, distance, angles and area.

Equivalent measurement systems, such as degrees and radians, can be used for angles to facilitate ease of calculation.

AHLPatterns can be identified in behaviours which can give us insight into appropriate strategies to model or solve them.

IB Content		Standards-Aligned Objectives. Instruction and assessment will align to the following objectives, with IB command terms in bold	
SL 3.1	The distance between two points in three-dimensional space, and their midpoint. Volume and surface area of three-dimensional solids including right-pyramid, right cone, sphere, hemisphere and combinations of these solids. The size of an angle between two intersecting lines or between a line and a plane.	Calculate the distance between two points in a three-dimensional space, and their midpoint using a variety of methods. Calculate the volume and surface area of three-dimensional solids. Hence determine the parameters for the three-dimensional solid.	
SL 3.2	Use of sine, cosine and tangent ratios to find the sides and angles of right-angled triangles. The sine rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$. The cosine rule: $c^2 = a^2 + b^2 - 2abcosC$; $cos C = \frac{a^2 + b^2 - c^2}{2ab}$.	Solve right-angled triangles using sine, cosine and tangent ratios. Determine the appropriate method for solving oblique triangles, and hence find all unknown sides and angles. Calculate the area of an non-right angled triangle.	

	I	
	Area of a triangle as $\frac{1}{2}$ absinC.	
SL 3.3	Applications of right and non-right angled trigonometry, including Pythagoras's theorem.	Apply properties of right and non-right angled triangle trigonometry to various real-world scenarios.
	Angles of elevation and depression. Construction of labelled diagrams from written statements.	Sketch right and non-right angled triangles to determine the appropriate method for solving a real-world problem.
SL 3.4	The circle: radian measure of angles; length of an arc; area of a sector.	Express angle measures in different units and compare and contrast the value of using each measure.
		Deduce the relationship between the area of a circle and the area of a sector; circumference of a circle and arc length.
SL 3.6	The Pythagorean identity $\cos^2 \theta + \sin^2 \theta = 1$.	Explore trigonometric ratios to deduce various identities.
	Double angle identities for sine and cosine.	Calculate trigonometric ratios using appropriate known formulae.
	The relationship between trigonometric ratios.	Simplify trigonometric expressions and solve trig equations using appropriate known formulae.
AHL 3.9	Definition of the reciprocal trigonometric ratios sec θ , cosec θ and cot θ .	Explore trigonometric ratios to deduce various identities.
	Pythagorean identities: $1 + \tan^2 \theta = \sec^2 \theta$ and $1 + \cot^2 \theta = \csc^2 \theta$.	Calculate trigonometric ratios using appropriate known formulae.
	Treat 0 - cosec 0.	Simplify trigonometric expressions and solve trig equations using appropriate known formulae.
	TOK Connections	International Mindedness Connections
SL 3.1: What is an axiomatic system? Are axioms self-evident to everybody? SL 3.2: Is it ethical that Pythagoras gave his name to a theorem that may not have been his own creation? What		SL 3.2: Diagrams of Pythagoras' theorem occur in early Chinese and Indian manuscripts. The earliest references to trigonometry are in Indian mathematics; the use of triangulation to find the curvature of the Earth in order to settle a dispute between England and France over Newton's gravity.
criteria might we use to make such a judgment? SL 3.3: If the angles of a triangle can add up to less than 180°, 180° or more than 180°, what does this tell us about the nature of mathematical knowledge?		SL 3.3: The use of triangulation to find the curvature of the Earth in order to settle a dispute between England and France over Newton's gravity.
SL 3.4 : Which is a better measure of angle: radian or degree? What criteria can/do/should mathematicians use to make such decisions?		
HL 3.9 : What is the relationship between concepts and facts? To what extent do the concepts that we use shape the conclusions that we reach?		

Unit 1: Foundational Knowledge Section 1.C: Function Analysis

sessions

IB Topic Essential Understandings:

Number and algebra allow us to represent patterns, show equivalencies and make generalizations which enable us to model real-world situations. Algebra is an abstraction of numerical concepts and employs variables which allow us to solve mathematical problems.

Duration: 2 HL, 7 SL

Models are depictions of real-life events using expressions, equations or graphs while a function is defined as a relation or expression involving one or more variables. Creating different representations of functions to model the relationships between variables, visually and symbolically as graphs, equations and tables represent different ways to communicate mathematical ideas.

Geometry and trigonometry allows us to quantify the physical world, enhancing our spatial awareness in two and three dimensions. This topic provides us with the tools for analysis, measurement and transformation of quantities, movements and relationships.

IB Content-Specific Conceptual Understandings

Formulae are a generalization made on the basis of specific examples, which can then be extended to new examples.

Different representations of functions, symbolically and visually as graphs, equations and tables provide different ways to communicate mathematical relationships.

Moving between different forms to represent functions allows for deeper understanding and provides different approaches to problem solving.

IB Content		Standards-Aligned Objectives. Instruction and assessment will align to the following objectives, with IB command terms in bold	
SL 2.1	Different forms of the equation of a straight line. Gradient; intercepts. Lines with gradients m₁ and m₂ • Parallel lines m₁ = m₂. • Perpendicular lines m₁ × m₂ = ¬ 1.	Compare and contrast different forms of linear functions in order to identify when to use each. Find and interpret gradients and intercepts for linear functions. Determine the relationship between different types of lines (including parallel and perpendicular).	
SL 2.2	Concept of a function, domain, range and graph. Function notation, for example $f(x)$, $v(t)$, $C(n)$. The concept of a function as a mathematical model. Informal concept that an inverse function reverses or undoes the effect of a function. Inverse function as a reflection in the line $y = x$, and the notation $f^{-1}(x)$.	Explore the properties of functions, including domain, range and graph. Represent functions in various forms and convert between them. Explore the relationship between a function and its inverse.	

SL 2.3	The graph of a function; its equation $y = f(x)$.	Represent functions in various forms and convert between them.
	Creating a sketch from information given or a context, including transferring a graph from screen to paper.	Sketch the graph of a function using various approaches.
	Using technology to graph functions including their sums and differences.	Compose functions using mathematical operations.
SL 2.4	Key features of graphs.	Determine key features of graphs.
	Point of intersection of two curves or lines using technology.	Find the point of intersection of two curves using technology.
SL 2.5	Composite functions.	Compose functions using mathematical operations
	Identity function. Finding the inverse function f ⁻¹ (x)	Verify inverse functions using composition
SL 2.6	The quadratic function $f(x) = ax^2 + bx + c$: its graph, y-intercept (0, c). Axis of symmetry.	Compare and contrast different forms of quadratic functions in order to identify various features of the function.
	The form $f(x) = a(x - p)(x - q)$, x-intercepts $(p, 0)$ and $(q,0)$.	Sketch the graphs of quadratic functions.
	The form $f(x) = a (x - h)^2 + k$, vertex (h, k).	
SL 2.7	Solution of quadratic equations and inequalities. The quadratic formula.	Solve quadratic functions using various methods.
	The discriminant $\Delta = b^2 - 4ac$ and the nature of the roots, that is, two distinct real roots, two equal real roots, no real roots.	Determine the nature of the roots of a quadratic based on the discriminant.
SL 2.10	Solutions of equations, both graphically and analyticall Use of technology to solve a variety of equations, including those where there is no appropriate analytic approach.	variety of methods for a variety of real-world scenarios.
	Applications of graphing skills and solving equations th relate to real-life situations.	nat
SL 2.11	 Transformations of graphs. Translations: y = f(x) + b; y = f(x − a). Reflections (in both axes): y = − f(x); y = f(− x) Vertical stretch with scale factor p: y = p f(x). Horizontal stretch with scale factor ¹/_q: y = f(qx) Composite transformations. 	from a variety of approaches, including analytic and
	TOK Connections	International Mindedness Connections
solved algebraically and vice versa. What does this tell us about mathematical representation and mathematical (Sw		SL 2.2: The development of functions by Rene Descartes (France), Gottfried Wilhelm Leibnitz (Germany) and Leonhard Euler (Switzerland); the notation for functions was developed by a number of different mathematicians in the 17th and 18th

- **SL 2.2:** Do you think mathematics or logic should be classified as a language?
- **SL 2.3:** Does studying the graph of a function contain the same level of mathematical rigour as studying the function algebraically? What are the advantages and disadvantages of having different forms and symbolic language in mathematics?
- **SL 2.5:** Do you think mathematics or logic should be classified as a language?
- **SL 2.6:** Are there fundamental differences between mathematics and other areas of knowledge? If so, are these differences more than just methodological differences? **SL 2.10:** What assumptions do mathematicians make when they apply mathematics to real-life situations?

centuries—how did the notation we use today become internationally accepted?

SL 2.4: Bourbaki group analytical approach versus the Mandlebrot visual approach

Unit 2 Overview: Further Trigonometry

Duration: 13 HL, 28 SL sessions

IB Topic Essential Understandings:

Geometry and trigonometry allows us to quantify the physical world, enhancing our spatial awareness in two and three dimensions. This topic provides us with the tools for analysis, measurement and transformation of quantities, movements and relationships.

IB Content-Specific Conceptual Understandings

The properties of shapes depend on the dimension they occupy in space.

Volume and surface area of shapes are determined by formulae, or general mathematical relationships or rules expressed using symbols or variables.

The relationships between the length of the sides and the size of the angles in a triangle can be used to solve many problems involving position, distance, angles and area.

Equivalent measurement systems, such as degrees and radians, can be used for angles to facilitate ease of calculation.

Different representations of the values of trigonometric relationships, such as exact or approximate, may not be equivalent to one another.

The trigonometric functions of angles may be defined on the unit circle, which can visually and algebraically represent the periodic or symmetric nature of their values.

AHL

Position and movement can be modelled in three-dimensional space using vectors.

The relationships between algebraic, geometric and vector methods can help us to solve problems and quantify those positions and movements.

Section and IB Sub-Topics	HL Duration	SL Duration
A: Spatial Geometry SL 3.1	2 sessions	3 sessions
B: Triangle Trigonometry SL 3.2, SL 3.3	2 sessions	4 sessions
C: Circle Trigonometry SL 3.4	2 sessions	4 sessions
D: Trigonometry Ratios SL 3.5, SL 3.6	2 sessions	5 sessions
E: Trigonometric Graphs and Applications SL 3.7, AHL 3.11	3 sessions	7 sessions
F: Algebra and Trigonometry SL 3.8, AHL 3.9, AHL 3.10	2 sessions	5 sessions

Unit 2: Further Trigonometry Section 2.A: Spatial Geometry

IB Topic Essential Understandings:

Geometry and trigonometry allows us to quantify the physical world, enhancing our spatial awareness in two and three dimensions. This topic provides us with the tools for analysis, measurement and transformation of quantities, movements and relationships.

Duration: 2 HL, 3 SL sessions

IB Content-Specific Conceptual Understandings

The properties of shapes depend on the dimension they occupy in space.

Volume and surface area of shapes are determined by formulae, or general mathematical relationships or rules expressed using symbols or variables.

IB Content			Standards-Aligned Objectives. Instruction and assessment will align to the following objectives, with IB command terms in bold
SL 3.1 The distance between two points in three-dimensional space, and their midpoint. Volume and surface area of three-dimensional solids including right-pyramid, right cone, sphere, hemisphere and combinations of these solids. The size of an angle between two intersecting lines or between a line and a plane.		Calculate the distance between two points in a three-dimensional space, and their midpoint using a variety of methods. Calculate the volume and surface area of three-dimensional solids. Hence determine the parameters for the three-dimensional solid.	
TOK Connections			International Mindedness Connections
SL 3.1: What is an axiomatic system? Are axioms self-evident to everybody? SL 3.1: Demonstrate Cavalieri's principle using a variety of materials. Explore how would we calculate the volume if the cross-sectional areas are not the same? Revisit when learning integral calculus.		SL 3.1: H	istory of the distance formula. ow were the Pyramids built and how long would it take for ople to construct with modern technology? What would it

Unit 2: Further Trigonometry
Section 2.B: Triangle Trigonometry

IB Topic Essential Understandings:

Geometry and trigonometry allows us to quantify the physical world, enhancing our spatial awareness in two and three dimensions. This topic provides us with the tools for analysis, measurement and transformation of quantities, movements and relationships.

Duration: 2 HL, 4 SL sessions

IB Content-Specific Conceptual Understandings

The relationships between the length of the sides and the size of the angles in a triangle can be used to solve many problems involving position, distance, angles and area.

morning position, distance, angles and area.			
IB Content		Standards-Aligned Objectives. Instruction and assessment will align to the following objectives, with IB command terms in bold	
SL 3.2	Use of sine, cosine and tangent ratios to find the sides and angles of right-angled triangles. The sine rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$. The cosine rule: $c^2 = a^2 + b^2 - 2ab\cos C$; $\cos C = \frac{a^2 + b^2 - c^2}{2ab}$.		Determine when to use which trig ratio based on given information Apply the sine rule and cosine rule to solve non-right triangles Calculate the area of a non-right triangle
SL 3.3	Area of a triangle as $\frac{1}{2}ab\sin C$.		Apply rules of right and non-right triangles to solve real world problems including problems involving angles of elevation and depression Construct diagrams that represent written statements
	Construction of labelled diagrams from written statements.		Construct diagrams that represent written statements
	TOK Connections		International Mindedness Connections
theorem that may not have been his own creation? What criteria might we use to make such a judgment? Ind		Indian m Indian m the Earth	iagrams of Pythagoras' theorem occur in early Chinese and anuscripts. The earliest references to trigonometry are in athematics; the use of triangulation to find the curvature of in order to settle a dispute between England and France vton's gravity.

Unit 2: Further Trigonometry Section 2.C: Circle Trigonometry

IB Topic Essential Understandings:

Geometry and trigonometry allows us to quantify the physical world, enhancing our spatial awareness in two and three dimensions. This topic provides us with the tools for analysis, measurement and transformation of quantities, movements and relationships.

Duration: 2 HL, 4 SL sessions

IB Content-Specific Conceptual Understandings

Equivalent measurement systems, such as degrees and radians, can be used for angles to facilitate ease of calculation.

Different representations of the values of trigonometric relationships, such as exact or approximate, may not be equivalent to one another.

IB Content		Standards-Aligned Objectives. Instruction and assessment will align to the following objectives, with IB command terms in bold
SL 3.4	The circle: radian measure of angles; length of an arc; area of a sector.	Investigate the relationship between central angle, arc length and sector area. Solve for missing values of a circle using known information about arc length and/or sector area.
	TOK Connections	International Mindedness Connections
SL 3.4: Which is a better measure of angle: radian or degree? What criteria can/do/should mathematicians use to make such decisions?		SL 3.4: Seki Takakazu calculating π to ten decimal places; Hipparchus, Menelaus and Ptolemy; Why are there 360 degrees in a complete turn? Links to Babylonian mathematics

Unit 2: Further Trigonometry Section 2.D: Trigonometry Ratios

IB Topic Essential Understandings:

Geometry and trigonometry allows us to quantify the physical world, enhancing our spatial awareness in two and three dimensions. This topic provides us with the tools for analysis, measurement and transformation of quantities, movements and relationships.

Duration: 2 HL, 5 SL sessions

IB Content-Specific Conceptual Understandings

Different representations of the values of trigonometric relationships, such as exact or approximate, may not be equivalent to one another.

IB Content			Standards-Aligned Objectives. Instruction and assessment will align to the following objectives, with IB command terms in bold
SL 3.5	Definition of $\cos\theta$, $\sin\theta$ in terms of the unit circle Definition of $\tan\theta$ as $\sin\theta/\cos\theta$ Exact values of trigonometric ratios of 0, $\pi/6$, $\pi/4$, $\pi/\pi/2$ and their multiples. Extension of the sine rule to the ambiguous case The Pythagorean identity $\cos^2\theta + \sin^2\theta = 1$. Double an identities for sine and cosine. The relationship between trigonometric ratios	3 ,	Define trigonometric ratios from known relationships on the unit circle. Hence find trig ratios for special angles. Extend understanding of sine rule to ambiguous case Use known relationships between trig ratios to simplify trigonometric expressions, solve trigonometric equations and show that trig expressions are equivalent.
	The relationship between trigonometric ratios		
	TOK Connections		International Mindedness Connections
SL 3.5: Trigonometry was developed by successive civilizations and cultures. To what extent is mathematical knowledge embedded in particular traditions or bound to particular cultures? How have key events in the history of mathematics shaped its current form and methods?			The first work to refer explicitly to the sine as a function of the list the Aryabhatiya of Aryabhata (ca 510).

Unit 2: Further Trigonometry

Section 2.E: Trigonometric Graphs and Applications

IB Topic Essential Understandings:

Geometry and trigonometry allows us to quantify the physical world, enhancing our spatial awareness in two and three dimensions. This topic provides us with the tools for analysis, measurement and transformation of quantities, movements and relationships.

Duration: 3 HL, 7 SL sessions

IB Content-Specific Conceptual Understandings

The relationships between the length of the sides and the size of the angles in a triangle can be used to solve many problems involving position, distance, angles and area.

IB Content			Standards-Aligned Objectives. Instruction and assessment will align to the following objectives, with IB command terms in bold
SL 3.7	The circular functions sinx, cosx, and tanx; amplitude, their periodic nature, and their graphs Composite functions of the form $f(x) = a\sin(b(x+c)) + d$.		Graph trigonometric functions for given domains.
			Transform trigonometric functions analytically and graphically
	Transformations.		Model real-world applications using trigonometric functions
	Real-life contexts.		
AHL 3.11	Relationships between trigonometric functions and th symmetry properties of their graphs.	e	Explore relationship between trigonometric functions and symmetry
	TOK Connections		International Mindedness Connections
this tell us	SL 3.7: Music can be expressed using mathematics. What does this tell us about the relationship between music and mathematics?		one applicable
AHL 3.11: Mathematics and knowledge claims: how can there be an infinite number of discrete solutions to an equation?			

Unit 2: Further Trigonometry

Section 2.F: Algebra and Trigonometry

IB Topic Essential Understandings:

Geometry and trigonometry allows us to quantify the physical world, enhancing our spatial awareness in two and three dimensions. This topic provides us with the tools for analysis, measurement and transformation of quantities, movements and relationships.

Duration: 2 HL, 5 SL sessions

IB Content-Specific Conceptual Understandings

The relationships between the length of the sides and the size of the angles in a triangle can be used to solve many problems involving position, distance, angles and area.

Equivalent measurement systems, such as degrees and radians, can be used for angles to facilitate ease of calculation.

Different representations of the values of trigonometric relationships, such as exact or approximate, may not be equivalent to one another.

IB Content			Standards-Aligned Objectives. Instruction and assessment will align to the following objectives, with IB command terms in bold
SL 3.8	Solving trigonometric equations in a finite interval, graphically and analytically Equations leading to quadratic equations in sinx, costanx.		Solve trigonometric equations graphically and analytically Represent trigonometric equations in the form of quadratic equations; hence use quadratic properties to solve trigonometric equations.
AHL 3.9	Definition of the reciprocal trigonometric ratios $\sec\theta$ cosec θ and $\cot\theta$. Pythagorean identities: $1 + \tan^2\theta = \sec^2\theta$ and $1 + \cos^2\theta$. The inverse functions $f(x) = \arcsin x$, $f(x) = \arccos x$, $f(x) = \arccos x$, their domains and ranges; their graphs	tot² θ =	Sketch inverse trig functions and compare and contrast their domains with their inverses. Use known relationships between trig ratios to simplify trigonometric expressions, solve trigonometric equations and show that trig expressions are equivalent.
AHL 3.10	Compound angle identities. Double angle identity for tan.		Investigate relationship between angle sums and differences. Use the double angle identity to find trigonometric ratios, simplify expressions, solve equations and verify equations.
	TOK Connections		International Mindedness Connections
facts? To v			9: The origin of degrees in the mathematics of Mesopotamia y we use minutes and seconds for time; the origin of the ne.

Unit 3 Overview: Further Functions Duration: 22 HL, 40 SL sessions

IB Topic Essential Understandings:

Models are depictions of real-life events using expressions, equations or graphs while a function is defined as a relation or expression involving one or more variables. Creating different representations of functions to model the relationships between variables, visually and symbolically as graphs, equations and tables represent different ways to communicate mathematical ideas.

IB Content-Specific Conceptual Understandings

Logarithm laws provide the means to find inverses of exponential functions which model real-life situations.

Different representations of functions, symbolically and visually as graphs, equations and tables provide different ways to communicate mathematical relationships.

The parameters in a function or equation correspond to geometrical features of a graph and can represent physical quantities in spatial dimensions.

Moving between different forms to represent functions allows for deeper understanding and provides different approaches to problem solving.

Our spatial frame of reference affects the visible part of a function and by changing this "window" can show more or less of the function to best suit our needs.

Equivalent representations of quadratic functions can reveal different characteristics of the same relationship.

Functions represent mappings that assign to each value of the independent variable (input) one and only one dependent variable (output).

AHL

Extending results from a specific case to a general form can allow us to apply them to a larger system.

Patterns can be identified in behaviours which can give us insight into appropriate strategies to model or solve them.

Section and IB Sub-Topics	HL Duration	SL Duration
A: Function Analysis SL 2.2, SL 2.5, AHL 2.14, AHL 2.15, AHL 2.16	5 sessions	13 sessions
B: Linear Functions SL 2.1, SL 2.3, SL 2.4, SL 2.10, SL 2.11	1 sessions	5 sessions
C: Quadratic and Other Polynomial Functions SL 2.3, SL 2.4, SL 2.6, SL 2.7, SL 2.10, SL 2.11, AHL 2.12	4 sessions	9 sessions
D: Logarithms and Exponential Functions SL 2.3, SL 2.4, SL 2.9, SL 2.10, SL 2.11	8 sessions	13 sessions
E: Rational Functions (HL only) SL 2.3, SL 2.4, SL 2.8, SL 2.10, SL 2.11, AHL 2.13	4 sessions	n/a

Unit 3: Further Functions Section 3.A: Function Analysis

IB Topic Essential Understandings:

Models are depictions of real-life events using expressions, equations or graphs while a function is defined as a relation or expression involving one or more variables. Creating different representations of functions to model the relationships between variables, visually and symbolically as graphs, equations and tables represent different ways to communicate mathematical ideas.

Duration: 5 HL, 13 SL sessions

IB Content-Specific Conceptual Understandings

Different representations of functions, symbolically and visually as graphs, equations and tables provide different ways to communicate mathematical relationships.

The parameters in a function or equation correspond to geometrical features of a graph and can represent physical quantities in spatial dimensions.

Moving between different forms to represent functions allows for deeper understanding and provides different approaches to problem solving.

Our spatial frame of reference affects the visible part of a function and by changing this "window" can show more or less of the function to best suit our needs.

Functions represent mappings that assign to each value of the independent variable (input) one and only one dependent variable (output).

AHL

Extending results from a specific case to a general form can allow us to apply them to a larger system.

Patterns can be identified in behaviours which can give us insight into appropriate strategies to model or solve them.

IB Content		Standards-Aligned Objectives. Instruction and assessment will align to the following objectives, with IB command terms in bold
SL 2.2	Concept of a function, domain, range and graph. Function notation, for example $f(x)$, $v(t)$, $C(n)$. The concept of a function as a mathematical model. Informal concept that an inverse function reverses or undoes the effect of a function. Inverse function as a reflection in the line $y = x$, and the notation $f^{-1}(x)$	Understand the concept of a function and its inverse that reverses the effect of the function. Model functions & inverse functions as a reflection in the line y=x.
SL 2.5	Composite functions Identity function. Finding the inverse function $f^{-1}(x)$.	Compose functions algebraically. Find inverse functions analytically. Verify inverse functions using compositions.

AHL 2.14	Odd and even functions Finding the inverse function, $f^{-1}(x)$, including dorrestriction.	main	Explore rotations and reflections of functions and hence classify them as odd, even or neither.	
	Self-inverse functions			
AHL 2.15	Solutions of $g(x) \ge f(x)$, both graphically and ana	lytically	Determine solutions of polynomial inequalities graphically and analytically.	
AHL 2.16	The graphs of the functions, $y = f(x) $ and $y = f(1/f(x))$, $y = f(ax + b)$, $y = [f(x)]^2$. Solution of modulus equations and inequalities	x), y =	Graph functions and Interpret key features of their graphs. Find the solutions of modulus equations and inequalities.	
TOK Connections Inte		Internatio	International Mindedness Connections	
classified as a language? Gott: (Swit AHL 2.14 If systems of notation and measurement are diffe		Gottfried (Switzerla different i	e development of functions by Rene Descartes (France), Wilhelm Leibnitz (Germany) and Leonhard Euler nd); the notation for functions was developed by a number of mathematicians in the 17th and 18th centuries—how did the we use today become internationally accepted?	
AHL 2.15 Are there differences in terms of value that different cultures ascribe to mathematics, or to the relative value that they ascribe to different areas of knowledge? AHL 2.15 Are there differences in terms of value that differences in terms of value that they ascribe to different areas of knowledge?		different i notation v	The notation for functions was developed by a number of mathematicians in the 17th and 18th centuries. How did the we use today become internationally accepted? The Bourbaki group analytic approach versus Mandlebrot proach.	

Unit 3: Further Functions Section 3.B: Linear Functions

IB Topic Essential Understandings:

Models are depictions of real-life events using expressions, equations or graphs while a function is defined as a relation or expression involving one or more variables. Creating different representations of functions to model the relationships between variables, visually and symbolically as graphs, equations and tables represent different ways to communicate mathematical ideas.

Duration: 1 HL, 5 SL sessions

IB Content-Specific Conceptual Understandings

Different representations of functions, symbolically and visually as graphs, equations and tables provide different ways to communicate mathematical relationships.

The parameters in a function or equation correspond to geometrical features of a graph and can represent physical quantities in spatial dimensions.

Moving between different forms to represent functions allows for deeper understanding and provides different approaches to problem solving.

Our spatial frame of reference affects the visible part of a function and by changing this "window" can show more or less of the function to best suit our needs.

Functions represent mappings that assign to each value of the independent variable (input) one and only one dependent variable (output).

AHL

Extending results from a specific case to a general form can allow us to apply them to a larger system.

Patterns can be identified in behaviours which can give us insight into appropriate strategies to model or solve them.

IB Conter	nt	Standards-Aligned Objectives. Instruction and assessment will align to the following objectives, with IB command terms in bold
SL 2.1	Different forms of the equation of a straight line.	Distinguish between the different forms of the equation of a straight line.
	Gradient; intercepts. Lines with gradients m_1 and m_2 .	Identify the key features of linear equations, including gradient and intercepts.
	Parallel lines $m_1 = m_2$. Perpendicular lines $m_1 \times m_2 = -1$.	Understand the relationship between parallel and perpendicular lines.
SL 2.3	The graph of a function; its equation y = f(x). Creating a sketch from information given or a context, including transferring a graph from screen to paper.	Graph linear equations by hand and through the use of technology.
	Using technology to graph functions including their sums and differences	

SL 2.4	Determine key features of graphs.		Determine key features of graphs.
	Finding the point of intersection of two curves or lines technology.	using	Find the points of intersection of two curves or lines using technology.
SL 2.10	Solving equations, both graphically and analytically. Use of technology to solve a variety of equations, inclutions where there is no appropriate analytic approach. Applications of graphing skills and solving equations the		Solve equations, both graphically and analytically and through the use of technology. Model and analyze real life applications.
	relate to real-life situations.		
SL 2.11	Transformations of graphs.		Demonstrate transformations of graphs.
	Translations: $y = f(x) + b$; $y = f(x - a)$. Reflections (in both axes): $y = -f(x)$; $y = f(-x)$. Vertical stretch with scale factor p: $y = p f(x)$. Horizontal stretch with scale factor 1 q: $y = f(qx)$.		Describe composite transformations.
	Composite transformations.		
	TOK Connections		International Mindedness Connections
SL 2.1 Descartes showed that geometric problems could be solved algebraically and vice versa. What does this tell us about mathematical representation and mathematical knowledge? SL 2.3 Does studying the graph of a function contain the same level of mathematical rigour as studying the function algebraically? What are the advantages and disadvantages of having different forms and symbolic language in mathematics? SL 2.10 What assumptions do mathematicians make when they apply mathematics to real-life situations?			Bourbaki group analytical approach versus the Mandlebrot pproach.

Unit 3: Further Functions

Section 3.C: Quadratic and Other Polynomial Functions

IB Topic Essential Understandings:

Models are depictions of real-life events using expressions, equations or graphs while a function is defined as a relation or expression involving one or more variables. Creating different representations of functions to model the relationships between variables, visually and symbolically as graphs, equations and tables represent different ways to communicate mathematical ideas.

Duration: 4 HL, 9 SL sessions

IB Content-Specific Conceptual Understandings

Different representations of functions, symbolically and visually as graphs, equations and tables provide different ways to communicate mathematical relationships.

The parameters in a function or equation correspond to geometrical features of a graph and can represent physical quantities in spatial dimensions.

Moving between different forms to represent functions allows for deeper understanding and provides different approaches to problem solving.

Our spatial frame of reference affects the visible part of a function and by changing this "window" can show more or less of the function to best suit our needs.

Equivalent representations of quadratic functions can reveal different characteristics of the same relationship.

Functions represent mappings that assign to each value of the independent variable (input) one and only one dependent variable (output).

AHL

Extending results from a specific case to a general form can allow us to apply them to a larger system.

Patterns can be identified in behaviours which can give us insight into appropriate strategies to model or solve them.

IB Content		Standards-Aligned Objectives. Instruction and assessment will align to the following objectives, with IB command terms in bold
SL 2.3	The graph of a function; its equation y = f(x). Creating a sketch from information given or a context, including transferring a graph from screen to paper. Using technology to graph functions including their sums and differences	Graph polynomial equations by hand and through the use of technology.
SL 2.4	Determine key features of graphs. Finding the point of intersection of two curves or lines using technology.	Determine key features of graphs. Find the point of intersection of two curves or lines using technology.

SL 2.6	The quadratic function $f(x) = ax^2 + bx + c$: its graph, -intercept (0, c). Axis of symmetry. The form $f(x) = a$	a(x - p)(x	Convert between forms of quadratic functions.
	- q), x-intercepts (p, 0) and (q, 0). The form f(x) = a + k, vertex (h, k)	(x - h) ²	Identify the key features of the graphs of quadratic functions
SL 2.7	Solution of quadratic equations and inequalities. The quadratic formula.	ne	Solve quadratic equations using the Quadratic Formula.
	The discriminant $\Delta = b^2 - 4ac$ and the nature of the that is, two distinct real roots, two equal real roots, roots.		Evaluate the discriminant of a quadratic in order to analyze the roots of a function.
SL 2.10	Solving equations, both graphically and analytically		Solve equations, both graphically, analytically and using technology where there is an absence of an analytic
	Use of technology to solve a variety of equations, in those where there is no appropriate analytic appro	_	approach.
	Applications of graphing skills and solving equation relate to real-life situations.	s that	Model and analyze real life applications.
SL 2.11	Transformations of graphs.		Demonstrate transformations of graphs.
	Translations: $y = f(x) + b$; $y = f(x - a)$. Reflections (in both axes): $y = -f(x)$; $y = f(-x)$. Vertical stretch with scale factor $p: y = p f(x)$. Horizontal stretch with scale factor $1 q: y = f(qx)$.		Describe composite transformations.
	Composite transformations.		
AHL 2.12	Polynomial functions, their graphs and equations; zoots and factors. The factor and remainder theore		Graph polynomial functions.
	Sum and product of the roots of polynomial equati	ons.	Identify the roots and zeros of polynomial functions using the remainder and factor theorem.
			Find the sum and product of the roots of polynomial equations.
	TOK Connections		International Mindedness Connections
SL 2.3 Does studying the graph of a function contain the same level of mathematical rigour as studying the function algebraically? What are the advantages and disadvantages of having different forms and symbolic language in mathematics?		visual ap SL 2.7 T ² /2 . Sulk	burbaki group analytical approach versus the Mandlebrot proach. The Babylonian method of multiplication: $ab = (a + b^2 - a^2 - ba)$ The Sutras in ancient India and the Bakhshali Manuscript and an algebraic formula for solving quadratic equations.
SL 2.6 Are there fundamental differences between mathematics and other areas of knowledge? If so, are these differences more than just methodological differences?		Containe	a an angest are formula for solving quadratic equations.

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Unit 3: Further Functions

Section 3.D: Logarithms and Exponential Functions

IB Topic Essential Understandings:

Models are depictions of real-life events using expressions, equations or graphs while a function is defined as a relation or expression involving one or more variables. Creating different representations of functions to model the relationships between variables, visually and symbolically as graphs, equations and tables represent different ways to communicate mathematical ideas.

Duration: 8 HL, 13 SL sessions

IB Content-Specific Conceptual Understandings

Logarithm laws provide the means to find inverses of exponential functions which model real-life situations.

Different representations of functions, symbolically and visually as graphs, equations and tables provide different ways to communicate mathematical relationships.

The parameters in a function or equation correspond to geometrical features of a graph and can represent physical quantities in spatial dimensions.

Moving between different forms to represent functions allows for deeper understanding and provides different approaches to problem solving.

Our spatial frame of reference affects the visible part of a function and by changing this "window" can show more or less of the function to best suit our needs.

Equivalent representations of quadratic functions can reveal different characteristics of the same relationship.

Functions represent mappings that assign to each value of the independent variable (input) one and only one dependent variable (output).

AHL

Extending results from a specific case to a general form can allow us to apply them to a larger system.

Patterns can be identified in behaviours which can give us insight into appropriate strategies to model or solve them.

IB Content		Standards-Aligned Objectives. Instruction and assessment will align to the following objectives, with IB command terms in bold	
SL 2.3	The graph of a function; its equation y = f(x). Creating a sketch from information given or a context, including transferring a graph from screen to paper. Using technology to graph functions including their sums and differences	Graph logarithmic and exponential equations by hand and through the use of technology.	
SL 2.4	Determine key features of graphs.	Determine key features of graphs.	

	Finding the point of intersection of two curves or line using technology.	S	Find the point of intersection of two curves or lines using technology.	
SL 2.9	Exponential functions and their graphs: $f(x) = a^x$, $a > 0$ $f(x) = e^x$),	Analyze exponential and logarithmic functions.	
	Logarithmic functions and their graphs: $f(x) = log_a x$, $x = f(x) = ln x$, $x > 0$.	> 0,		
SL 2.10	Solving equations, both graphically and analytically. Use of technology to solve a variety of equations, including those where there is no appropriate analyti approach. Applications of graphing skills and solving equations t relate to real-life situations.		Solve equations, both graphically, analytically and using technology where there is an absence of an analytic approach. Model and analyze real life applications.	
SL 2.11	SL 2.11 Transformations of graphs.		Demonstrate transformations of graphs.	
	Translations: $y = f(x) + b$; $y = f(x - a)$. Reflections (in both axes): $y = -f(x)$; $y = f(-x)$. Vertical stretch with scale factor $p: y = p(x)$. Horizontal stretch with scale factor $p: y = p(x)$.		Describe composite transformations.	
	Composite transformations.			
	TOK Connections	International Mindedness Connections		
SL 2.3 Does studying the graph of a function contain the same level of mathematical rigour as studying the function algebraically? What are the advantages and disadvantages of having different forms and symbolic language in mathematics? SL 2.9 What role do "models" play in mathematics? Do they play a different role in mathematics compared to their role in other areas of knowledge?		SL 2.4 Bourbaki group analytical approach versus the Mandlebrot visual approach.		
SL 2.10 What assumptions do mathematicians make when they apply mathematics to real-life situations?				

Unit 3: Further Functions

Section 3.E: Rational Functions (HL only)

Duration: 4 HL, n/a SL sessions

IB Topic Essential Understandings:

Models are depictions of real-life events using expressions, equations or graphs while a function is defined as a relation or expression involving one or more variables. Creating different representations of functions to model the relationships between variables, visually and symbolically as graphs, equations and tables represent different ways to communicate mathematical ideas.

IB Content-Specific Conceptual Understandings

Different representations of functions, symbolically and visually as graphs, equations and tables provide different ways to communicate mathematical relationships.

The parameters in a function or equation correspond to geometrical features of a graph and can represent physical quantities in spatial dimensions.

Moving between different forms to represent functions allows for deeper understanding and provides different approaches to problem solving.

Our spatial frame of reference affects the visible part of a function and by changing this "window" can show more or less of the function to best suit our needs.

Equivalent representations of quadratic functions can reveal different characteristics of the same relationship.

Functions represent mappings that assign to each value of the independent variable (input) one and only one dependent variable (output).

AHL

Extending results from a specific case to a general form can allow us to apply them to a larger system.

Patterns can be identified in behaviours which can give us insight into appropriate strategies to model or solve them.

IB Content		Standards-Aligned Objectives. Instruction and assessment will align to the following objectives, with IB command terms in bold	
SL 2.3	The graph of a function; its equation y = f(x). Creating a sketch from information given or a context, including transferring a graph from screen to paper. Using technology to graph functions including their sums and differences	Graph rational equations by hand and through the use of technology.	
SL 2.4	Determine key features of graphs. Finding the point of intersection of two curves or lines using technology.	Determine key features of graphs. Find the point of intersection of two curves or lines using technology.	

The reciprocal function $f(x) = \frac{1}{x}$, $x \ne 0$: its graph and self-inverse nature. Rational functions of the form $f(x) = \frac{ax+b}{cx+d}$ and their graphs. Equations of vertical and horizontal asymptotes		Analyze the reciprocal function. Identify the equations of the vertical and horizontal asymptotes.
Equations of vertical and horizontal asymptotes		
Solving equations, both graphically and analytically. Use of technology to solve a variety of equations, including those where there is no appropriate analytic approach. Applications of graphing skills and solving equations that relate to real-life situations.		Solve equations, both graphically, analytically and using technology where there is an absence of an analytic approach. Model and analyze real life applications.
.11 Transformations of graphs. Translations: y = f(x) + b; y = f(x - a). Reflections (in both axes): y = - f(x); y = f(-x). Vertical stretch with scale factor p: y = p f(x). Horizontal stretch with scale factor 1 q: y = f(qx). Composite transformations.		Demonstrate transformations of graphs. Describe composite transformations.
Rational functions of the form $f(x) = \frac{ax + b}{cx^2 + dx + e}$, and $f(x) = \frac{ax^2 + bx + c}{dx + e}$	(x) =	Analyze rational functions.
TOK Connections	International Mindedness Connections	
SL 2.3 Does studying the graph of a function contain the same level of mathematical rigour as studying the function algebraically? What are the advantages and disadvantages of having different forms and symbolic language in mathematics? SL 2.8 What are the implications of accepting that mathematical knowledge changes over time? SL 2.10 What assumptions do mathematicians make when they apply mathematics to real-life situations? AHL 2.13 Does studying the graph of a function contain the same level of mathematical rigour as studying the function		Bourbaki group analytical approach versus the Mandlebrot approach.
- : h / r	Use of technology to solve a variety of equations, including those where there is no appropriate analyt approach. Applications of graphing skills and solving equations relate to real-life situations. Transformations of graphs. Translations: $y = f(x) + b$; $y = f(x - a)$. Reflections (in both axes): $y = -f(x)$; $y = f(-x)$. Vertical stretch with scale factor $p: y = p(x)$. Horizontal stretch with scale factor $p: y = p(x)$. Composite transformations. Rational functions of the form $p(x) = \frac{ax + b}{cx^2 + dx + e}$, and $p(x) = \frac{ax^2 + bx + c}{dx + e}$. TOK Connections Studying the graph of a function contain the same mematical rigour as studying the function? What are the advantages and disadvantages of tent forms and symbolic language in mathematics? are the implications of accepting that all knowledge changes over time? It assumptions do mathematicians make when they matics to real-life situations?	Use of technology to solve a variety of equations, including those where there is no appropriate analytic approach. Applications of graphing skills and solving equations that relate to real-life situations. Transformations of graphs. Translations: $y = f(x) + b$; $y = f(x - a)$. Reflections (in both axes): $y = -f(x)$; $y = f(-x)$. Vertical stretch with scale factor $p: y = p(x)$. Horizontal stretch with scale factor $p: y = p(x)$. Composite transformations. Rational functions of the form $p(x) = \frac{ax + b}{cx^2 + dx + e}$, and $p(x) = \frac{ax^2 + bx + c}{dx + e}$ TOK Connections Studying the graph of a function contain the same nematical rigour as studying the function? What are the advantages and disadvantages of ent forms and symbolic language in mathematics? are the implications of accepting that all knowledge changes over time? It assumptions do mathematicians make when they matics to real-life situations? es studying the graph of a function contain the function contain the function? What are the advantages and disadvantages of

Unit 4 Overview: Complex Numbers (HL Only)

Duration: 18 HL, 0 SL sessions

IB Topic Essential Understandings:

Number and algebra allow us to represent patterns, show equivalencies and make generalizations which enable us to model real-world situations. Algebra is an abstraction of numerical concepts and employs variables which allow us to solve mathematical problems.

IB Content-Specific Conceptual Understandings

AHL

Section and IB Sub-Topics	HL Duration	SL Duration
A: Properties of & Operations with Complex Numbers AHL 1.12, AHL 1.13	5 sessions	N/A
B: Graphs of Complex Numbers AHL 1.12	7 sessions	N/A
C: Roots of Complex Numbers AHL 1.14	6 sessions	N/A

Unit 4: Complex Numbers (HL only)

Section 4.A: Properties of & Operations with Complex Numbers

IB Topic Essential Understandings:

Number and algebra allow us to represent patterns, show equivalencies and make generalizations which enable us to model real-world situations. Algebra is an abstraction of numerical concepts and employs variables which allow us to solve mathematical problems.

Duration: 5 HL, 0 SL sessions

IB Content-Specific Conceptual Understandings

AHL

IB Content		Standards-Aligned Objectives. Instruction and assessment will align to the following objectives, with IB command terms in bold	
AHL 1.12	Complex numbers: the number i, where $i^2 = -1$.	Understand complex numbers as a distinct set of numbers.	
	Cartesian form z = a + bi; the terms real part, imaginary part, conjugate, modulus and argument.	Represent complex numbers in cartesian form and distinguish the real parts from the imaginary parts.	
AHL 1.13	Modulus–argument (polar) form: $z = r(\cos\theta + i\sin\theta) = rcis\theta$	Represent complex numbers in cartesian, polar and Euler forms and transform between each.	
	Euler form: z = re ^{iθ} Sums, products and quotients in Cartesian, polar or Euler forms and their geometric interpretation.	Interpret geometrically the sums, products and quotients of complex numbers in Cartesian, polar or Euler forms.	
	TOK Connections	International Mindedness Connections	
AHL 1.12 How does language shape knowledge? For example, do the words "imaginary" and "complex" make the concepts more difficult than if they had different names?		n/a	
AHL 1.13 Why might it be said that $e^{i\pi} + 1 = 0$ is beautiful? What is the place of beauty and elegance in mathematics? What about the place of creativity?			

Unit 4 : Complex Numbers (HL only)
Section 4.B: Graphs of Complex Numbers

IB Topic Essential Understandings:

Number and algebra allow us to represent patterns, show equivalencies and make generalizations which enable us to model real-world situations. Algebra is an abstraction of numerical concepts and employs variables which allow us to solve mathematical problems.

Duration: 7 HL, 0 SL sessions

IB Content-Specific Conceptual Understandings

AHL

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IB Content		Standards-Aligned Objectives. Instruction and assessment will align to the following objectives, with IB command terms in bold	
AHL 1.12	The complex plane.	Compare and contrast the complex plane with the coordinate plane. Represent complex numbers on a coordinate plane.	
TOK Connections		International Mindedness Connections	
AHL 1.12 How does language shape knowledge? For example, do the words "imaginary" and "complex" make the concepts more difficult than if they had different names?		n/a	

Unit 4: Complex Numbers (HL only)
Section 4.C: Roots of Complex Numbers

IB Topic Essential Understandings:

Number and algebra allow us to represent patterns, show equivalencies and make generalizations which enable us to model real-world situations. Algebra is an abstraction of numerical concepts and employs variables which allow us to solve mathematical problems.

Duration: 6 HL, 0 SL sessions

IB Content-Specific Conceptual Understandings

AHL

IB Content		Standards-Aligned Objectives. Instruction and assessment will align to the following objectives, with IB command terms in bold		
AHL 1.14	Complex conjugate roots of quadratic and polynomial equations with real coefficients. Complex roots occur in conjugate pairs. De Moivre's theorem and its extension to rational exponents. Powers and roots of complex numbers.	Solve polynomials with complex roots Explain why complex roots of polynomials occur in conjugate pairs. Apply De Moivre's theorem to simplify rational expressions Simplify complex numbers with rational exponents		
TOK Connections		International Mindedness Connections		
AHL 1.14 Could we ever reach a point where everything important in a mathematical sense is known? Reflect on the creation of complex numbers before their applications were known.		n/a		

Unit 5 Overview: Combinatorics Duration: 10 HL, 20 SL sessions

IB Topic Essential Understandings:

Number and algebra allow us to represent patterns, show equivalencies and make generalizations which enable us to model real-world situations. Algebra is an abstraction of numerical concepts and employs variables which allow us to solve mathematical problems

IB Content-Specific Conceptual Understandings

Patterns in numbers inform the development of algebraic tools that can be applied to find unknowns.

The binomial theorem is a generalization which provides an efficient method for expanding binomial expressions.

AHL

Proof serves to validate mathematical formulae and the equivalence of identities.

Section and IB Sub-Topics	HL Duration	SL Duration
A: Counting Principles, including Permutations & Combinations AHL 1.10	5 sessions	9 sessions
B: Pascal's Triangle SL 1.9	2 sessions	3 sessions
C: Binomial Theorem SL 1.9, AHL 1.10	3 sessions	8 sessions

Unit 5: Combinatorics

Section 5.A: Counting Principles, including Permutations & Combinations

IB Topic Essential Understandings:

Number and algebra allow us to represent patterns, show equivalencies and make generalizations which enable us to model real-world situations. Algebra is an abstraction of numerical concepts and employs variables which allow us to solve mathematical problems

Duration: 5 HL, 9 SL sessions

IB Content-Specific Conceptual Understandings

Patterns in numbers inform the development of algebraic tools that can be applied to find unknowns.

AHL

IB Content		Standards-Aligned Objectives. Instruction and assessment will align to the following objectives, with IB command terms in bold	
AHL 1.10	Counting principles, including permutations and combinations.	Apply counting principles. Distinguish between real world situations that can be represented using combinations and permutations.	
	TOK Connections	International Mindedness Connections	
1	What counts as understanding in mathematics? Is it just getting the right answer?	AHL 1.10 The properties of "Pascal's triangle" have been known in a number of different cultures long before Pascal (for example the Chinese mathematician Yang Hui).	

Unit 5 : Combinatorics Section 5.B: Pascal's Triangle

IB Topic Essential Understandings:

Number and algebra allow us to represent patterns, show equivalencies and make generalizations which enable us to model real-world situations. Algebra is an abstraction of numerical concepts and employs variables which allow us to solve mathematical problems

Duration: 2 HL, 3 SL sessions

IB Content-Specific Conceptual Understandings

Patterns in numbers inform the development of algebraic tools that can be applied to find unknowns.

The binomial theorem is a generalization which provides an efficient method for expanding binomial expressions.

IB Content		Standards-Aligned Objectives. Instruction and assessment will align to the following objectives, with IB command terms in bold	
SL 1.9 Use of Pascal's triangle and nCr		Generate Pascal's triangle and apply in order to identify binomial coefficients.	
TOK Connections			
	TOK Connections	International Mindedness Connections	

Unit 5 : Combinatorics

Section 5.C: Binomial Theorem Duration: 3 HL, 8 SL sessions

IB Topic Essential Understandings:

Number and algebra allow us to represent patterns, show equivalencies and make generalizations which enable us to model real-world situations. Algebra is an abstraction of numerical concepts and employs variables which allow us to solve mathematical problems

IB Content-Specific Conceptual Understandings

Patterns in numbers inform the development of algebraic tools that can be applied to find unknowns.

The binomial theorem is a generalization which provides an efficient method for expanding binomial expressions.

AHL

IB Content		Standards-Aligned Objectives. Instruction and assessment will align to the following objectives, with IB command terms in bold	
SL 1.9	The binomial theorem: expansion of $(a + b)^n$, $n \in \mathbb{N}$.	Apply Pascal's triangle and the Binomial Theorem to expand binomials.	
AHL 1.10 Extension of the binomial theorem to fractional & negative indices, ie $(a + b)^n$, $n \in Q$.		Extend the Binomial Theorem to expand binomials with fractional and negative indices.	
TOK Connections		International Mindedness Connections	
SL 1.9 How have notable individuals shaped the development of mathematics as an area of knowledge? Consider Pascal and "his" triangle.			
of mathen	natics as an area of knowledge? Consider Pascal	AHL 1.10 The properties of "Pascal's triangle" have been known in a number of different cultures long before Pascal (for example the Chinese mathematician Yang Hui).	

Unit 6 Overview: Reasoning and Proof (HL only)

Duration: 18 HL, 0 SL sessions

IB Topic Essential Understandings:

Number and algebra allow us to represent patterns, show equivalencies and make generalizations which enable us to model real-world situations. Algebra is an abstraction of numerical concepts and employs variables which allow us to solve mathematical problems.

IB Content-Specific Conceptual Understandings

Numbers and formulae can appear in different, but equivalent, forms, or representations, which can help us to establish identities.

AHL

Section and IB Sub-Topics	HL Duration	SL Duration
A: Proof by Deduction SL 1.6	5 sessions	N/A
B: Proof by Contradiction & Counterexample AHL 1.15	6 sessions	N/A
C: Proof by Mathematical Induction AHL 1.15	7 sessions	N/A

Unit 6: Reasoning and Proof Section 6.A: Proof by Deduction

IB Topic Essential Understandings:

Number and algebra allow us to represent patterns, show equivalencies and make generalizations which enable us to model real-world situations. Algebra is an abstraction of numerical concepts and employs variables which allow us to solve mathematical problems.

Duration: 5 HL, 0 SL sessions

IB Content-Specific Conceptual Understandings

Numbers and formulae can appear in different, but equivalent, forms, or representations, which can help us to establish identities

AHL

IB Content		Standards-Aligned Objectives. Instruction and assessment will align to the following objectives, with IB command terms in bold	
SL 1.6	Simple deductive proof, numerical and algebraic; how to lay out a left-hand side to right-hand side (LHS to RHS) proof. The symbols and notation for equality and identity.	Perform simple deductive proofs. Understand symbols and notations used in equations and identities.	
TOK Connections		International Mindedness Connections	
SL 1.6 Is mathematical reasoning different from scientific reasoning, or reasoning in other Areas of Knowledge?		n/a	

Unit 6: Reasoning and Proof

Section 6.B: Proof by Contradiction & Counterexample

IB Topic Essential Understandings:

Number and algebra allow us to represent patterns, show equivalencies and make generalizations which enable us to model real-world situations. Algebra is an abstraction of numerical concepts and employs variables which allow us to solve mathematical problems.

Duration: 6 HL, 0 SL sessions

IB Content-Specific Conceptual Understandings

AHL

IB Content		Standards-Aligned Objectives. Instruction and assessment will align to the following objectives, with IB command terms in bold	
AHL 1.15	Proof by contradiction. Use of a counterexample to show that a statement i not always true.	Prove that a statement is true using contradiction or a counterexample	

TOK Connections	International Mindedness Connections
AHL 1.15 What is the role of the mathematical community in determining the validity of a mathematical proof? Do proofs provide us with completely certain knowledge? What is the difference between the inductive method in science and proof by induction in mathematics?	AHL 1.15 How did the Pythagoreans find out that 2 is irrational?

Unit 6: Reasoning and Proof

Section 6.C: Proof by Mathematical Induction

Duration: 7 HL, 0 SL sessions

IB Topic Essential Understandings:

Number and algebra allow us to represent patterns, show equivalencies and make generalizations which enable us to model real-world situations. Algebra is an abstraction of numerical concepts and employs variables which allow us to solve mathematical problems

IB Content-Specific Conceptual Understandings

AHL

Thos serves to variable matternation formulae and the equivalence of facilities.			
IB Content		Standards-Aligned Objectives. Instruction and assessment will align to the following objectives, with IB command terms in bold	
AHL 1.15 Proof by mathematical induction.		Prove statements to be true using mathematical induction.	
TOK Connections		International Mindedness Connections	
AHL 1.15 What is the role of the mathematical community in determining the validity of a mathematical proof? Do proofs provide us with completely certain knowledge? What is the difference between the inductive method in science and proof by induction in mathematics?		AHL 1.15 How did the Pythagoreans find out that 2 is irrational?	

Unit 7 Overview: Linear Algebra (HL only)

Duration: 8 HL, 0 SL sessions

IB Topic Essential Understandings:

Number and algebra allow us to represent patterns, show equivalencies and make generalizations which enable us to model real-world situations. Algebra is an abstraction of numerical concepts and employs variables which allow us to solve mathematical problems.

IB Content-Specific Conceptual Understandings

AHL

Representing partial fractions and complex numbers in different forms allows us to easily carry out seemingly difficult calculations.

The solution for systems of equations can be carried out by a variety of equivalent algebraic and graphical methods.

Section and IB Sub-Topics	HL Duration	SL Duration
A: Solving Systems using Matrices AHL 1.16	5 sessions	N/A
B: Partial Fractions AHL 1.11	3 sessions	N/A

Unit 7: Linear Algebra (HL only)

Section 7.A: Solving Systems using Matrices

IB Topic Essential Understandings:

Number and algebra allow us to represent patterns, show equivalencies and make generalizations which enable us to model realworld situations. Algebra is an abstraction of numerical concepts and employs variables which allow us to solve mathematical problems.

Duration: 5 HL, 0 SL sessions

IB Content-Specific Conceptual Understandings

The solution for systems of equations can be carried out by a variety of equivalent algebraic and graphical methods.			
IB Content		Standards-Aligned Objectives. Instruction and assessment will align to the following objectives, with IB command terms in bold	
AHL 1.16	Solutions of systems of linear equations (a maximum of three equations in three unknowns), including cases where there is a unique solution, an infinite number of solutions or no solution.	Calculate solutions of systems of linear equations (a maximum of three equations in three unknowns), including cases where there is a unique solution, an infinite number of solutions or no solution.	
	TOK Connections	International Mindedness Connections	
find solution	Mathematics, Sense, Perception and Reason: If we can ons in higher dimensions can we reason that these of beyond our sense perception?	n/a	

Unit 7: Linear Algebra

Section 7.B: Partial Fractions

Duration: 3 HL, 0 SL sessions

IB Topic Essential Understandings:

Number and algebra allow us to represent patterns, show equivalencies and make generalizations which enable us to model real-world situations. Algebra is an abstraction of numerical concepts and employs variables which allow us to solve mathematical problems.

IB Content-Specific Conceptual Understandings

AHL

Representing partial fractions and complex numbers in different forms allows us to easily carry out seemingly difficult calculations.

Representing partial fractions and complex numbers in different forms allows us to easily early out seemingly difficult calculations.			
IB Content		Standards-Aligned Objectives. Instruction and assessment will align to the following objectives, with IB command terms in bold	
AHL 1.11	Partial fractions.	Convert rational expressions into partial fractions.	
TOK Connections		International Mindedness Connections	
n/a		n/a	

Unit 8 Overview: Vectors (HL Only)

Duration: 25 HL, 0 SL sessions

IB Topic Essential Understandings:

Geometry and vectors allow us to quantify the physical world, enhancing our spatial awareness in two and three dimensions. This topic provides us with the tools for analysis, measurement and transformation of quantities, movements and relationships.

IB Content-Specific Conceptual Understandings

AHL

Position and movement can be modelled in three-dimensional space using vectors.

Section and IB Sub-Topics	HL Duration	SL Duration
A: Vector Properties AHL 3.12	6 sessions	N/A
B: Vector Operations AHL 3.12, AHL 3.13, AHL 3.16	6 sessions	N/A
C: Coordinate Geometry and Vectors AHL 3.14, AHL 3.15, AHL 3.17	7 sessions	N/A
D: Application of Vectors AHL 3.17,AHL 3.18	6 sessions	N/A

Unit 8: Vectors (HL Only)
Section 8.A: Vector Properties

IB Topic Essential Understandings:

Geometry and vectors allow us to quantify the physical world, enhancing our spatial awareness in two and three dimensions. This topic provides us with the tools for analysis, measurement and transformation of quantities, movements and relationships.

Duration: 6 HL, 0 SL sessions

IB Content-Specific Conceptual Understandings

AHL

obligation?

Position and movement can be modelled in three-dimensional space using vectors.

IB Content		Standards-Aligned Objectives. Instruction and assessment will align to the following objectives, with IB command terms in bold
AHL 3.12	Concept of a vector; position vectors; displacement vectors. Representation of vectors using directed line segments. Base vectors i, j, k. Components of a vector: $v = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = v_1 i + v_2 j + v_3 k.$	Understand the concept of a vector; position vectors; displacement vectors. Represent vectors using directed line segments. Identify the components of a vector.
TOK Connections		International Mindedness Connections
AHL 3.12 Vectors are used to solve many problems in position location. This can be used to save a lost sailor or destroy a building with a laser-guided bomb. To what extent does possession of knowledge carry with it an ethical		n/a

Unit 8 : Vectors (HL Only)
Section 8.B: Vector Operations

IB Topic Essential Understandings:

Geometry and vectors allow us to quantify the physical world, enhancing our spatial awareness in two and three dimensions. This topic provides us with the tools for analysis, measurement and transformation of quantities, movements and relationships.

Duration: 6 HL, 0 SL sessions

IB Content-Specific Conceptual Understandings

AHL

Position and movement can be modelled in three-dimensional space using vectors.

movements.		
IB Content		Standards-Aligned Objectives. Instruction and assessment will align to the following objectives, with IB command terms in bold
AHL 3.12	Algebraic and geometric approaches to the following: • the sum and difference of two vectors • the zero vector 0, the vector $-v$ • multiplication by a scalar, kv, parallel vectors • magnitude of a vector $ v $; unit vectors $\frac{v}{ v }$ • position vectors $\overrightarrow{OA} = a$, $\overrightarrow{OB} = b$ • Displacement vector $\overrightarrow{AB} = b - a$ Proofs of geometrical properties using vectors	Apply properties of vectors to perform operations on vectors. Prove vector properties using a variety of methods.
AHL 3.13	The definition of the scalar product of two vectors. The angle between two vectors. Perpendicular vectors; parallel vectors.	Understand the definition of the scalar product of two vectors. Calculate the scalar product and angle between two vectors. Classify vectors as parallel, perpendicular, or skew.
AHL 3.16	The definition of a vector product of two vectors Properties of the vector product Geometrical interpretation of $ v \times w $	Understand the definition of a vector product of two vectors. Apply the properties of the vector product.
	TOK Connections	International Mindedness Connections
AHL 3.12 Vectors are used to solve many problems in position location. This can be used to save a lost sailor or destroy a building with a laser-guided bomb. To what extent does possession of knowledge carry with it an ethical obligation? AHL 3.13 The nature of mathematics: why this definition of scalar product?		n/a

AHL 3.16 To what extent is certainty attainable in mathematics? Is certainty attainable, or desirable, in other areas of knowledge?	

Unit 8 : Vectors (HL Only)

Section 8.C: Coordinate Geometry and Vectors

IB Topic Essential Understandings:

Geometry and vectors allow us to quantify the physical world, enhancing our spatial awareness in two and three dimensions. This topic provides us with the tools for analysis, measurement and transformation of quantities, movements and relationships.

Duration: 7 HL, 0 SL sessions

IB Content-Specific Conceptual Understandings

AHL

Position and movement can be modelled in three-dimensional space using vectors.

IB Content		Standards-Aligned Objectives. Instruction and assessment will align to the following objectives, with IB command terms in bold
AHL 3.14	Vector equation of a line in two and three dimensions $= \mathbf{a} + \lambda \mathbf{b}$. The angle between two lines. Simple applications to kinematics	Write equations of vectors in various forms.Calculate the angle between two lines.Apply properties of kinematics to solve real world problems.
AHL 3.15	Coincident, parallel, intersecting and skew lines, distinguishing between these cases. Points of intersection.	Classify lines as coincident, parallel, intersecting and skew. Find the points of intersection of systems of equations.
AHL 3.17	Vector equations of a plane:	Write equations of planes in various forms. Calculate and interpret dot products of vector equations.
	TOK Connections	International Mindedness Connections

TOK Connections	International Mindedness Connections
AHL 3.14 Why might it be argued that one form of representation is superior to another? What criteria might a mathematician use in making such an argument?	n/a
AHL 3.15 How can there be an infinite number of discrete solutions to an equation? What does this suggest about the nature of mathematical knowledge and how it compares to knowledge in other disciplines?	

Unit 8 : Vectors (HL Only)

Section 8.D: Application of Vectors

Duration: 6 HL, 0 SL sessions

IB Topic Essential Understandings:

Geometry and vectors allow us to quantify the physical world, enhancing our spatial awareness in two and three dimensions. This topic provides us with the tools for analysis, measurement and transformation of quantities, movements and relationships.

IB Content-Specific Conceptual Understandings

AHL

Position and movement can be modelled in three-dimensional space using vectors.

IB Content		Standards-Aligned Objectives. Instruction and assessment will align to the following objectives, with IB command terms in bold
AHL 3.17	Vector equations of a plane: $ r = a + \lambda b + \mu c, \text{ where } b \text{ and } c \text{ are non-parallel } \\ \text{vectors within the plane.} $ $ r \cdot n = a \cdot n, \text{ where } n \text{ is a normal to the plane and } a \text{ is the position vector of a point on the plane.} $ Cartesian equation of a plane ax + by + cz = d.	Write equations of planes in various forms. Calculate and interpret dot products of vector equations.
AHL 3.18	Intersections of: a line with a plane; two planes; three planes. Angle between: a line and a plane; two planes.	Find the intersection of lines and planes, two planes and three planes. Find the angle between a line and plane and between two
		planes.
TOK Connections		International Mindedness Connections
AHL 3.18 Mathematics and the knower: are symbolic representations of three-dimensional objects easier to deal with than visual representations? What does this tell us about our knowledge of mathematics in other dimensions?		n/a