

MATHEMATICS PAPER 1 UCE

SECTION A

1. Express $\frac{8\sqrt{2}}{\sqrt{6}-\sqrt{2}}$ in the form $a\sqrt{b} + c$, hence state the values of a, b, and c.

$$\begin{aligned}\frac{8\sqrt{2}}{\sqrt{6}-\sqrt{2}} &= \frac{8\sqrt{2}(\sqrt{6}+\sqrt{2})}{(\sqrt{6}-\sqrt{2})(\sqrt{6}+\sqrt{2})} \\ &= \frac{8\sqrt{2}(\sqrt{6}+\sqrt{2})}{(\sqrt{6})^2 - (\sqrt{2})^2} \\ &= \frac{8\sqrt{2}(\sqrt{6}+\sqrt{2})}{4} \\ &= 2\sqrt{2}(\sqrt{6} + \sqrt{2}) \\ &= 2\sqrt{12} + 4 \\ &= 4\sqrt{3} + 4\end{aligned}$$

Compare with $a\sqrt{b} + c$

$$= a = 4, b = 3, c = 4$$

2. Given that $\mathbf{OA} = \begin{pmatrix} 4 \\ 8 \end{pmatrix}$, $\mathbf{OB} = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$, M is the mid-point of AB and G divides OM in the ratio 2:1. Find the position vector of G.

$$\overrightarrow{OA} = \begin{pmatrix} 4 \\ 8 \end{pmatrix}, \overrightarrow{OB} = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$$

$$\mathbf{AB} = \mathbf{OB} - \mathbf{OA}$$

$$= \begin{pmatrix} 2 \\ 4 \end{pmatrix} - \begin{pmatrix} 4 \\ 8 \end{pmatrix} = \begin{pmatrix} -2 \\ -4 \end{pmatrix}$$

$$\mathbf{OG} = \mathbf{OA} + \mathbf{AG}$$

$$= \begin{pmatrix} 4 \\ 8 \end{pmatrix} + \frac{1}{2} \begin{pmatrix} -2 \\ -4 \end{pmatrix}$$

$$= \left(\frac{4}{8} \right) + \left(\frac{-1}{-2} \right)$$

$$Om = \left(\frac{3}{6} \right)$$

$$OG = \frac{2}{3} om$$

$$= \frac{2}{3} \left(\frac{3}{6} \right)$$

$$OG = \left(\frac{2}{4} \right)$$

3. Given $f(x) = \frac{3+2x}{x-4}$, find the value of x for which $f^{-1}(x)$ is meaningless.

$$f(x) = \frac{3+2x}{x-4}$$

$$f^{-1}(x)$$

$$\text{Let } y = \frac{3+2x}{x-4}$$

$$y(x - 4) = 3 + 2x.$$

$$yx - 4y = 3 + 2x.$$

$$yx - 2x = 3 - 4y$$

$$x(y-2) = 3 - 4y$$

$$x = \frac{3-4y}{y-2}$$

$$\therefore f^{-1}(x) = \frac{3-4x}{x-2}$$

$F^{-1}(x)$ is meaningless when

$$x - 2 = 0$$

$$\therefore x = 2$$

4. Factorize completely

$$(i) \quad ax^2 + 3abx - 3x - 9b$$

$$ax^2 + 3abx - 3x - 9b$$

$$ax^2 - 3x + 3abx - 9b$$

$$= x(a x - 3) + 3b(ax - 3)$$

$$= (ax - 3)(x + 3b)$$

$$(ii) \quad 8x^2 - 32y^2$$

$$8x^2 - 32y^2$$

$$= 8(x^2 - 4y^2)$$

$$= 8(x + 2y)(x - 2y).$$

5. Find the equation of a line which is perpendicular to the line $3y + 4x = 5$ and passing through the mid-point of the $\overline{AB}$ where $A(-2,3)$ and $B(4,5)$.

$$3y + 4x = 5$$

$$3y = -4x + 5$$

$$y = -\frac{4}{3}x + \frac{5}{3}$$

$$m_1 = \frac{-4}{3}$$

$$m_2 = \frac{3}{4}, \text{ Grad of } \dots\dots\dots \text{ line}$$

$$\text{Midpoint } m \left(\frac{-2+4}{2}, \frac{3+5}{2} \right)$$

$$m(1,4).$$

$$y = mx + c$$

$$4 = \frac{3}{4} \times 1 + c$$

$$4 = \frac{3}{4} + c$$

$$c = 4 - \frac{3}{4} = \frac{16-3}{4} = \frac{13}{4}$$

$\therefore$ Eqn is

$$y = \frac{3}{4}x + \frac{13}{4}.$$

$$\text{Or } 4y = 3x + 13$$

6. Mr. Ouma deposited 1.321 million shillings on his bank account at a compound interest rate of 7.5% per annum. Determine the number of years it will take to accumulate 1.764 million shillings.

$$A = P \left(1 + \frac{r}{100}\right)^n$$

$$1.764 = 1.321 \left(1 + \frac{7.5}{100}\right)^n$$

$$\frac{1.764}{1.321} = (1 + 0.075)^n$$

$$1.3354 = (1.075)^n$$

$$n = \frac{\log 1.3354}{\log 1.075}$$

$$n = 3.9993$$

$$n \cong 4 \text{ years}$$

7. Simplify $\frac{1}{3} \log_{10} 64 + \log_{10} 80 - 5 \log_{10} 2$

$$\frac{1}{3} \log_{10} 64 + \log_{10} 80 - 5 \log_{10} 2$$

$$\Rightarrow \log_{10} 64^{\frac{1}{3}} + \log_{10} 80 - \log_{10} 2^5$$

$$\Rightarrow \log_{10} 4 + \log_{10} 80 - \log_{10} 32$$

$$\Rightarrow \log_{10} \left(\frac{4 \times 80}{32}\right)$$

$$\Rightarrow \log_{10} 10$$

$$\Leftrightarrow \underline{1}$$

8. Given that $A = \{\text{the first five multiples of two}\}$

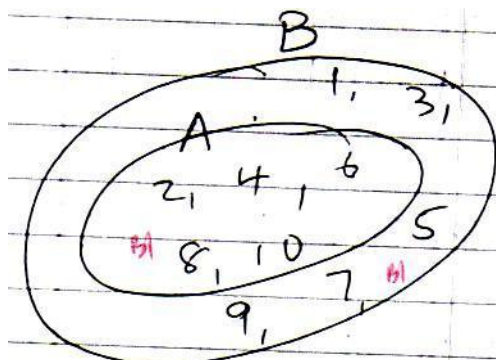
$$B = \{\text{the first ten counting numbers}\}$$

- (i) List the members of sets A and B

$$A = \{2, 4, 6, 8, 10\}$$

$$B = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$$

- (ii) Represent the two sets on a Venn- diagram.



9. Find the lowest common multiple (L.C.M) and the highest common factor (H.C.F) of 54 and 84

2	54	84
2	27	42
3	27	21
7	9	7
9	9	1
	1	1

$$\therefore \text{H.C.F} = 2 \times 3$$

$$= 6$$

$$\text{L.C.M} = 2^2 \times 3 \times 7 \times 9$$

$$= 756$$

10. Find the value of x given that $3x + 4) - (3x - 1) = 1$.

$$C = K \text{ and } C \times N$$

$$C = K \text{ and } C = RN$$

$$\underline{\therefore C = K + RN.}$$

SECTION B

11. The cost per week of running a private school is partly a constant and partly varies directly with the number of pupils. It costs shs. 350,000 a week when there are 50 pupils and shs. 600,000 a week when there are 100 pupils in the school if C is the cost and N is the number of Pupils.

a) Find the cost of running the school with 250 pupils a week.

$$350,000 = K + 50R \text{ -- (i)}$$

$$600,000 = K + 100R \text{ -- (ii)}$$

Solving simultaneously,

$$250,000 = 50R$$

$$R = 5000$$

$$350,000 = K + 250,000$$

$$K = 100,000$$

$$\therefore C = 100,000 + 5000 N.$$

$$\therefore \text{For } N = 250,$$

$$C = 100,000 + 5000 \times 250$$

$$= \text{Shs. } 1,350,000$$

b) If in a certain week shs. 750,000 was spent, determine the number of pupils that attended that week

$$C = 750,000, N = ?$$

$$C = 100,000 + 5000 N$$

$$750,000 = 100,000 + 5000 N$$

$$5000N = 650,000$$

$$N = 130$$

$\therefore$ 130 pupils

12. A group of 36 boys were asked which games they played from foot ball (F), Volley ball (V) and tennis (T) it was discovered that 12 play F, of which 2 play F and T only. An equal number of boys play V and T. 6 play F and V while 5 play all the games. Those who play V and T only are one less than those who play F only and half those who did not play any of the three games.

a) Represent this information on a Venn diagram

$$n(F) \text{ only} = 12 - (2 + 5 + 1) = 4$$

$$n(V) = n(T)$$

$$y + 1 + 5 + 3 + x + 2 + 5 + 3$$

$$y + 9 = x + 10$$

$$y - x = 1 \quad (1)$$

$$\Rightarrow y = x + 1$$

Also

$$4 + 1 + 2 + 5 + 3 + x + y + 6 = 36$$

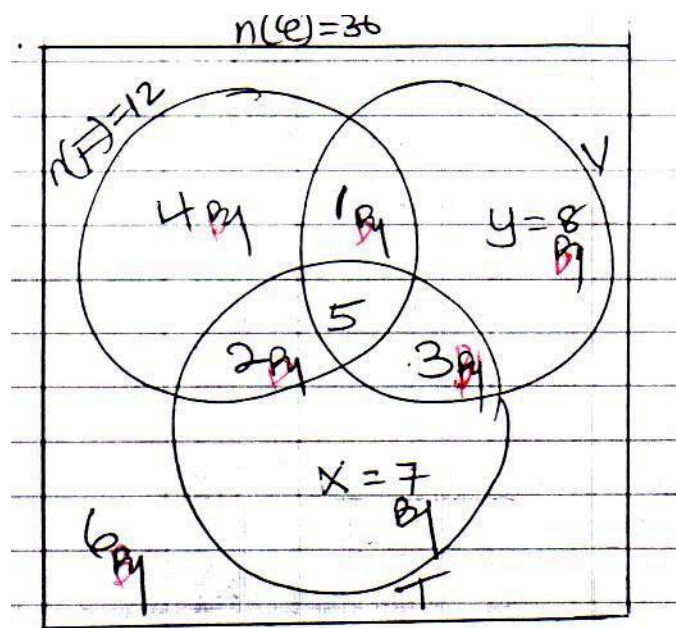
$$21 + x + x + 1 = 36$$

$$2x + 22 = 36$$

$$2x = 14$$

$$x = 7$$

$$\therefore x = 7, y = 8$$



b) Find the number of boys who play

(i) Tennis

17 boys

(ii) Only one game

19 boys

c) What is the probability that a boy chosen at random plays at most two games.

n (play at most one game)

$$=4+8+7+6$$

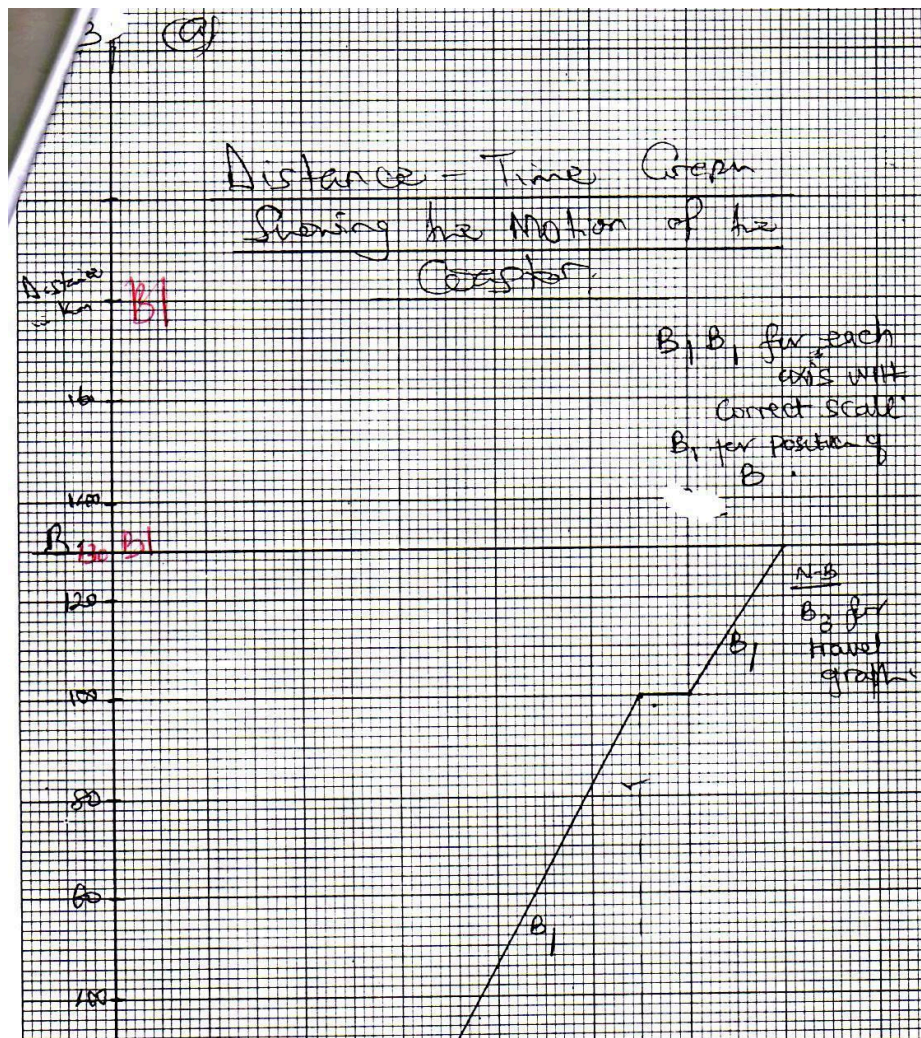
$$=25$$

$$\text{Prob} = \frac{25}{36}$$

13. Town A is 130km from town B. A coaster left town A at 9.30am and because of jam it moved at a speed of 10kmh^{-1} for the first 1 hour. It then covered the next 90km in $1\frac{1}{4}$ hours before developing a mechanical problem which was rectified in 15 minutes. The coaster then sets off again and arrived at town B at 12:30pm.

a) Draw a distance – time graph showing the motion of the coaster.

(Use, scales of 1cm: 10km and 4cm: 1 hr)



b) Find the speed of the coast

(i) Before getting a mechanical problem

$$\text{Speed} = \frac{D}{T}$$

$$= 1\frac{1}{4}$$

$$\underline{\quad\quad} = 72 \text{ km h}^{-1}$$

(ii) After rectifying the mechanical problem.

$$\text{Speed} = \frac{30}{\frac{1}{2}}$$

$$= \frac{130}{3}$$

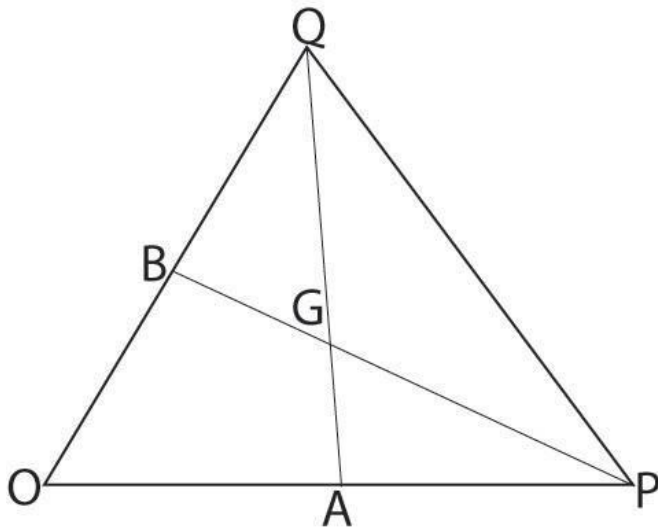
$$= 43\frac{1}{3} \text{ km h}^{-1}$$

c. Find the average speed of the coaster over the whole journey.

$$\text{Average speed} = \frac{\text{Total distance}}{\text{Total time}}$$

$$= \frac{130}{3}$$

$$= 43\frac{1}{3} \text{ km h}^{-1}$$



14.

In the figure above

OPQ is a triangle such that

$OA = 3a$, $OB = 3b$ and $OP = 2OA$, $OQ = 2OB$, $3AG = 2AQ$ and $5PG = 3PG$

a) Find in terms of **a** and **b** the vector

(i) **$AB = AO + OB$, or $OB - OA$**

$$= -3a + 3b$$

Or $3b - 39$

(ii) $\mathbf{PQ} = \mathbf{PO} + \mathbf{OQ}$ OR $\mathbf{OQ} - \mathbf{OP}$

$$= -20\mathbf{A} + 20\mathbf{B}$$

$$= -6\mathbf{a} + 6\mathbf{b}$$

Or $6b - 6a$

(iii) $\mathbf{OG} = \mathbf{OA} + \mathbf{AQ}$

but from $3\mathbf{AG} = 2\mathbf{AQ}$

$$\mathbf{AG} = \frac{2}{3} \mathbf{AQ}$$

and

$$\mathbf{AQ} = \mathbf{AO} + \mathbf{OQ}$$

$$= -3\mathbf{a} + 6\mathbf{b}$$

$$\therefore \mathbf{AG} = \frac{2}{3} (-3\mathbf{a} + 6\mathbf{b})$$

$$= -2\mathbf{a} + 4\mathbf{b}.$$

$$\therefore \mathbf{OG} = 3\mathbf{a} + (-2\mathbf{a}) + 4\mathbf{b}$$

$$= \mathbf{a} + 4\mathbf{b}$$

$$= \underline{\mathbf{a} + 4\mathbf{b}}$$

From $3\mathbf{AG} = 2\mathbf{AQ}$

$$\mathbf{AQ} = \frac{3}{2} \mathbf{AG}$$

$$= \frac{3}{2} (\mathbf{OG} - \mathbf{OA})$$

$$= \frac{3}{2} [(a + 4b) - 3a]$$

$$\mathbf{AQ} = \frac{3}{2} (a + 4b - 3a)$$

$$= \frac{3}{2} (-2a + 4b)$$

$$= 3(-a + 2b)$$

$$= \underline{-3a + 6b}.$$

b) Show that A, G and Q are collinear.

A, G, Q collinear

$$\vec{AQ} = -3\vec{a} + 6\vec{b}$$

$$\vec{AG} = \frac{2}{3} \vec{AQ}$$

$$= \frac{2}{3}(-3\vec{a} + 6\vec{b}).$$

$\vec{AG}$ and $\vec{AQ}$ are parallel.

Since $\vec{AG}$ and $\vec{AQ}$ have common point, A, G, Q are collinear.

15. The table below shows the tax structure on taxable income of a certain working class of people

Income (shs) per month	Rate of tax (%)
15,001 – 40,000	8.00
40,001 – 70,000	12.50
70,001 – 120,000	15.50
120,001 – 200,000	24.00
200,001 – 350,000	30.00
350,001 – 420,000	35.00

420,001 – 500,000	45.00
Above 500,000	48.00

An employee earning a gross income of shs 7,200,000 per annum is allowed the following;

Allowance	Amount
Transport and lunch	45,000 per month
Housing	960,000 per annum
Water and electricity	25,000 per month
Medical care	900,000 per annum
Marriage	$\frac{1^{th}}{20}$ of gross monthly income

The employee is allowed a family allowance for any three children according to the age distribution

Age	Amount (shs)
0 – 12	9000
13 – 18	5000
19 – 21	3500

22 – 25	2000
---------	------

Given that this employee has a family of five children aged 23 years, 20 years, 17 years, 15 years and 8 years.

a) Calculate the employee's income tax.

$$\text{Gross monthly income} = \frac{7,200,000}{12}$$

$$= \text{shs. } 600,000$$

Allowances

Transport and lunch 45,000

Housing $\frac{960,000}{12}$ 80,000

Water & electricity 25,000

Medical $\frac{900,000}{12} +$ 75,000

Marriage $\frac{1}{2} \times 600,000$ 30,000

Children $9000 + 2 \times 5000$ 19,000

Total allowances = 274,000

Taxable allowances =

Gross income – Allowances

$$= 600,000 - 274,000$$

$$= \underline{\text{shs.}326,000.}$$

Income tax;

Taxable income

Tax

The 1st 15,000/=

0

$$\text{On next } 25,000/= \Rightarrow \frac{8}{100} \times 25,000 = 2,000$$

$$\text{Next } 30,000/= \quad \frac{12.5}{100} \times 30,000 = 3,750$$

$$\text{Next } 50,000/= \Rightarrow \frac{15.5}{100} \times 50,000 = 7,750$$

$$\text{Next } 80,000/= \Rightarrow \frac{24}{100} \times 80,000 = 19,200$$

$$\text{Next } 126,000/= \Rightarrow \frac{30}{100} \times 126,000 = 37,800$$

$$\therefore \text{Total income tax} = 2000 + 3,750 + 7,750 + 19,200 + 37,800$$

$$= \text{Shs } 70,500$$

b) Determine the percentage of the employees taxable income that goes to tax

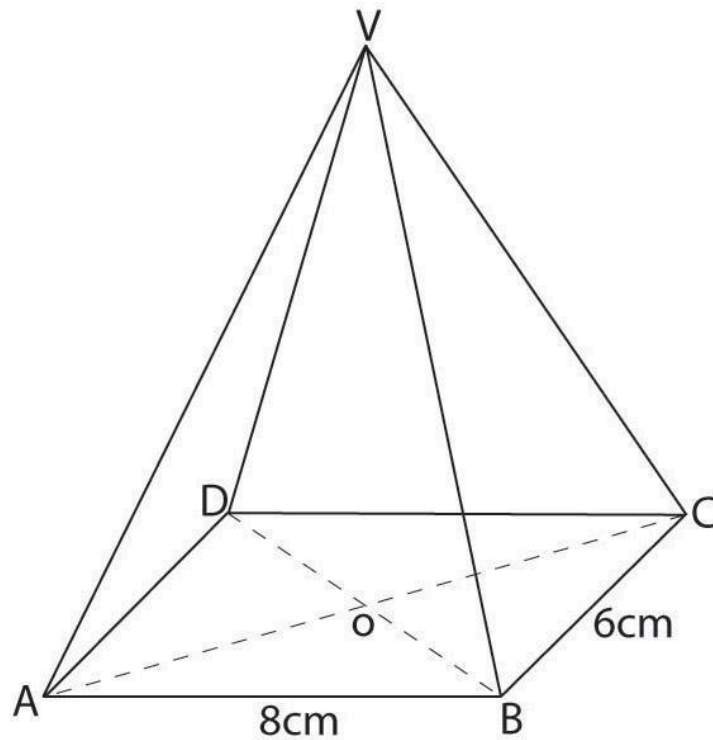
$$= \frac{\text{tax}}{\text{taxable income}} \times 100\%$$

$$= \frac{70,500}{326,000} \times 100\%$$

$$= 21.6258\%$$

$$\cong 21.63\%$$

16.

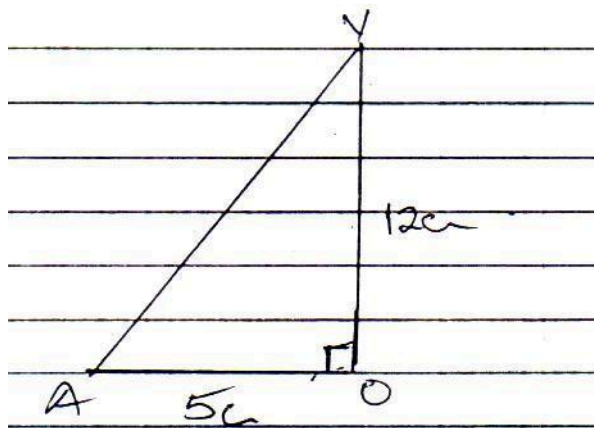


In the figure above $\overline{VA} = \overline{VB} = \overline{VC} = \overline{VD}$

$\overline{AB} = 8\text{cm}$, $\overline{BC} = 6\text{cm}$ and V is vertically above O. If $\overline{VO} = 12\text{cm}$, find

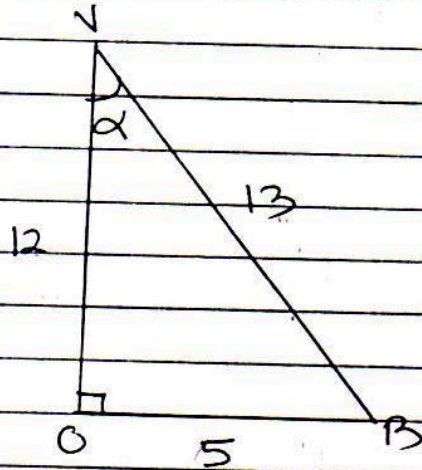
(i) The length VA

$$\begin{aligned} AC &= \sqrt{8^2 + 6^2} \\ &= \sqrt{100} \\ &= 10\text{cm} \end{aligned}$$



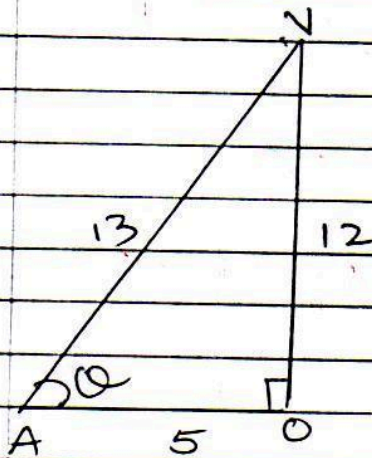
$$\begin{aligned} VA &= \sqrt{12^2 + 5^2} \\ &= \sqrt{144 + 25} \\ &= \sqrt{169} \end{aligned}$$

(ii)



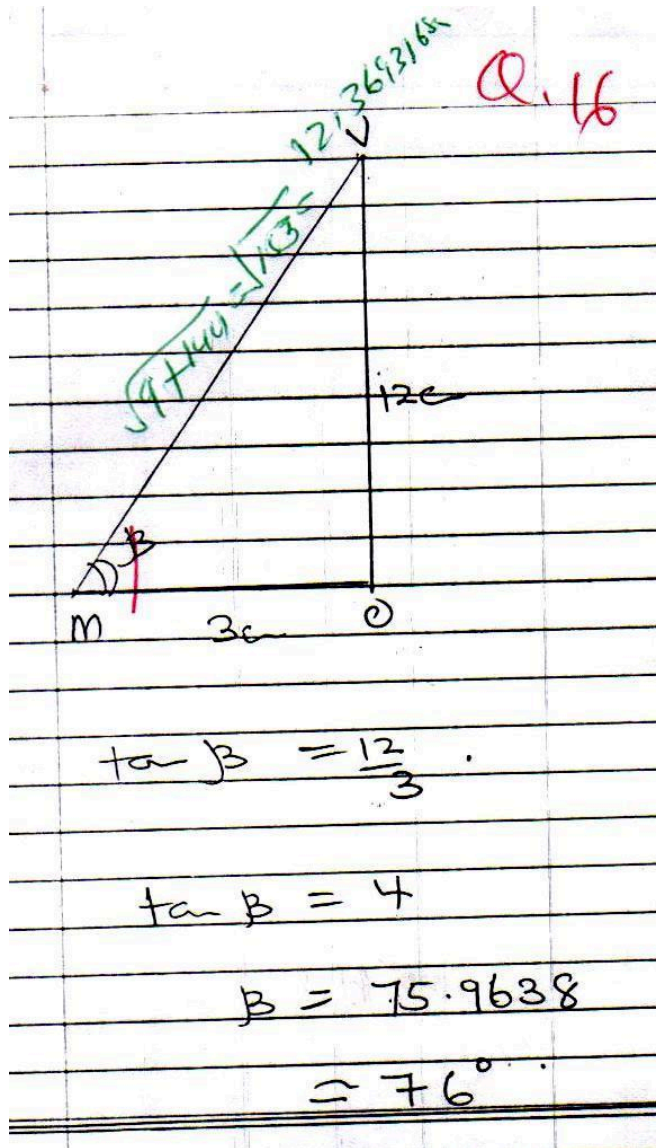
$$\begin{aligned}\tan \alpha &= \frac{5}{12} \\ &= 22.6215^\circ \\ &\approx \underline{\underline{22.62^\circ}}\end{aligned}$$

(iii)



$$\begin{aligned}\tan \theta &= \frac{12}{5} \\ &= 67.3801 \\ &= \underline{\underline{67.38^\circ}}\end{aligned}$$

- (iv) The angle between the planes VAB and ABCD



17. (a) Given the functions $f(x) = \frac{x+3}{2}$ and $g(x) = \frac{1-2x}{5}$, determine the values of x for which

$$fg(x) = \frac{9+24x+8x^2}{10}$$

$$f(x) = \frac{x+3}{2}$$

$$g(x) = \frac{1-2x}{5}$$

$$fg(x) = f \frac{(1-2x)}{5}$$

$$= \{1 - 2x \mid 5 + 3 \div 2$$

$$= \frac{1-2x+15}{5} \times \frac{1}{2}$$

$$= \frac{16-2x}{10}.$$

$$\frac{16-2x}{10} = \frac{9+24x+8x^2}{10}$$

$$16-2x=9+24x+8x^2$$

$$8x^2 + 24x + 9 = 16 - 2x$$

$$8x^2 + 24x + 2x + 9 - 16 = 0$$

$$x = -\frac{26 \pm}{26}$$

$$x = -\frac{26}{26} \pm$$

(b) If P:Q = 3:4 and Q:R = 1:2 and that P+Q+R = 30,000. Find

(i) The ratio P:Q:R

$$P: Q = 3:4$$

$$Q:R=1:2$$

$$\frac{P}{Q} = \frac{3}{4}$$

$$\frac{Q}{R} = \frac{1 \times 4}{2 \times 4} = \frac{4}{8}$$

$$P: Q: R = \underline{3:4:8.}$$

(ii) The proportional parts of P,Q and R

$$A + B + C = 30,000$$

$$\text{Total share} = 15$$

$$A \text{ gets } \frac{3}{15} \times 36,000$$

$$= \text{shs } 6,000$$

$$B \text{ gets } \frac{4}{15} \times 30,000$$

$$= \text{shs } 8,000$$

$$C \text{ gets } \frac{8}{15} \times 30,000$$

$$\underline{= \text{shs } 16,000}$$

END