Pre-Calculus Unit 4/5 Trigonometric Functions & Identities
13-14 Weeks
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<table>
<tr><th>Unit Goals (Students will…)</th><th></th></tr>
<tr><td>Explore the concept of radians, and will use the unit circle to extend the definitions of sine, cosine, tangent, and the reciprocal and inverse trigonometric functions to all special reference (and related) angles.
Define amplitude, period and phase shift, and will examine periodic functions in depth, focusing on sinusoidal functions.
Apply transformations to model periodic situations; apply modeling techniques to periodic functions.
Derive and use key trigonometric identities along with the Laws of Sines and Cosines.
Explore sum/difference and double angle formulas for sine, cosine and tangent.</td><td></td></tr>
<tr><th>Enduring Understandings (Student will understand that…)</th><th>Essential Questions</th></tr>
<tr><td>There is a relationship between degree and radian measures of angles.
Trigonometric functions can help us define ratios between side length and angle measurements of triangles; There is a fixed ratio between the lengths of sides and angles of special right triangles.
Trigonometry can be used to analyze triangles (both right and non-right).
Technology (graphing calculators, GSP) can be used to aid in the exploration of mathematics and graphs.
Periodic functions can be used to model many real world situations.
Trigonometric identities can be derived from the definitions of the trigonometric functions.
Trigonometric identities can be used to relate trigonometric functions to one another, to simplify expressions, solve equations, and analyze trigonometric graphs.</td><td>What are some of the real world applications of trigonometric functions?
How are trigonometric functions used to model data?
What is the unit circle and what is the relationship between degree and radian measure?
What are the relationships between graphs of periodic trigonometric functions and the unit circle?
How can transformations be used to demonstrate links between algebra and geometry?
How can trigonometry be used to analyze triangles?
What are some significant trigonometric identities?
Where do these identities come from?
How can such identities be used to simplify expressions, solve equations, and analyze trigonometric graphs?</td></tr>
<tr><th>Knowledge (Student will know…)</th><th>Skills (Student will be able to…)</th></tr>
<tr><td>The long run (end) behavior of functions; vertical and horizontal asymptotes.
Special right triangles from geometry and the relationships of the sides and angles of these triangles.
The Pythagorean Theorem.
How to transform a parent function to generate shifts in the graphs of the functions.
equations.</td><td>Define the primary, reciprocal and inverse trigonometric functions.
Generate the unit circle and use it to evaluate trigonometric functions at various angle measurements.
Convert between degree and radian measures.
Graph trigonometric functions.
Solve trigonometric equations.
Apply the Law of Sines and Law of</td></tr>
</table>

	Cosines to analyze the relationship between the sides and angles of any triangle. ● Derive and use the sum/difference and double angle formulas to solve trigonometric equations
Familiarity *(Student will be familiar with…)*	**Assessments**
● Restrictions and limitations to (inverse) trigonometric functions. ● Solving advanced trigonometric equations. ● Various trigonometric identity proofs.	● Daily homework sets ● GSP graphing activity ● Standards quizzes ● Unit circle (fill-in-the-blanks) relay race ● Real World Application: Trigonometric word problems (extended homework/mini-project) ● Quarter 2 Benchmark: Modeling with Sinusoidal Functions ● Quarter 3 Trigonometric Live Derive

Core Values:

Inquiry:

Students will work individually or with classmates to examine essential questions, mainly:

- What are some of the real world applications of trigonometric functions?
- How are trigonometric functions used to model data?
- What is the unit circle and what is the relationship between degree and radian measure?
- What are the relationships between graphs of periodic trigonometric functions and the unit circle?
- How can transformations be used to demonstrate links between algebra and geometry?
- How can trigonometry be used to analyze triangles?
- What are some significant trigonometric identities?
- Where do these identities come from?
- How can such identities be used to simplify expressions, solve equations, and analyze trigonometric graphs?

Research:

- Students will research proofs for specific trigonometric identities in order to complete a group project.
- Students will research the relationship between degree and radian angle measures.

For Quarter 2 Benchmark project, students will:

- Research the monthly average temperatures of a city for the last two years.
- Research the monthly average sunset times of a city for the last two years.

Collaboration:

Students will work with classmates regularly to examine key concepts in the course. This includes:

- Small group activities and investigations (groups of 4).
- Partner activities and problem sets.
- Small group “guest teachers” presentations.

In the 3rd quarter Benchmark on trigonometric functions and formulas, students work in groups to create a compilation of real-world word problems using laws of sines, cosines, etc.

Presentation:

Students will be expected to present their knowledge of key concepts in the following ways:

- Presentation of solutions to daily warm-up/homework review problems.
- Individual presentation of solutions to examples problems.
- Small group presentations and mini-lessons on key problems or concepts.
- Group mini-project presentation (keynote, iWeb, movie, flash, song, etc), including graphical representation and analysis of domain and range.

Reflection:

Students will reflect on their progress through the following:

- Class journaling, including questions before and after a lesson.
- Class discussions reflecting on key course topics.

PA Common Core Standards

- CC.2.2.HS.C.7 - Apply radian measure of an angle and the unit circle to analyze the trigonometric functions.
- CC.2.2.HS.C.8 - Choose trigonometric functions to model periodic phenomena and describe the properties of the graphs.
- CC.2.2.HS.C.9 - Prove the Pythagorean identify and use it to calculate trigonometric ratios.
- CC.2.3.HS.A.7 - Apply trigonometric ratios to solve problems involving right triangles.

SLA Mathematics Course Standards:

A-Computation and operations--Students can perform computational and algebraic operations to the appropriate level of course.

B-Visual-- Students can visually represent mathematical situations through graphs and diagrams.

C-Verbal and written communication skills-- Students can clearly communicate mathematical problem solving process.

D-Problem solving- Choose and apply various problem-solving strategies to model and solve a wide variety of problems.

Unit 4 Pre-Calculus Standards:

42. Construct a trigonometric unit circle (B)
43. Evaluate trigonometric functions (A)
44. Understand the relationship between reference and related angles (C)
45. Convert between degree and radian angle measures (A)
46. Define the primary and reciprocal trigonometric functions (C)
47. Graph trigonometric functions (B)
48. Describe the characteristics of a trigonometric function/equation (C)
49. Describe the characteristics of a trigonometric graph (C)
50. Model a periodic scenario using trigonometric functions (D)
51. Evaluate inverse trigonometric functions (A)
52. Graph (restricted) inverse trigonometric functions (B)
53. Solve bounded trigonometric equations (A)
54. Solve unbounded trigonometric equations (A)
55. Prove various trigonometric identities (C)
56. Simplify trigonometric expressions (D)
57. Use sum and difference identities to evaluate trigonometric functions (A)
58. Solve a triangle using the law of sines (A)

59. Solve a triangle using the law of cosines (A)
60. Use double angle formulas to evaluate trigonometric functions (A)
61. Solve word problems using trigonometric identities and formulas (D)

Quarter 2 Pre-Calculus Benchmark
Modeling with Transformations of Trigonometric Functions

Project Assigned: 01/06/14 (data collection assigned 12/19/13)

Project Due: 01/15/14 – printed hardcopy AND uploaded to *Canvas* before the start of class.
Late projects will be docked 5% daily, including each day of the weekend.

Essential Questions:

- What are periodic functions?
- What are some real world examples of periodic functions?
- What are the impacts of various types of transformations on the graphs of functions?
- What are sinusoidal functions?
- How can transformations be applied to the graphs of $y = \sin(x)$ and/or $y = \cos(x)$? What about to the graph of $y = \tan(x)$?
- How can trigonometric functions be used to model real world situations?

Project Details:

The second quarter benchmark will involve using transformations of trigonometric functions to model two sets of periodic data. You will be working on this term's project *individually* to gather and analyze your findings across the following tasks:

- Monthly Sunset Times (Sinusoidal Model: Sine/Cosine) – Your first task will require you to choose one American city (to be determined during class) and research the sunset times in that city for the first day of each month over the last two years (January 1, 2012–December 1, 2013). You will then use your pre-calculus skills to develop, graph and analyze a sinusoidal equation that will (closely) fit your set of 24 data points (note: your raw data should also be plotted on your graph(s) for comparison).
    - When organizing and graphing your data, let $x = 0$ correspond to January of the first year, $x = 1$ correspond to February of the first year, and continue this pattern for the remaining values of your independent variable.
    - Once you have gathered your research, you ***must*** convert values of your dependent variable from raw hour and minute sunset times (of the form HH:MM) into decimal hours, which will be easier to plot on a graph.
        - For example, a time of 4:46pm would become 4 + 46/60, or approximately 4.77 hours (note: you are permitted to round your final decimal hours to the nearest hundredth, but must indicate doing so in your project).
    - You will be using a sinusoidal parent function, either $f(x) = \sin(x)$ or $f(x) = \cos(x)$,

    (the choice is yours!) as the foundation of this task. The equation you derive to (closely) model your data should therefore be a transformation of the parent function of the form $g(x) = a\sin(bx - c) + d$ (or of similar form if using the cosine function).

- Length of a Shadow (Tangent Model) – Your next task (unrelated to the first) will require some intense fieldwork in order to collect your second set of data. Using self-constructed materials placed outside in the sunlight, you will record the length of an object's shadow <u>each half-hour</u> over an eight-hour period of time (8:00am–4:00pm). This will produce 17 pieces of data, which you are then expected to copy as though you collected the same data on a subsequent day. To clarify, you are allowed to assert in your project an assumption that the measurements you recorded would not change by a significant factor over the same 8-hour period in the same location on the following day of sunlight. You *should*, however, address in your project why this is a reasonable assumption to make.

  With your 34 pieces of data, you will then once again use your knowledge of function transformations and trigonometric parent functions to model your data with a tangent equation. As in the first task, you will develop this equation and its graph, analyzing how closely it fits your data (<u>note</u>: your raw data should also be plotted on your graph(s) for comparison).

  - When organizing and graphing your data, let $x = 0$ correspond to the time of your first measurement (8:00am) on day 1, $x = 1$ correspond to 8:30am on day 1, and continue this pattern for the remaining values of your independent variable. Don't forget, you will repeat your cycle of data a second time to represent "day 2" of your data collection...but be careful, 8:00am on day 2 should *not* be represented by $x = 18$.
    - <u>Note</u>: If you originally recorded lengths across different days, include those dates in your raw presentation of the data (data table), but discuss treating the data as though you found lengths on the same day.
- Once you have gathered your research, you ***<u>must</u>*** convert values of your dependent variable from raw lengths into lengths with respect to the shadow length at noon. The shadow length at noon will represent our base length, which means lengths found before noon should be made negative with respect to the baseline, and lengths after noon, positive. (<u>Note</u>: think about why you're being asked to do this and include your reasoning in your paper!).
  - For example, if the length of your object's shadow at 11:30am was 9.5", the length at noon was 9", and the length at 12:30pm was 9.4", your *converted* lengths would become -0.5", 0", and 0.4", respectively.
- You will be using the tangent parent function, $f(x) = \tan(x)$, as the foundation of this task. The equation you derive to (closely) model your data should therefore be a transformation of the parent function of the form $g(x) = a\tan(bx - c) + d$.

<u>Finished Product:</u>

Your work for this project will be completed individually, although collaborative class and/or peer discussions might occur. The final product should be in the form of a *polished* paper. In other words, you are composing a formal report, which means organizing your work logically, clearly, and with several critical revisions to mitigate errors in your math and written expressions.

Your final product should include:

- An introduction section in which you provide a brief overview of our two most-recent units, your benchmark task(s), and the processes you took to acquire your data.
- Two detailed and *labeled* data tables (one for each task).
    - Exceptional tables will include labeled columns, with proper units, for each day and time of data collection, raw data collected, converted data values, and corresponding (x, y) coordinates, as plotted in your graph(s).
- (At least*) Two detailed and *labeled* graphs (one for each task). Color is also appreciated.
    - Each must include your full set of data points, the equation you generated, and your model function (the illustrative curve defined by your equation).
    - *You may also want to include parent functions (equations and graphs) and/or intermediary transformations of the parent functions as references.
- Detailed descriptions and corresponding calculations (properly formatted!) of the process that you took to generate each of your models. In other words, how you found the values of the parameters (a, b, c, and d) in each of your derived equations.
    - You should include references to specific vocabulary we studied in both our transformations unit and our trigonometry unit (*e.g. outside vs. inside change, horizontal translation, vertical dilation, amplitude, etc.*).
    - You should include references to specific formulas we studied in both our transformations unit and our trigonometry unit (*e.g.* $g(x) = kf(x)$ *for vertical stretch, period formulas, phase shift formula, etc.*).
- Detailed analysis/discussion of how well your models fit your data.
    - You may want to use a lot of the same transformations and/or trigonometric vocabulary mentioned in the last bullet point.
    - Finely labeled (and colored!) graphs could really pack a punch, here.
- A conclusion section in which you summarize what you learned.
    - A solid conclusion will include a response to the "*So What…?*" question. Why does this matter? What is special about this type of modeling and these types of functions? Provide higher order thinking, not simply surface level answers.
- A <u>properly constructed</u> *References* section with organized, <u>complete</u> citations.
- Any other element(s) of your choosing that will enhance the quality of your project for reader comprehension (and not simply to "bulk up" your submission).

Your final product should include or satisfy the following formatting guidelines:

- A cover page at the top/front of your work with the traditional title page information. Please DO NOT include a table of contents.
- Papers should be printed with 1" margins on all sides and with page numbers centered on the bottom of each page (but not on the cover page).

- Papers should be typed and double-spaced using *Times New Roman* 12-point font, with appropriate section headers (if your organization style requires their use).
- Mathematical calculations should flow logically with vertical orientation and be typed using *Equation Editor* (or another acceptable program) to construct accurate and visually appealing math text.
    - Explanations of basic arithmetic and algebra need not be included if calculations/solutions are presented correctly.
- Feel free to include appendices of additional graphs or diagrams, as needed.
- You should NOT have huge sections of white/empty space in your paper.

Trigonometric Live Derive

Q3 Pre-Calculus Benchmark

Benchmark Overview:

In class we covered a lot of material involving trigonometric angles, functions, and properties. We have also derived many important formulas and identities together. There were, however, many other essential formulas and identities that we did not derive and/or prove as a class.

For your Quarter 3 Benchmark, you and a group of partners will work together to research and *group-think* your way through understanding a solid derivation (or derivations) of one of these formulas or identities. Since many derivations exist, you will need to apply higher-order thinking skills to analyze your research and study the proof you find to be most helpful (in terms of understanding the nature of your identity). You are then *individually* expected to adapt the step-by-step processes of the proof to your own style so that you will be able to effectively and efficiently communicate the derivation to your teacher in a live oral presentation.

In your 1:1 presentation you will first derive one of the formulas we have yet to develop together (sum formula for sine, sum formula for cosine, difference formula for cosine, law of cosines), and then you will put the formula into practice by solving 1-2 problems that will be given to you.

Benchmark Objectives:

Through this benchmark you will be polishing your skills in the following areas:

(1) Performing thorough academic research on mathematics.

(2) Collaborating with other scholars to discuss ideas and achieve a common goal.

(3) Comprehension of where the math we are studying comes from and why it is undisputed truth through the evolution of its proof.

(4) Analyzing your metacognitive processes, meaning discovering how you learn best through thinking about which of your researched derivations speak to your talents and levels of understanding.

(5) Live oral presentation and communication of the math concepts at play.

Benchmark Process Details:

- You and your classmates will be divided into research teams and assigned one of the trigonometric identities that we have used in class, but have not yet derived. In your teams, perform *professionally adequate* research until you have a bank of proofs that you can discuss as a group.
- Work in your teams to develop a solid understanding of the components involved in each of the proofs you collected. Decide which processes may be too advanced (as in proofs at the college or professional level using techniques you have not yet learned, nor understand), and eliminate those from your pool of derivations. With the derivation methods that remain, choose one or two that you really "get," and make sure that your whole team has a comfortable familiarity with each stage of the proof(s).
- As an individual, re-design/re-imagine the proof(s) you have studied in your team so that the processes involved in the derivation are easy for you to communicate to someone else. In other words, think about how you can most effectively present the derivation using your own skill set. Perhaps one derivation is easier for you to follow than another. Perhaps the steps that you found presented online or by a teammate are not in an order that makes sense to you, and you can reorganize the process so it suits your level of understanding best.
- As a team, practice your oral presentation of the proof to each other, from start to finish. Give each other feedback on what works well, or what places in your delivery need polish.
- As a team, find or create practice problems (beyond the problems given to you in class) for each other using your identity. Peer-edit your solutions to find mistakes and learn from them. Your problems should be varied. Use *both* degree and radian measurements when practicing, along with straight calculation problems and problems that may require the use of a calculator. Practice until you feel confident that you would be able to obtain a correct solution when given any problem where your formula would solve the question at hand.
- As an individual you will be assigned a 5–10 minute 1:1 benchmark appointment with your teacher. At your meeting you will need to present your identity and its derivation from scratch within 5 minutes (most likely with original diagrams and/or using pen and paper or whiteboard), and its use in solving application problems. The level of mastery and ease with which you can successfully communicate these things (with or without *self-designed* notes or reference sheets) will determine your benchmark grade. The specific criteria that will be used to assess your presentation are outlined below (and on your benchmark rubric).

Benchmark Timeline:

- 3/13 (A) & 3/14 (BD) – Project description distribution
- 3/15-3/16 Weekend – BEGIN WORK ON PROJECTS
- 3/20 (A) & 3/21 (BD) – Work period

- 3/22-3/23 Weekend – WORK ON PROJECTS
- 3/24 – Benchmark work *following* Trigonometry Quiz 5.2
- 3/25 (AB) & 3/26 (D) – Benchmark work *following* review of quiz
- 3/26 (AB) & 3/27 (D) – Work period
- 3/27 (A) & 3/28 (BD) – Work period
- 3/29-3/30 Weekend – WORK ON PROJECTS
- 3/31-4/4 – BENCHMARK PRESENTATION WEEK (See corresponding schedule below)

<u>Benchmark Teams:</u>

- Sine Sum Formula
    - A Band: Amani, Keyaira, Julian, Rae, Alyssa
    - B Band: Roberto, Karly, Jamira, Stephen, Owen, Nomi, Darya, My
    - D Band: Ruby, Melanie, Chaveliz, Lauren
- Cosine Sum Formula
    - A Band: Jamie, Jaaz, Jasir, Sadie, Mingxue
    - B Band: Nikki, Nuri, Karoline, Khari, Abou, Marlyn, Kenny, Phoenix
    - D Band: Azaria, Klarissa, Tyanna
- Cosine Difference Formula
    - A Band: Bailey, Haji, Joshua, Aaron
    - B Band: Warda, Anthony, Amy, Jackie, Pierce, Gabriel, Sabian, Sieanna
    - D Band: Monisha, Thomas, Andrew
- Law of Cosines
    - A Band: Kennedy, Dylan, Keith, Chris
    - B Band: Isaac, Kenyatta, Liza, Micah, Brandon, Haneef, Ryan
    - D Band: Michelle, Eric, Jalisa

<u>Benchmark Assessment:</u>

- Is the presented derivation accurate?
- Are mathematical arguments in the derivation presented in logical order?
- Does the presenter reach appropriate conclusions from one step in the proof to the next?
- Is the presenter's focus sharp and to the point?
- How well does the presenter engage the audience?
- Is the presenter's response to questions competent?
- Is there use of creativity? Is the presentation original and/or beyond expectations?

<u>Benchmark Grading:</u>

- Exceeds Expectations: A Level – 95%
    - Design – Derivation arguments are structured in logical order with appropriate conclusions across all stages of the process.
    - Knowledge – All derivation steps are presented without error and *without* referencing notes. Effective and focused delivery of the proof demonstrates mastery of concepts and techniques.
    - Application – Benchmark identities are chosen (*without* referencing notes) and applied correctly to

solve provided problems without error.
    - Process – In-class research work time is maximized. Leadership is observed in collaboration with team. Oral presentation is completed with ease, and keeps well within the 5-minute limit.
    - Presentation – Professional communication skills are observed during 1:1 meeting. Engagement with audience is maintained. Delivery of all relevant information is efficient and does *not* require the use of reference aides (notes, help from teacher, etc.).
- Meets Expectations: B Level – 85%
    - Design – Derivation arguments are structured in logical order with appropriate conclusions across all stages of the process, perhaps with one misstep or flaw in sequencing along the way.
    - Knowledge – All derivation steps are presented without error, perhaps with one misstep (*without* referencing notes) or *all* steps are presented without error (*with* referencing notes *only once*). Effective and focused delivery of the proof demonstrates solid understanding of concepts and techniques.
    - Application – Benchmark identities are chosen (*without* referencing notes) and applied correctly to solve provided problems, perhaps with one minor error. Otherwise, identities are chosen and applied correctly (*with* notes) to solve provided problems with *no* errors.
    - Process – In-class research work time is utilized well. Collaboration with team is observed to a high degree. Oral presentation is strong, and keeps within the 5-minute limit.
    - Presentation – Strong communication skills are observed during 1:1 meeting. Engagement with audience is maintained. Delivery of all relevant information is satisfied, perhaps with *one* reference to notes or teacher guidance at one stage in the process.
- Approaches Expectations: C Level – 75%
    - Design – Derivation arguments are generally structured with appropriate conclusions, though perhaps with 2-3 missteps or flaws in sequencing along the way.
    - Knowledge – Derivation is presented with 2-3 errors (*without* referencing notes) or is presented without error (*with* referencing notes *more than once*). Assisted construction of the proof suggests moderate understanding of concepts and techniques.
    - Application – Benchmark identities are chosen (*without* referencing notes) and applied correctly to solve provided problems with some errors. Otherwise, identities are chosen and applied correctly (*with* notes) to solve provided problems with 1-2 *minor* errors.
    - Process – In-class research work time is not always utilized well. Collaboration with team is not always observed. Oral presentation is completed with some rough spots along the way and/or is not completed within the 5-minute limit.
    - Presentation – Many communication skills are observed during 1:1 meeting, perhaps with some room for practice. Engagement with audience is not always maintained. Delivery of all relevant information is somewhat incomplete and/or requires *more than one* reference to notes or teacher guidance throughout in the process.
- Not Yet Approaches Expectations: D Level – 65%
    - Design – Derivation arguments are poorly organized and/or lead to incorrect conclusions.
    - Knowledge – Derivation is presented with many errors (*with or without* referencing notes), suggesting minimal understanding of concepts and techniques.
    - Application – Benchmark identities are chosen (*with or without* referencing notes), and/or applied to solve provided problems incorrectly.
    - Process – In-class research work time is not utilized well. Collaboration with team is mostly not observed. Oral presentation is completed with difficulty and is not completed within the 5-minute limit.
    - Presentation – Communication skills are weak during 1:1 meeting and/or engagement with audience is not maintained. Delivery of all relevant information is incomplete even *with* reference to notes or teacher guidance throughout in the process.
- Below Expectations: F Level 1 – 55%

- Design – Derivation arguments are not organized and/or conclusions are not reached.
- Knowledge – Derivation is mostly incorrect (*with or without* referencing notes), suggesting very low understanding of concepts and techniques.
- Application – Benchmark identities are chosen (*with or without* referencing notes), and applied incorrectly to solve provided problems incorrectly.
- Process – Most in-class research work time is not utilized well. Collaboration with team is not observed. Oral presentation is left incomplete and/or terminated.
- Presentation – Communication skills are very weak during 1:1 meeting and engagement with audience is poor. Delivery of all relevant information is incomplete even *with* reference to notes or teacher guidance throughout in the process.

- Below Expectations: F Level 2 – 25%
    - Benchmark appointment was never scheduled or not attended
    - In-class work research time was utilized poorly.
    - Team collaboration was minimal and/or generally unproductive.
    - Unable to solve provided problems requiring the use of provided benchmark identities.
- Not Assessed/No Evidence to Score: F Level 3 – 0%
    - Benchmark appointment was never scheduled or not attended
    - In-class research work time was not utilized.
    - Team collaboration was not observed.
    - No attempt to solve provided problems requiring the use of benchmark identities.