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B.Sc. (Hons.) Mathematics (Semester : 2nd)
ANALYSIS-II
Subject Code: BMATS1203
Paper ID: 19131208

Time: 03 Hours **Maximum Marks: 60**

Instructions for candidates:

1. Section A is compulsory. It consists of 10 parts of two marks each.
2. Section B consists of 5 questions of 5 marks each. The student has to attempt any 4 questions out of it.
3. Section C consists of 3 questions of 10 marks each. The student has to attempt any 2 questions out of it.

Section – A **(2 marks each)**

Q1. Attempt the following:

a. Let $f(x) = x$ for $x\in[0,\ 1]$ and let $P = \{0, \frac{1}{3}, \frac{2}{3}, 1\}$ be a partition of $[0,\ 1]$. Then Compute $U(P,\ f)$ and $L(P,\ f)$.

b. Using limit of sum, show that $\int_{1}^{2} 2x + 3dx = 6$.

c. Using the trapezoidal rule, find the area enclosed by the function $f(x) = x$ between $x = 0$ to $x = 3$ with 3 intervals.

d. Show that the sequence $< f_n >$ where $f_n = \frac{x^n}{n}$, $0\leq x\leq 1$ converges uniformly to 0.

e. State Dini's theorem.

f. Show that if 0 < r < 1, then the series $\sum_{n=1}^{\infty} r^n \cos cos\ nx$ is uniformly convergent.

g. Test for uniformly convergence and continuity for the sequence $< f_n >$ where

$$f_n(x) = x^n,\ 0\leq x\leq 1.$$

h. Show that the sequence $< f_n >$ where $f_n(x) = \frac{nx}{1+n^2x^2}$, $0\leq x\leq 1$ cannot be differentiated term by term at $x = 0$.

i. Let X be a non-empty set and $d: X\times X\rightarrow R$ is metric on X. The show that $d': X\times X\rightarrow R$ defined by $d'(x, y) = \{1,\ d(x, y)\}$ is also a metric on X.

j. Let F be a subset of a metric space $(X,\ d)$. Prove that the set of limit points of F is closed subset of $(X,\ d)$.

Section – B **(5 marks each)**

Q2. Show that a bounded function $f: [a,\ b] \rightarrow R$ is integrable on $[a,\ b]$ if f is continuous on $[a,\ b]$.

Q3. State and prove first mean value theorem for Riemann integration.

Q4. Show that if $f_n(x)$, be continuous and converge uniformly on [a, b]. Then show that $\int_{a}^{b} f_n(x)dx\ = \int_{a}^{b} f_n(x)\ dx$.

Q5. If the metric space M has the Heine-Borel property, then prove that M is compact.

Q6. Show that a subset A of a metric space X is open if and only if its complement Ac is closed.

Section – C **(10 marks each)**

Q7. If f is Riemann integrable on $[a,\ b]$ and m, M are the infimum and supremum of f in $[a,\ b]$, then show that $\int_{a}^{b} f(x)dx\ =\ \mu(b\ -\ a)$ where $\mu\in [m,\ M]$.

Q8. State and prove bounded convergence theorem.

Q9. Assume that $f : X \to Y$ is a continuous function between two metric spaces. If $K \subset X$ is compact, then show that $f(K)$ is a compact subset of Y.