The Division Progression and Teacher Notes

For Hands-on Division with Fractions for Student Understanding Curriculum

Overview and Goals

The *Hands-on Division with Fractions for Student Understanding* curriculum is designed to enhance students' conceptual understanding of division with fractions through real-world contexts and hands-on and visual models. It explores modeling division with fractions for grades four through eight. The concept of division (and its two meanings) can be hard for many students, and division with fractions can be an added struggle. This curriculum provides students a chance to explore hands-on methods of solving division problems with fractions. The purpose is to promote students' mathematical reasoning, problem-solving, and critical thinking skills. The curriculum is designed to implement manipulatives and real-world problems as well as align with the Common Core State Standards for Mathematics. This project provides two different methods that can be used to model division with fractions which include connecting fraction tiles and an online virtual manipulative, *Polypad*. The curriculum is based on *The Division Progression*, which is a progression of 8 student objectives (details below). The curriculum is in the form of 8 handouts (one for each objective) to be utilized by instructors and students in a classroom setting. Each handout includes example problems, homework items, and an assessment item.

The handouts and teacher information documents are provided as Google docs. This is intended to be a supplemental curriculum.

The Division Progression

The Division Progression is a listing of 8 objectives involving division when fractions are involved (as part of a problem statement, answer, or both). Each objective is a problem type. This list is prepared from the Common Core State Standards for Mathematics (CCSSM) and the CCSSM standard numbers are listed. Grades 4 through 6 are listed. However, considering the challenging nature of these problems, these problem types should probably be used frequently through the rest of middle school and into high school.

Assumptions

- When we say "fractions," it could be a fraction or a mixed number.
- We want *both* meanings of division (and students should have seen Group Size Unknown and Number of Groups Unknown before diving into the objectives below). For the objectives below, one meaning is often a bit easier than the other, but students should gain experience with both meanings.
- We want context. The goal is to help students connect (a) real-world context (problem), (b) meaning of the operation, (c) diagram, (d) verbal explanation, (e) number sentence and answer to the problem.

- The teacher and the students will be doing multiplication (and probably addition and subtraction) hand-in-hand with the division questions. Our purpose here is to enhance understanding of the division questions.
    - The 8 handouts are designed as supplementary curriculum. They do not replace a fuller curriculum.

Student Objectives

4th Grade

1. Divide whole numbers with a remainder, leading to a fraction. 4.OA.3

5th Grade

2. Connecting fractions with division. Whole divided by a whole. Answer: boom! 5.NF.3
3. Unit fraction divided by a whole number. 5.NF.7abc
4. Whole number divided by a unit fraction. 5.NF.7abc

6th Grade 6.NS.1

5. Fraction (not a unit fraction) divided by a whole number.
6. Whole number divided by a (non-unit) fraction.
7. Fraction divided by a fraction that comes out to a whole number.
8. Fraction divided by a fraction that comes out to a fraction.

Teacher notes for The Division Progression

Objective	Teacher notes
1. Divide whole numbers with a remainder, leading to a fraction. 4.OA.3	Each meaning presents it's own challenge. ● In <u>group size unknown</u>, there will be left-over items (from the resource) which need to be divided into fractional parts. ● In <u>number of groups unknown</u> the challenge is handling the remainder as a *partial group*. For number of groups unknown, simple counters can be used, because they don't need to be divided into fractional parts.
2. Connecting fractions with division. Whole divided by a whole. Answer: Boom! 5.NF.3	● The problems themselves are the same as Objective 1. ● The key here is turning the division into a fraction *and* turning it into a mixed number.

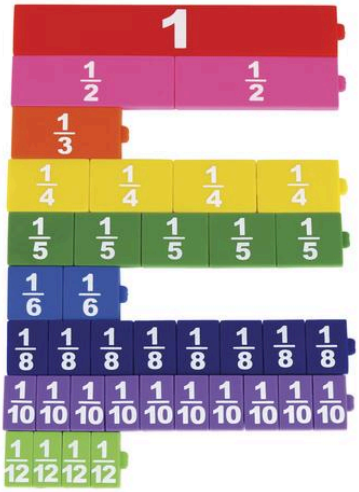
3. Unit fraction divided by a whole number. 5.NF.7abc	● Group size unknown is somewhat easier. ● Number of groups unknown is often harder, because because we have a small fraction of a group (and not even one group).
4. Whole number divided by a unit fraction. 5.NF.7abc	● Number of groups unknown is somewhat *easier*. ● Group size unknown is *harder*, because we do not have a full group. What we are given is a fraction of a group (not even one group). That is, we start with a fraction of a group.*
5. Fraction (not a unit fraction) divided by a whole number. 6.NS.1	● Number of groups unknown may be harder because the remainder will be a partial group. To determine what fraction of a group it is, compare it to a whole group.
6. Whole number divided by a (non-unit) fraction. 6.NS.1	Each meaning presents it's own challenge. ● In group size unknown, the number of groups (given) contains a fraction, so you will have a partial group. To 'fairly divide,' fewer are put in the partial group than a whole group. For example, if the partial group is ½, for every 2 in a whole group put 1 in the half group.* ● In number of groups unknown the challenge is handling the remainder as a partial group. Compare the remainder to a full group. For example, if you have ⅖ and a full group is ⅗, then you have ⅔ of a group.*
7. Fraction divided by a fraction that comes out to a whole number. 6.NS.1	Same notes as Objective 6.
8. Fraction divided by a fraction that comes out to a fraction. 6.NS.1	Same notes as Objective 6.

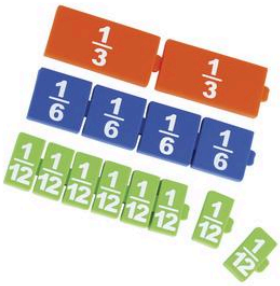
More information about the Connecting Fraction Tiles (physical manipulative) and Polypad Fraction Bars can be found on the following pages.

Connecting Fraction Tiles Notes

We may want to convert (trade), for example, 3 wholes into 24 eighths. Therefore:

1. You'll need *more than one set*. 8 eighths is not enough, and
2. It may be best to *sort* the tiles *by type*. For example, have a bag of eighths.

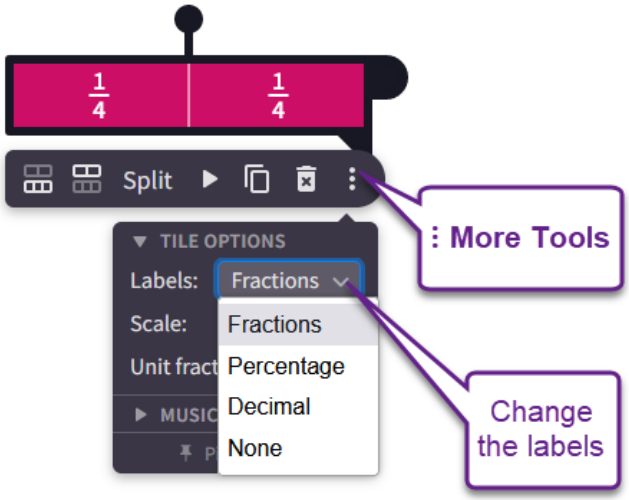




*** Polypad Fraction Bar notes on the Next Page ***

Polypad Notes - Fraction Bars

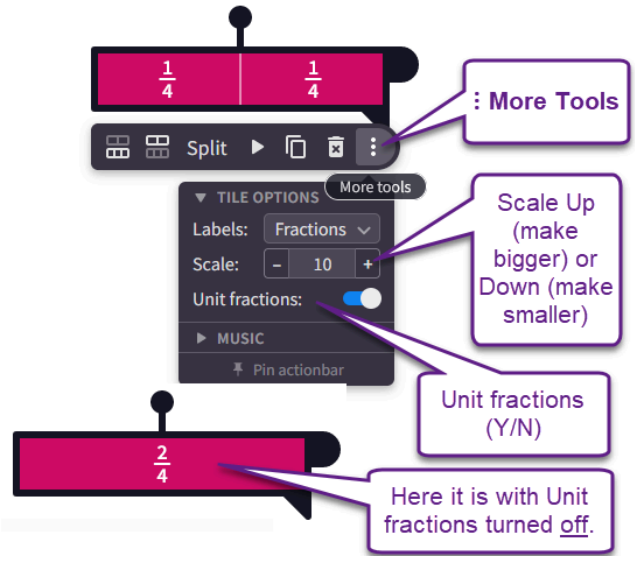
We prefer the Fraction Bars over the Fraction Circles.



1. *Split* and *Merge* - General processes in *Polypad.* This is Ungroup and Group objects. For a fraction bar, you'll probably Split first. Note for Merge: Even if the fraction pieces are *not lined up,* Merge will line them up!

2. *Rename* - both directions. To a larger denominator (more pieces) or a smaller denominator (fewer pieces), if possible.

3. More Tools -
   a. Change the labels (to percentage, for example)
   b. Change the scale. Changing the scale is nice so that you fraction bars can fit on the page. (However, do make them all the *same scale*. ;-)
   c. Unit fractions - toggle on or off. Converts, for example $\frac{1}{4}$ and $\frac{1}{4}$ to $\frac{2}{4}$.



See also:

- Comments on the Two Meanings of Division - A few comments on *Group size unknown* and *Number of groups unknown*
- Key Take-Aways document - This describes what students learn from the problems in 8 objectives of the progression.
- Curriculum handouts

Page by Jim Olsen, JR-Olsen@wiu.edu
Updated: 9-24-24