

2021-22 NCTS Undergraduate Research Project

Title: Inverse scattering transform and Riemann-Hilbert Problem in the integrable system

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Students:

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Background and Motivation:

In the context of inverse scattering transform, Zakharov and Manakov studied the large-time asymptotic behaviors of solutions to nonlinear Schrodinger equation through an oscillatory Riemann–Hilbert problem. It is called nonlinear steepest descent method now. Deift and Zhou later provided a detailed rigorous proof to calculate the large-time asymptotic behaviors of the nonlinear evolution equations in integrable system. This approach has been successfully applied in random matrix and orthogonal polynomial. Also, the numerical Riemann-Hilbert problem is developed in recent years.

In this project, we study the inverse scattering transform for Kdv equation and nonlinear Schrodinger equation. We also study the linear steepest descent method, and some elementary properties and methods for scalar Rieamn-Hilbert problem. In the end of this project, we study the Deift-Zhou's nonlinear steepest descent method for kdv equation.

Plan:

2021/10-2021/12, Stage 1: From Inverse Scattering transform to Riemann-Hilbert Problem([1] [3])

We study how to obtain the Riemann-Hilbert Problem from inverse scattering transform and its rigorous argument.

2021/01-2022/02 Stage 2: Asymptotic Analysis and Riemann-Hilbert Problem ([2], [6])

We study the linear steepest descent method and its application for Airy's equation. Also, we study some basic properties for scalar Riemann-Hilbert problem.

2022/03-2022/06, Stage 3: Deift-Zhou's nonlinear steepest descent method for kdv equation([4],[5])

In this stage, we follow [4][5] to study how to apply the Deift-Zhou's approach to obtain the asymptotic behavior of kdv equation. Among others, we study the factorization of Riemann-Hilbert problem and linear complex differential equations and its connection formula.

Reference:

- [1] M. Ablowitz and H. Segur. Solitons and the inverse scattering transform. Society for Industrial and Applied Mathematics, 1981.
- [2] M. Ablowitz and A. Fokas. Complex variables: introduction and applications. Cambridge University Press, 2003.
- [3] P. Deift and E. Trubowitz. "Inverse scattering on the line." Communications on Pure and Applied Mathematics 32.2 (1979): 121-251.
- [4] P. Deift and X. Zhou, A steepest descent method for oscillatory Riemann-Hilbert problems. Asymptotics for the MKdV equation, Annals of Mathematics 137 (2), 295-368
- [5] I. Egorova, Z. Gladka, and V. Kotlyarov, G. Teschel, Long-time asymptotics for the Korteweg-de Vries equation with steplike initial data, with Nonlinearity 26, 1839-1864 (2013)
- [6] P. Miller, Applied asymptotic analysis. Vol. 75. American Mathematical Soc., 2006.

