**MARKING SCHEME FOR BASIC APPLIED MATHEMATICS PRE- NATIONAL EXAMINATION**

1 (a) (i) 0.9929 (02marks)

(ii) 0.9917 ( 02marks)

(b) 0.4167 (03marks)

(c) $x_1$ =0.14286 ( 01marks)f

$x_2$ = -0.10526 (01marks)

$x_3$ =0.07692 ( 01marks)

2 (a) (i)Given that f(x)= $x^2 + bx + c$ and $g(x) = \frac{3}{x+2} : f(2) = 12$ and $fog(1) = 6$

$$f(2)=4+2b+c$$
$$12=4+2b+c$$
$$2b+c=8.........\,.........\,.........\,.........\,.........\,........(i)(01marks)$$
$$fog(x)=f(g(x))$$
$$=f\left(\frac{3}{x+2}\right)$$
$$=\left(\frac{3}{x+2}\right)^2+b\left(\frac{3}{x+2}\right)+c(01marks)$$
$$fog(1)=\left(\frac{3}{3}\right)^2+b\left(\frac{3}{3}\right)+c$$
$$6=1+b+c$$
$$b+c=5.........\,.........\,.........\,.........\,.........\,..........\,....(ii)(01marks)$$
$$solving\ (i)and(ii)$$
$$b=3(0.5marks)$$
$$c=2(0.5marks)$$
$$2(a)(ii)\,fog(x)=\left(\frac{3}{x+2}\right)^2+b\left(\frac{3}{x+2}\right)+c$$
$$fog(7)=\left(\frac{3}{9}\right)^2+3\left(\frac{3}{9}\right)+2$$
$$=\left(\frac{1}{3}\right)^2+3\left(\frac{1}{3}\right)+2$$
$$=\frac{1}{9}+1+2$$
$$=3\frac{1}{9}$$
$$=\frac{28}{9}(01marks)$$
$$2(b)(i)Givenf\ (x)=]\frac{2x+3}{x-1}$$

Vertical asymptote: $x-1=0$ (0.5marks)

$X=1$

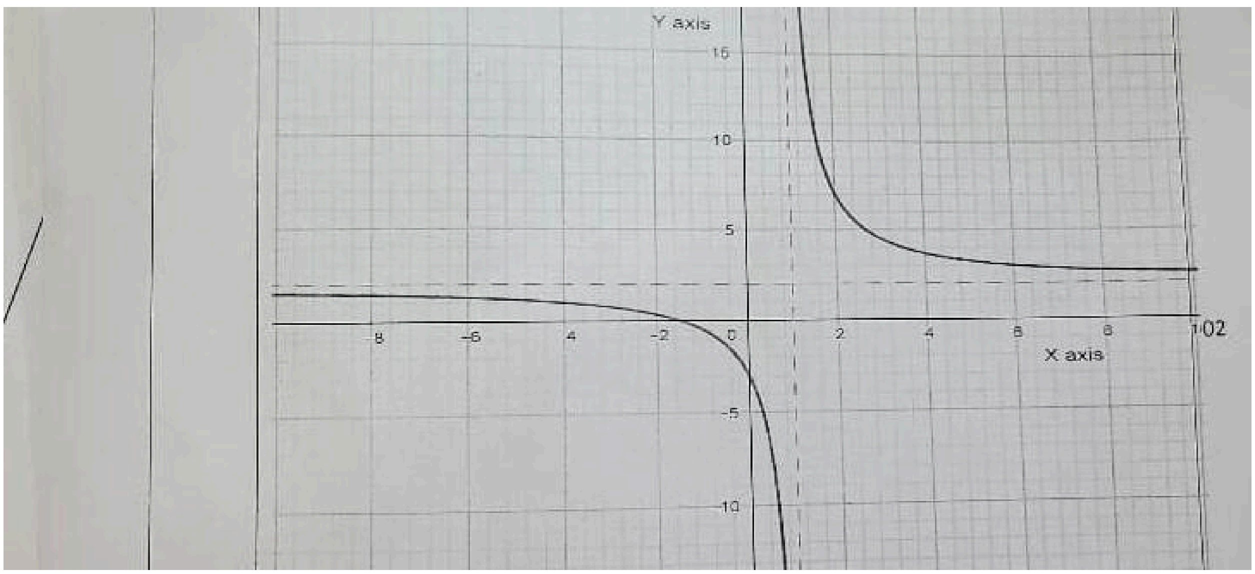
$$y = \frac{2 + \frac{3}{x}}{1 - \frac{1}{x}}$$

Horizontal asymptote: as $x \to \infty$, y=2 (0.5marks)

Intercepts: When x =0 , y=-3 (01marks)

When y=0, x= $\frac{-3}{2}$

A sketch of graph of $f(x) = \frac{2x+3}{x-1}$




2(b) (ii) Domain = All real numbers except 1 (0.5marks)

Range = All real numbers except 2 (0.5marks)

3(a) Let the number be x and y

A.M= $\frac{x+y}{2} = 34$ and G.M= $\sqrt{xy}$ =16

$$x + y = 64 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(i)$$
$$xy = 256 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(ii)$$

Therefore (0.5marks)

From (i) y=64-x, substitute in equation(ii)

$x(68-x)=256$
$68x-x^2-256=0$
$x^2-68x+256=0$
$(x-4)(x-64)=0$
$x=4 or x=64$ (01marks )

And y=64 or y=4

The numbers are 4 and 64 (0.5marks)

3(b) $A_2, A_5, A_{11}$ are in G.P i.e $G_1 = A_2, G_2 = A_5, and G_3 = A_{11}$

$$\frac{G_2}{G_1}=\frac{G_3}{G_2} \Rightarrow \frac{A_5}{A_1}=\frac{A_{11}}{A_5} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (0.5marks)$$
$$\frac{A_1+4d}{A_1+d}=\frac{A_1+10d}{A_1+4d}$$
$$(A_1+4d)^2=(A_1+d)(A_1+10d)$$
$$A_1^2+8A_1d+16d^2=A_1^2+11A_1d+10d^2 \ldots\ldots\ldots\ldots (01marks)$$
$$3A_1d=6d^2$$
$$A_1=2d \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (0.5marks)$$
$$r=\frac{A_1+4d}{A+d_1}=\frac{2d+4d}{2d+d}$$
$$=\frac{6d}{3d}$$
$$r=2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (01marks)$$
$$3(b(ii))A_1=2d \Rightarrow d=\frac{A_1}{2} given A_1=8$$
$$d=4 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (01marks)$$
$$3(b)(iii)S_n=\frac{G_1(r^n-1)}{r-1} \ldots\ldots\ldots\ldots\ldots\ldots (0.5marks)$$
$$But G_1=A_2=A_1+d=8+4=12 \ldots\ldots\ldots\ldots\ldots (0.5marks)$$
$$S_n=\frac{12(2^6-1)}{2-1}$$
$$S_n=12\times 63$$
$$S_n=756 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (01marks)$$

=

3(c ) Let W to represent number of workers and M for number of months

$$W \alpha \frac{1}{M}$$

$$W = \frac{k}{M}$$

$$k = MW$$

$$k = 12 \times 58 = 696 \text{.......... .......... .......... .......... .......... .......... ..........} (01 marks)$$

$$from W = \frac{k}{M}$$

$$= \frac{696}{8} \text{.......... .......... .......... .......... .......... .......... .......... .......... ........} (0.5 marks)$$

$$w = 87$$

Therefore the number of workers added =87-58=29……………………………………………..(0.5 marks)

$$f'(x) = \lim_{\to 0} \frac{f(x+h) - f(x)}{h} \text{...........................................................................} (0.5 marks)$$

$$= \lim_{h \to 0} \frac{\frac{1}{(x+h)^2} - \frac{1}{x^2}}{h}$$

$$\lim_{h \to 0} \frac{x^2 - (x+h)^2}{hx^2 (x+h)^2} \text{...........................................................................} (01 marks)$$

$$= \lim_{h \to 0} \frac{x^2 - (x^2 + 2hx + h^2)}{hx^2 (x+h)^2}$$

$$= \lim_{h \to 0} \frac{-2xh - h^2}{hx^2 (x+h)^2}$$

$$= \lim_{h \to 0} \frac{-2x - h}{x^2 (x+h)^2} \text{...........................................................................} (01 marks)$$

$$as h \to 0,$$

$$f'(x) = \frac{-2x}{x^2 (x)^2}$$

$$f'(x) = \frac{-2}{x^3} \text{...........................................................................} (01 marks)$$

4(b) Given $y^2 + 2xy - 3x^2 - 5 = 0$

$$\frac{d}{dx}\left(y^2 + 2xy - 3x^2 - 5\right) = \frac{d}{dx}(0)\text{.......................................................}(01marks)$$

$$2y\frac{dy}{dx} + 2\left(y + x\frac{dy}{dx}\right) - 6x = 0\text{.......................................................}(01marks)$$

$$(2y + 2x)\frac{dy}{dx} = 6x - 2y$$

$$\frac{dy}{dx} = \frac{6x - 2y}{2x + 2y}\text{.......................................................}(01.5marks)$$

$$\frac{dy}{dx} = \frac{3x - y}{x + y}$$

$$\frac{dy}{dx} = \frac{3(1) - 2}{1 + 2} = \frac{1}{3}\text{..........................................}(01marks)$$

at the poiunt (1,2)

$$C(x) = x^3 - 6x^2 + 9x + 15$$

4 (C ) $m\arg inal \cos tC'(x) = 3x^2 - 12x + 9\text{..........................................}(01marks)$

At optimum cost $C'(x) = 0$

$$0 = 3x^2 - 12x + 9$$

$$0 = x^2 - 4x + 3 \Rightarrow x = 1 or x = 3$$

$$C''(x) = 6x - 12$$

$$x = 1, C''(x) = 6 - 12 = -6\text{..........................................}(01marks)$$

$$When x = 3, C''(x) = 18 - 12 = 6$$

When

Therefore the total cost is maximum when x=1 and its minimum when x=3 ………………………

$01marks$

5 (a) (I ) Given $\int x^3\sqrt{x^4 + 5}dx$

$$u = x^4 + 5$$

$$du = 4x^3 dx \Rightarrow \frac{1}{4} du = x^3 dx \text{..........................................................................} (01 marks)$$

$$\int \left(\sqrt{x^4 + 5}\right) x^3 dx = \int \sqrt{u} \frac{du}{4}$$

$$= \frac{1}{4} \int u^{\frac{1}{2}} du$$

$$\frac{2}{4} \left( \frac{u^{\frac{3}{2}}}{3} \right) + c$$

Let

$$= \frac{\left(x^4 + 5\right)^{\frac{3}{2}}}{6} + c \text{..............................................................................................} (01 marks)$$

$$\int_0^1 \left(x^3 + 4\right)^2 dx = \int_0^1 \left(x^6 + 8x^3 + 16\right) dx$$

5(a ) (ii)

$$= \left[ \frac{x^7}{7} + \frac{8x^4}{4} + 16x \right]_0^1 \text{.....................................................} (01 marks)$$

$$= \left( \frac{1}{7} + 2 + 16 \right) - 0$$

$$= 18\frac{1}{7}$$

$$= 18.143 \text{...................................................................} (01 marks)$$

$$\int_2^4 \left( \frac{4x^6 - 2x^4 + p}{x^3} \right) dx = 225$$

5(b) Given

$$\left(4x^3 - 2x + px^{-3}\right)dx = 225$$

$$\left[\frac{4x^4}{4} - \frac{2x^2}{2} + \frac{px^{-2}}{-2}\right]_2^4 \text{..................................................}(01marks)$$

$$\int_2^4 \left(256 - 16 - \frac{p}{32}\right) - \left(16 - 4 - \frac{p}{8}\right) = 225 \text{..................................................}(01marks)$$

$$228 + \frac{3p}{32} = 225$$

$$\frac{3p}{32} = -3$$

$$p = -32 \text{..................................................}(01marks)$$

5(c ) $V = \prod\int_a^b y^2 dx$

$$= \prod\int_0^2 \left(x^2 + 3\right)^2 dx \text{..................................................}(01marks)$$

$$= \prod \int_0^2 \left(x^4 + 6x^2 + 9\right)dx$$

$$= \prod\left[\frac{x^5}{5} + \frac{6x^3}{3} + 9x\right]_0^2 \text{..................................................}(01marks)$$

$$\prod\left(\frac{32}{5} + 16 + 18\right) - 0$$

$$= \prod(40.4)$$

$$V = 126.92 cubicunits \text{..................................................}(01marks)$$

7(a ) A=3

N= 2

Total letters = 8

Number of ways of arranging TANZANIA $= \frac{8!}{3!2!}$ ..................................$(01marks)$

=3360 ways ……………………………..(01 marks)

7(b) (I) $P(RR) = P(R) \times P(R)$..............................................$(01marks)$

$$= \frac{3}{5} \times \frac{2}{4}$$

$$= \frac{3}{10} \text{.........................................................} (01 marks)$$

$$P(RW or WR) = P(RW) + P(WR) \text{...........................................} (01 marks)$$

=

$$= \frac{3}{5} \times \frac{2}{4} + \frac{2}{5} \times \frac{3}{4}$$

$$= \frac{3}{10} + \frac{3}{10}$$

$$= \frac{3}{5} \text{...............................................................} (01 marks)$$

7(c )(i ) $P(A \cup B) = P(A) + P(B)$..................................................$(01 marks)$

$$= \frac{2}{5} + \frac{1}{5}$$

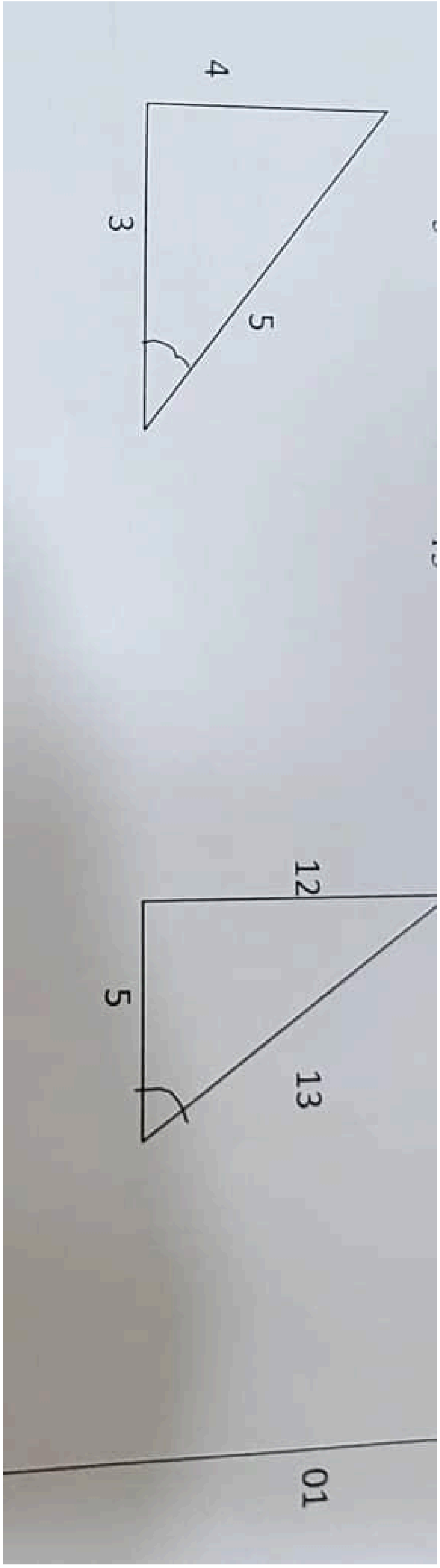
$$= \frac{3}{5} \text{..........................................................................} (01 marks)$$

(ii) $P(A \cap B) = P(A) \times P(B)$.................................................................$(01 marks)$

$$= \frac{2}{5} \times \frac{1}{5}$$

$$= \frac{2}{25} \text{.......................................................................} (01 marks)$$

8 (a ) Given $SinA = \frac{4}{5}$ and $CosB = \frac{5}{13}$


4
3
5
12
5
13

$\Rightarrow \tan A = \frac{4}{3}$ and $\tan B = \frac{12}{5}$ ..............................................................$(01 marks)$

$\Rightarrow \tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B} = \frac{\frac{4}{3}+\frac{12}{5}}{1-\frac{4}{3}\times\frac{12}{5}}$ ......... ......... ......... ......... ......... .........$(01 marks)$

$\frac{56}{15} \div \frac{-11}{5} = \frac{56}{15} \times \frac{5}{-11}$

$\therefore \tan(A+B) = \frac{-56}{33} = -1.697$......................................$(01 marks)$

8 (b ) $\sqrt{\frac{1+\cos\vartheta}{1-\cos\theta}} = \sqrt{\frac{(1+\cos\theta)(1+\cos\theta)}{(1-\cos\theta)(1+\cos\theta)}} = \frac{1+\cos\theta}{\sqrt{1-\cos^2\theta}}$ ......... ......... ......... ......... ..........$(01 marks)$

$\frac{1+\cos\theta}{\sqrt{\sin^2\theta}}$......................................................$(01 marks)$

$= \frac{1}{\sin\theta} + \frac{\cos\theta}{\sin\vartheta}$

$= \cos ec\theta + \cot\theta proved$......... ......... ......... ..........$(01 marks)$

$1+\cos^2\theta = 2\sin^2\theta \Rightarrow 1+\cos\theta = 2(1-\cos^2\theta)$

8(c )

$1+\cos\theta = 2 - 2\cos^2\theta$

$2\cos^2\theta + \cos\theta - 1 = 0$....................................................................$(01 marks)$

$\cos\theta = -1, or Cos\theta = \frac{1}{2}$

$\Rightarrow \cos\theta = -1, \theta = 180^\circ$...........................................................................$(01 marks)$

$\Rightarrow \cos\theta = \frac{1}{2}, \vartheta = 60^\circ, 300^\circ$

$\theta = 60^\circ, 180^\circ and 300^\circ$...........................................................................$(01 marks)$

$$A^T = \begin{pmatrix} 1 & 2 \\ 0 & -1 \end{pmatrix}$$

$$B^T = \begin{pmatrix} -2 & 3 \\ 1 & 0 \end{pmatrix}$$

$$A^T + B^T = \begin{pmatrix} 1 & 2 \\ 0 & -1 \end{pmatrix} + \begin{pmatrix} -2 & 3 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} -1 & 5 \\ 1 & -1 \end{pmatrix}$$

$$\left(A^T + B^T\right)^2 = \begin{pmatrix} -1 & 5 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} -1 & 5 \\ 1 & -1 \end{pmatrix}$$

…………………………………

$$\begin{pmatrix} 1+5 & -5-5 \\ -1-1 & 5+1 \end{pmatrix}$$

$$= \begin{pmatrix} 6 & -10 \\ -2 & 6 \end{pmatrix} \text{.......................................}(01marks)$$

=

10 let x be number of days for printer N to operate

y be number of days for printer T to operate

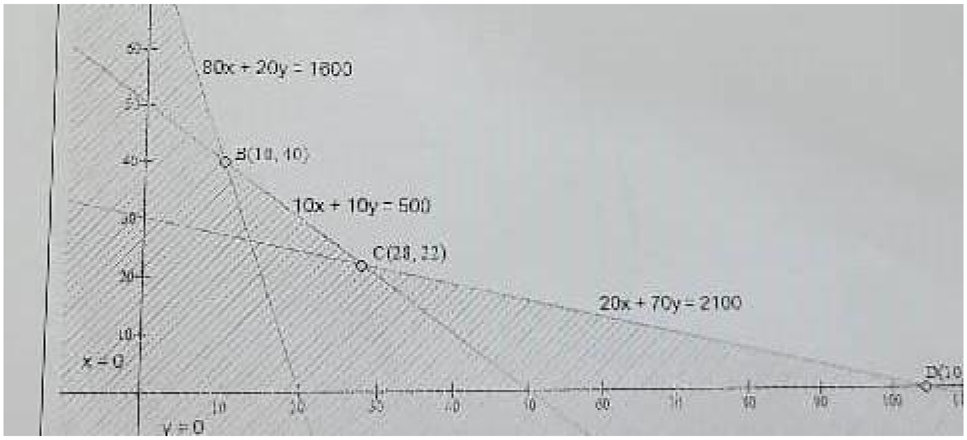
Objective function :mini $f(x, y) = 10{,}00x + 20{,}000y$..................................................$(01marks)$

$$80x + 20y \geq 1600$$
$$10x + 10y \geq 500$$
$$20x + 70y \geq 2100$$

Constraints : $x \geq 0, andy \geq 0$ ……………………………………………………..(02marks)

Table of values

| | 80x+20y=1600 | | 10x+10y=500 | | 20x+70y=2100 | |
|---|---|---|---|---|---|---|
| x | 0 | 20 | 0 | 50 | 0 | 105 |
| y | 80 | 0 | 50 | 0 | 30 | 0 |




| Corner point | F(x,y)=10000x+20000y |
|---|---|
| A(0,80) | 1600000 |
| B(10,40) | 900000 |
| C(28,22) | 720000 |
| (D(105,0) | 1050000 |
| | |

…………………………………………………………….(02marks)

To minimize cost ,printer N should operate 28 days and printer T should operate 22 days ……………………(01marks