MARKING SCHEME FOR BASIC APPLIED MATHEMATICS PRE- NATIONAL EXAMINATION

1 (a) (i) 0.9929 (02marks)

(ii) 0.9917 (02marks)

(b) 0.4167 (03marks)

(c) x_1 =0.14286 (01marks)f

x_2 = -0.10526 (01marks)

x_3 =0.07692 (01marks)

2 (a) (i)Given that f(x)= $x^2 + bx + c$ and $g(x) = \frac{3}{x+2} : f(2) = 12$ and $fog(1) = 6$

$f(2) = 4 + 2b + c$

$12 = 4 + 2b + c$

$2b + c = 8 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (i)(01marks)$

$fog(x) = f(g(x))$

$= f\left(\frac{3}{x+2}\right)$

$= \left(\frac{3}{x+2}\right)^2 + b\left(\frac{3}{x+2}\right) + c(01marks)$

$fog(1) = \left(\frac{3}{3}\right)^2 + b\left(\frac{3}{3}\right) + c$

$6 = 1 + b + c$

$b + c = 5 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (ii)(01marks)$

$solving\ (i) and\ (ii)$

$b = 3(0.5marks)$

$c = 2(0.5marks)$

$2(a)(ii)\ fog(x) = \left(\frac{3}{x+2}\right)^2 + b\left(\frac{3}{x+2}\right) + c$

$fog(7) = \left(\frac{3}{9}\right)^2 + 3\left(\frac{3}{9}\right) + 2$

$= \left(\frac{1}{3}\right)^2 + 3\left(\frac{1}{3}\right) + 2$

$= \frac{1}{9} + 1 + 2$

$= 3\frac{1}{9}$

$= \frac{28}{9}(01marks)$

$2(b)(i) Given f(x) =] \frac{2x+3}{x-1}$

Vertical asymptote: x-1=0 (0.5marks)

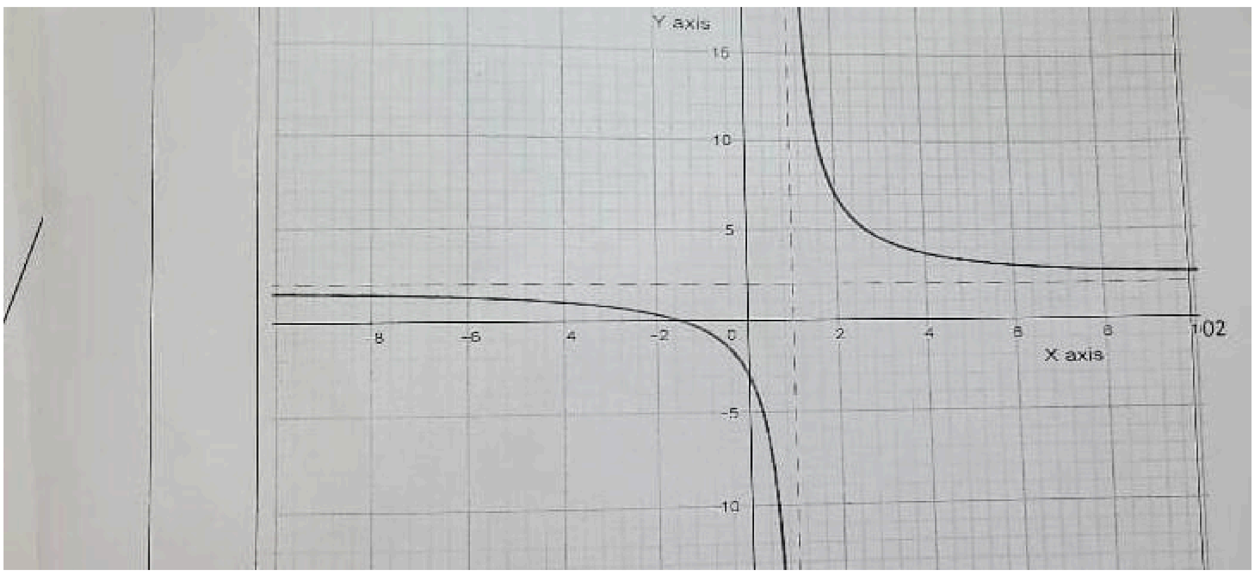
X=1

Horizontal asymptote: $y=\dfrac{2+\frac{3}{x}}{1-\frac{1}{x}}$ as $x \to \infty$, y=2 (0.5marks)

Intercepts: When x =0 , y=-3 (01marks)

When y=0, x= $\dfrac{-3}{2}$

A sketch of graph of $f(x)=\dfrac{2x+3}{x-1}$



2(b) (ii) Domain = All real numbers except 1 (0.5marks)

Range = All real numbers except 2 (0.5marks)

3(a) Let the number be x and y

A.M= $\dfrac{x+y}{2}=34$ and G.M= $\sqrt{xy}$ =16

$$x+y=64 \text{...} (i)$$
$$xy=256 \text{...} (ii)$$

Therefore (0.5marks)

From (i) y=64-x, substitute in equation(ii)

$x(68-x)=256$

$68x-x^2-256=0$

$x^2-68x+256=0$

$(x-4)(x-64)=0$

$x=4 or x=64$ (01marks)

And y=64 or y=4

The numbers are 4 and 64 (0.5marks)

3(b) A_2, A_5, A_{11} are in G.P i.e $G_1=A_2, G_2=A_5, and G_3=A_{11}$

$\frac{G_2}{G_1}=\frac{G_3}{G_2}\Rightarrow\frac{A_5}{A_1}=\frac{A_{11}}{A_5}$..... .... $(0.5marks)$

$\frac{A_1+4d}{A_1+d}=\frac{A_1+10d}{A_1+4d}$

$(A_1+4d)^2=(A_1+d)(A_1+10d)$

$A_1^{\,2}+8A_1d+16d^2=A_1^{\,2}+11A_1d+10d^2$..... ....$(01marks)$

$3A_1d=6d^2$

$A_1=2d$..... ....$(0.5marks)$

$r=\frac{A_1+4d}{A+d_1}=\frac{2d+4d}{2d+d}$

$=\frac{6d}{3d}$

$r=2$..... ....$(01marks)$

$3(b(ii))A_1=2d\Rightarrow d=\frac{A_1}{2}\, given A_1=8$

$d=4$..... ....$(01marks)$

$3(b)(iii)S_n=\frac{G_1(r^n-1)}{r-1}$..... ....$(0.5marks)$

$But G_1=A_2=A_1+d=8+4=12$..... ....$(0.5marks)$

$S_n=\frac{12(2^6-1)}{2-1}$

$S_n=12\times 63$

$S_n=756$..... ....$(01marks)$

=

3(c) Let W to represent number of workers and M for number of months

$$W \alpha \frac{1}{M}$$
$$W = \frac{k}{M}$$
$$k = MW$$
$$k = 12 \times 58 = 696 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (01 marks)$$
$$from W = \frac{k}{M}$$
$$= \frac{696}{8} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (0.5 marks)$$
$$w = 87$$

Therefore the number of workers added =87-58=29…………………………………………..(0.5 marks)

$$f'(x) = \lim_{\to 0} \frac{f(x+h) - f(x)}{h} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (0.5 marks)$$
$$= \lim_{h \to 0} \frac{\frac{1}{(x+h)^2} - \frac{1}{x^2}}{h}$$
$$\lim_{h \to 0} \frac{x^2 - (x+h)^2}{hx^2 (x+h)^2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (01 marks)$$
$$= \lim_{h \to 0} \frac{x^2 - (x^2 + 2hx + h^2)}{hx^2 (x+h)^2}$$
$$= \lim_{h \to 0} \frac{-2xh - h^2}{hx^2 (x+h)^2}$$
$$= \lim_{h \to 0} \frac{-2x - h}{x^2 (x+h)^2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (01 marks)$$
$$as h \to 0,$$
$$f'(x) = \frac{-2x}{x^2 (x)^2}$$
$$f'(x) = \frac{-2}{x^3} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (01 marks)$$

4(b) Given $y^2 + 2xy - 3x^2 - 5 = 0$

$$\frac{d}{dx}\left(y^2+2xy-3x^2-5\right)=\frac{d}{dx}(0)\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(01marks)$$

$$2y\frac{dy}{dx}+2\left(y+x\frac{dy}{dx}\right)-6x=0\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(01marks)$$

$$(2y+2x)\frac{dy}{dx}=6x-2y$$

$$\frac{dy}{dx}=\frac{6x-2y}{2x+2y}\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(01.5marks)$$

$$\frac{dy}{dx}=\frac{3x-y}{x+y}$$

$$\frac{dy}{dx}=\frac{3(1)-2}{1+2}=\frac{1}{3}\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(01marks)$$

at the poiunt (1,2)

$$C(x)=x^3-6x^2+9x+15$$

4 (C) $$marginal\ cost\ C'(x)=3x^2-12x+9\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(01marks)$$

At optimum cost $C'(x)=0$

$$0=3x^2-12x+9$$

$$0=x^2-4x+3\Rightarrow x=1 or x=3$$

$$C''(x)=6x-12$$

$$x=1, C''(x)=6-12=-6\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(01marks)$$

$$When x=3, C''(x)=18-12=6$$

When

Therefore the total cost is maximum when x=1 and its minimum when x=3 ……

$01marks$

5 (a) (I) Given $\int x^3\sqrt{x^4+5}dx$

$u = x^4 + 5$

$du = 4x^3 dx \Rightarrow \frac{1}{4} du = x^3 dx$.....(01marks)

$\int \left(\sqrt{x^4 + 5}\right) x^3 dx = \int \sqrt{u} \frac{du}{4}$

$= \frac{1}{4} \int u^{\frac{1}{2}} du$

$\frac{2}{4}\left(\frac{u^{\frac{3}{2}}}{3}\right) + c$

$= \frac{\left(x^4 + 5\right)^{\frac{3}{2}}}{6} + c$.....(01marks)

Let

$$\int_0^1 \left(x^3 + 4\right)^2 dx = \int_0^1 \left(x^6 + 8x^3 + 16\right) dx$$

5(a) (ii)

$= \left[\frac{x^7}{7} + \frac{8x^4}{4} + 16x\right]_0^1$.....(01marks)

$= \left(\frac{1}{7} + 2 + 16\right) - 0$

$= 18\frac{1}{7}$

$= 18.143$.....(01marks)

$$\int_2^4 \left(\frac{4x^6 - 2x^4 + p}{x^3}\right) dx = 225$$

5(b) Given

$\left(4x^{3}-2x+px^{-3}\right)dx=225$

$\left[\frac{4x^{4}}{4}-\frac{2x^{2}}{2}+\frac{px^{-2}}{-2}\right]_{2}^{4}$..$(01 marks)$

$\int_{2}^{4}\left(256-16-\frac{p}{32}\right)-\left(16-4-\frac{p}{8}\right)=225$..$(01 marks)$

$228+\frac{3p}{32}=225$

$\frac{3p}{32}=-3$

$p=-32$..$(01 marks)$

5(c) $V=\prod\int_{a}^{b}y^{2}dx$

$=\prod\int_{0}^{2}\left(x^{2}+3\right)^{2}dx$..$(01 marks)$

$=\prod\int_{0}^{2}\left(x^{4}+6x^{2}+9\right)dx$

$=\prod\left[\frac{x^{5}}{5}+\frac{6x^{3}}{3}+9x\right]_{0}^{2}$..$(01 marks)$

$\prod\left(\frac{32}{5}+16+18\right)-0$

$=\prod(40.4)$

$V=126.92 cubic units$..$(01 marks)$

7(a) A=3

N= 2

Total letters = 8

Number of ways of arranging TANZANIA $=\frac{8!}{3!2!}$....................................$(01 marks)$

=3360 ways ……………………………..(01 marks)

7(b) (I) $P(RR)=P(R)\times P(R)$...$(01 marks)$

$$= \frac{3}{5} \times \frac{2}{4}$$

$$= \frac{3}{10} \text{..} (01 marks)$$

$$P(RW or WR) = P(RW) + P(WR) \text{..} (01 marks)$$

=

$$= \frac{3}{5} \times \frac{2}{4} + \frac{2}{5} \times \frac{3}{4}$$

$$= \frac{3}{10} + \frac{3}{10}$$

$$= \frac{3}{5} \text{..} (01 marks)$$

7(c)(i) $P(A \cup B) = P(A) + P(B)$...(01$marks$)

$$= \frac{2}{5} + \frac{1}{5}$$

$$= \frac{3}{5} \text{..} (01 marks)$$

(ii) $P(A \cap B) = P(A) \times P(B)$..(01$marks$)

$$= \frac{2}{5} \times \frac{1}{5}$$

$$= \frac{2}{25} \text{..} (01 marks)$$

8 (a) Given $SinA = \frac{4}{5}$ and $CosB = \frac{5}{13}$

4
3
5
12
5
13
01

$\Rightarrow \tan A = \frac{4}{3}$ and $\tan B = \frac{12}{5}$ .....(01 marks)

$\Rightarrow \tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B} = \frac{\frac{4}{3} + \frac{12}{5}}{1 - \frac{4}{3} \times \frac{12}{5}}$ ....(01 marks)

$\frac{56}{15} \div \frac{-11}{5} = \frac{56}{15} \times \frac{5}{-11}$

$\tan(A+B) = \frac{-56}{33} = -1.697$.....(01 marks)

8 (b) $\sqrt{\frac{1+\cos\vartheta}{1-\cos\theta}} = \sqrt{\frac{(1+\cos\theta)(1+\cos\theta)}{(1-\cos\theta)(1+\cos\theta)}} = \frac{1+\cos\theta}{\sqrt{1-\cos^2\theta}}$ ....(01 marks)

$\frac{1+\cos\theta}{\sqrt{\sin^2\theta}}$.....(01 marks)

$= \frac{1}{\sin\theta} + \frac{\cos\theta}{\sin\vartheta}$

$= \cos ec\theta + \cot\theta proved$ ....(01 marks)

$1 + \cos^2\theta = 2\sin^2\theta \Rightarrow 1 + \cos\theta = 2(1 - \cos^2\theta)$

8(c)

$1 + \cos\theta = 2 - 2\cos^2\theta$

$2\cos^2\theta + \cos\theta - 1 = 0$.....(01 marks)

$\cos\theta = -1, or Cos\theta = \frac{1}{2}$

$\Rightarrow \cos\theta = -1, \theta = 180^\circ$.....(01 marks)

$\Rightarrow \cos\theta = \frac{1}{2}, \vartheta = 60^\circ, 300^\circ$

$\theta = 60^\circ, 180^\circ and 300^\circ$.....(01 marks)

$$A^T = \begin{pmatrix} 1 & 2 \\ 0 & -1 \end{pmatrix}$$

$$B^T = \begin{pmatrix} -2 & 3 \\ 1 & 0 \end{pmatrix}$$

$$A^T + B^T = \begin{pmatrix} 1 & 2 \\ 0 & -1 \end{pmatrix} + \begin{pmatrix} -2 & 3 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} -1 & 5 \\ 1 & -1 \end{pmatrix}$$

$$\left(A^T + B^T\right)^2 = \begin{pmatrix} -1 & 5 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} -1 & 5 \\ 1 & -1 \end{pmatrix}$$

……

$$\begin{pmatrix} 1+5 & -5-5 \\ -1-1 & 5+1 \end{pmatrix}$$

$$= \begin{pmatrix} 6 & -10 \\ -2 & 6 \end{pmatrix} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (01 marks)$$

=

10 let x be number of days for printer N to operate

y be number of days for printer T to operate

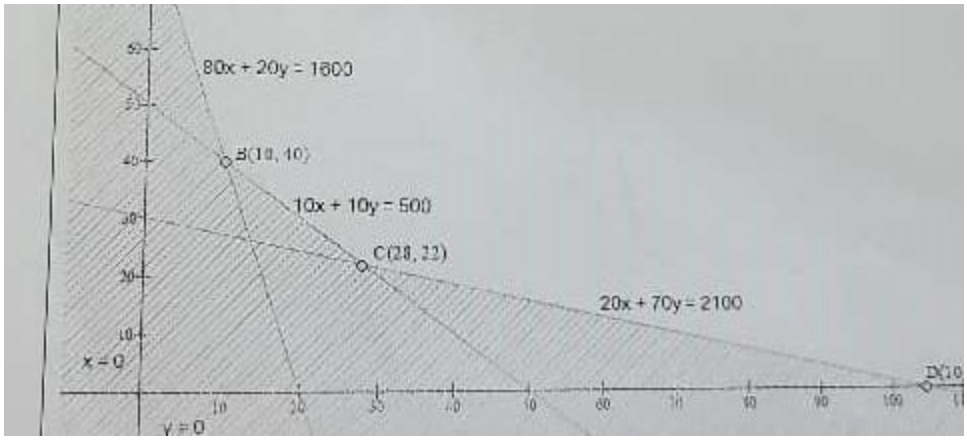
Objective function :mini $f(x, y) = 10{,}00x + 20{,}000y \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (01 marks)$

$$80x + 20y \geq 1600$$
$$10x + 10y \geq 500$$
$$20x + 70y \geq 2100$$

Constraints : $x \geq 0, and y \geq 0$ ……..(02marks)

Table of values

	80x+20y=1600		10x+10y=500		20x+70y=2100	
x	0	20	0	50	0	105
y	80	0	50	0	30	0



Corner point	F(x,y)=10000x+20000y
A(0,80)	1600000
B(10,40)	900000
C(28,22)	720000
(D(105,0)	1050000

……..(02marks)

To minimize cost ,printer N should operate 28 days and printer T should operate 22 days

……..(01marks