# Derivative and Integral Rules
# For Basics and Trig Functions

Here are useful rules to help you work out the derivatives of many functions (with examples below). Note: the little mark ′ means **derivative of**, and f and g are functions.

| Common Functions | Function | Derivative |
|---|---|---|
| Constant | $c$ | $0$ |
| Line | $x$ | $1$ |
| | $ax$ | $a$ |
| Square | $x^2$ | $2x$ |
| Square Root | $\sqrt{x}$ | $(½)x^{-½}$ |
| Exponential | $e^x$ | $e^x$ |
| | $a^x$ | $\ln(a)\, a^x$ |
| Logarithms | $\ln(x)$ | $1/x$ |
| | $\log_a(x)$ | $1 / (x \ln(a))$ |
| Trigonometry (x is in radians) | $\sin(x)$ | $\cos(x)$ |
| | $\cos(x)$ | $-\sin(x)$ |
| | $\tan(x)$ | $\sec^2(x)$ |
| Inverse Trigonometry | $\sin^{-1}(x)$ | $1/\sqrt{(1-x^2)}$ |
| | $\cos^{-1}(x)$ | $-1/\sqrt{(1-x^2)}$ |
| | $\tan^{-1}(x)$ | $1/(1+x^2)$ |

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| Rules | Function | Derivative |
|---|---|---|
| Multiplication by constant | $cf$ | $cf'$ |
| Power Rule | $x^n$ | $nx^{n-1}$ |
| Sum Rule | $f + g$ | $f' + g'$ |
| Difference Rule | $f - g$ | $f' - g'$ |
| Product Rule | $fg$ | $f\,g' + f'\,g$ |
| Quotient Rule | $f/g$ | $\frac{f'\,g - g'\,f}{g^2}$ |
| Reciprocal Rule | $1/f$ | $-f'/f^2$ |
| | | |
| Chain Rule (as "Composition of Functions") | $f \circ g$ | $(f' \circ g) \times g'$ |
| Chain Rule (using ′ ) | $f(g(x))$ | $f'(g(x))g'(x)$ |
| Chain Rule (using $\frac{d}{dx}$ ) | $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$ | |

# Derivative and Integral Rules
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| Common Functions | Function | Integral |
|---|---|---|
| Constant | $\int a\,dx$ | $ax + C$ |
| Variable | $\int x\,dx$ | $x^2/2 + C$ |
| Square | $\int x^2\,dx$ | $x^3/3 + C$ |
| Reciprocal | $\int (1/x)\,dx$ | $\ln\|x\| + C$ |
| Exponential | $\int e^x\,dx$ | $e^x + C$ |
| | $\int a^x\,dx$ | $a^x/\ln(a) + C$ |
| | $\int \ln(x)\,dx$ | $x\ln(x) - x + C$ |
| Trigonometry (x in radians) | $\int \cos(x)\,dx$ | $\sin(x) + C$ |
| | $\int \sin(x)\,dx$ | $-\cos(x) + C$ |
| | $\int \sec^2(x)\,dx$ | $\tan(x) + C$ |
| | | |
| **Rules** | **Function** | **Integral** |
| Multiplication by constant | $\int cf(x)\,dx$ | $c\int f(x)\,dx$ |
| Power Rule ($n \neq -1$) | $\int x^n\,dx$ | $\frac{x^{n+1}}{n+1} + C$ |
| Sum Rule | $\int (f + g)\,dx$ | $\int f\,dx + \int g\,dx$ |
| Difference Rule | $\int (f - g)\,dx$ | $\int f\,dx - \int g\,dx$ |

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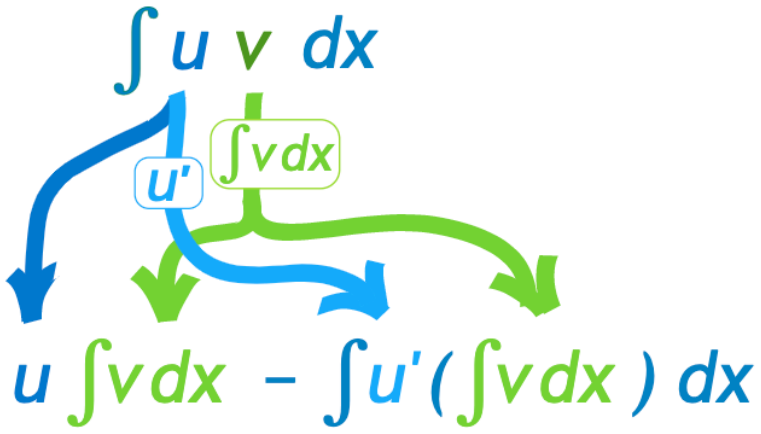
## Integration by Parts

$$\int u\,v\,dx = u\int v\,dx - \int u'\left(\int v\,dx\right)dx$$

- **u** is the function u(x)
- **v** is the function v(x)
- **u'** is the derivative of the function u(x)

The rule as a diagram:



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## Integration by Substitution

The first and most vital step is to be able to write our integral in this form:

$$\int f(g(x))\ g'(x)\ dx$$

Note that we have **g(x)** and its derivative **g'(x)**

Like in this example:

$$\int \cos(x^2)\ 2x\ dx$$

Here **f=cos**, and we have **g=x²** and its derivative **2x**
This integral is good to go!

When our integral is set up like that, we can do **this substitution**:

$$\int f(g(x))\ g'(x)\,dx$$

$$\int f(u)\ du$$

Then we can **integrate f(u)**, and finish by **putting g(x) back as u**.

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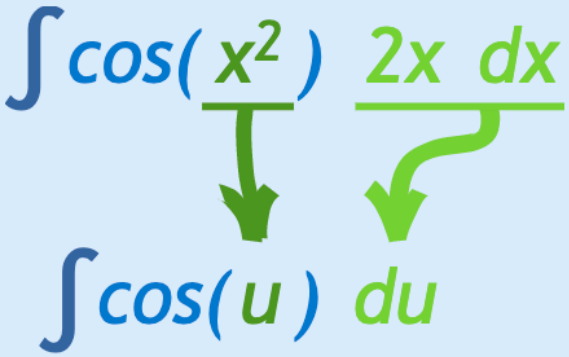
We know (from above) that it is in the right form to do the substitution:




Now integrate:

$$\int \cos(u)\, du = \sin(u) + C$$

And finally put $\mathbf{u=x^2}$ back again:

$$\sin(x^2) + C$$