

# Superconductivity theory

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### *1. Ginzburg-Landau theory*

Free energy and the Ginzburg-Landau equations. Current and gradient invariance. Quantization of magnetic flux. Penetration depth and coherence length. Proximity effect. S-N interface energy and two types of superconductors. Critical field and critical current of a thin film.

### *2. Type II superconductivity: basics*

Solution of GL equations for a single vortex and the lower critical field. Abrikosov solution for a vortex lattice and the upper critical field. Interaction of vortices (in the London approximation). Vortex in a thin film.

### *3. Type II superconductivity: fluctuation effects*

Fluctuations near the transition temperature (estimate of the width of the fluctuation region, diamagnetism, paraconductivity). Berezinskii-Kosterlitz-Thouless transition in a thin film.

### *4. Weak superconductivity: phenomenology*

Stationary Josephson effect. Types of weak contacts. Non-stationary Josephson effect. Resistive characteristics. Critical current and "return current". Tunnel junction in a magnetic field. Josephson vortices. SQUIDs.

### *5. Fluctuation effects in weak contacts*

Thermal fluctuations in a Josephson junction, phase slip and  $I(V,T)$ . Macroscopic quantum effects: "phase tunneling" and transition to a resistive state. Macroscopic quantum coherence in SQUIDs.

### *6. Gor'kov Functions and the Ginzburg-Landau Functional:*

#### *Derivation by the Functional Integral Method*

Statistical Sum as an Integral over Grassmann Variables. Functional Integral for a Fermi Gas with Attraction in an Arbitrary Channel. Effective Action and Gor'kov Functions. Expansion in Powers of  $\Delta$ : Local Terms. Gradient Term and Correlation Length.

### *7. Temperature and magnetic field effects*

Self-consistency equation for  $\Delta$  and its solution at low temperatures;  $\Delta(T)$  for nontrivial pairing. Heat capacity and spin susceptibility of a superconductor. Paramagnetic limit for a singlet superconductor. Diamagnetic response, field penetration depth and its temperature dependence.

### *8. Inhomogeneous superconducting states*

Bogolyubov - de Gennes equations. Andreev reflection and Andreev levels in S-N-S contact. Localized electron levels in the center of Abrikosov vortex.

### *9. Superconductors with impurities*

Potential impurities and their influence on usual s-pairing. Dependence of critical magnetic fields on disorder. Spin-orbit impurity scattering and paramagnetic limit. Violation of

T-invariance and depairing factor. Suppression of superconductivity by magnetic field. Gapless superconductivity. Destruction of "unusual" superconducting phases by potential impurities.

*10. Microscopic Theory of Weak Superconducting Contacts*

Binding energy in S-I-S contact and critical current. Temperature dependence of current, Ambegaokar-Baratov formula for symmetric S-I-S. Andreev levels and supercurrent in S-N-S contacts. Beenakker formula for short contact. Analysis of limiting cases of high and low transparency. Thermal coherence length, introduction to the method of semiclassical Green's functions. Contact of superconductors through a ferromagnet (S-F-S) and phase inversion ( $\pi$ -contact).

*11\*. Eliashberg theory (not sure it will fit in allowed time)*

Hamiltonian of the electron-phonon interaction and the Dyson equations. Formal solution for Green's functions, spectral representation and averaging over the Fermi surface. Integral equation for the order parameter  $\Delta(\omega)$  and analysis of its solution. The role of the Coulomb interaction. Simple application: isotope effect in superconductivity.