The Shortest Path Problem

Background:
The Shortest Path Problem is almost what it sounds like:

\[ \text{Given a graph of nodes and edges, where the weights of the edges represent some cost (distance, time, money, etc.), find the lowest-cost path between any two nodes.} \]

Typically, we think of the shortest path problem like a map with roads between towns or different points in a big city and we’re trying to find the shortest/fastest way to get from one place to another. (This is what map apps that give you driving directions do.) But the shortest path problem doesn’t have to represent traveling anywhere at all. You could use a graph of a social network to figure out how closely connected you are to some other person, for example, how many friend connections separate me from the president of the United States?

Example:
On the Internet, of course, we’re interested in finding the fastest way to route information from one place to another. The graph below shows the connections between routers on a small segment of a network, and the amount of time (in milliseconds) it takes a packet to get between them.

If we are in charge of router A, we want to know the fastest way to get information from router A to router C. But, just because A is directly connected to C doesn’t mean it’s the fastest way to get information there. We need to find the shortest path.

The shortest path from A to C is highlighted.

Try It Out!
For all of the examples below, you’re also going to try to find the shortest path from A to C.
The graphs all look the same, but warning: the weights are different. You are encouraged to mark up these diagrams as you go.

...but you should be thinking...

- **Remember**: in computer science, “solving a problem” doesn’t mean finding an answer to an instance of a problem; it means finding an algorithm that might be able to solve any instance of that problem.
- As you look at the graphs on the next page, your brain is working to find a solution. You might think you’re just trying “random stuff” but you’re not. You are using your human intelligence to help you.
- **Think about your own thinking process.**
- **Could you express what you are thinking to solve this problem as an algorithm?**
**Directions**

- **FIRST**, find the shortest path from A to C for each of the graphs below. Highlight the path and make a note of the total distance.
- When you're done, compare with a partner to see if you found the same things.
- **NEXT**, after you have identified the shortest path from A to C, go back and find the shortest path from A to B, D, and E.
- In the “Algorithms Notes” area jot down a few ideas for how an algorithm to find the shortest path might work. Maybe make a few notes about what's potentially tricky: what things do you want to be sure to remember?

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<tr>
<th>Graph 1:</th>
<th>Graph 2:</th>
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