VCE School Assessed Coursework: SAC

Students Name:

Sacred Heart College Yarrawonga

VCE Study:	Further Mathematics
Unit:	4
Outcomes:	1, 2 and 3
Assessment Task	Matrices SAC – Lesson 2
Date:	Friday 30th June 2018
Time:	5 mins reading 60 mins writing
Instructions:	Answer all questions in the spaces provided.
Conditions:	Silent, individual work
Permitted Materials:	Pens, Pencils, Ruler, Eraser, TIInspire CAS calculator, Bound Notes
Marks allocated:	29 marks

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the room.

I understand I must not intentionally or unintentionally disclose any details on this SAC or imply what is or is not included, or in any way gain an unfair advantage for myself over other students. If I do, I understand that disciplinary action will occur and my result will be

downgraded. In fairness to fellow students it is my responsibility to inform the VCE office if I am aware that information about the SAC is being passed on, or that a student has gained unfair advantage.

Question 1 (5 marks)

Tungamah and Rennie have completed an 18 game season. The results were:

Rennie (R): 8 wins, 2 draws and 8 losses

Tungamah (T): 10 wins, 1 draw and 7 losses

For a win they received four (4) points, a draw two (2) points and a loss one (1) point.

(a) Represent the season results as a 2 x 3 matrix

$$P = \begin{matrix} \\ R \\ T \end{matrix} \begin{bmatrix} & & \\ & & \end{bmatrix} \begin{matrix} W & D & L \end{matrix}$$

(1 mark)

(b) Represent the points awarded as a 3 x 1 matrix.

$$Q = \begin{bmatrix} \\ \\ \\ \end{bmatrix} \begin{matrix} W \\ D \\ L \end{matrix}$$

(1 mark)

(c) Is it possible to find the product matrix QP?

(1 mark)

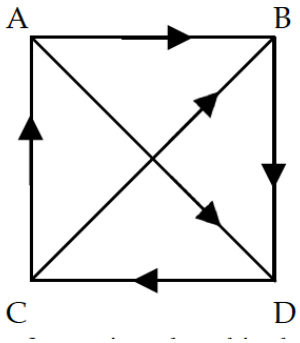
(d) Calculate the total premiership points for each team by calculating PQ.

(2 marks)

Question 2 (4 marks)

During the season, the coach of the Bears divided his team into four squads – named for the captain of each squad **(Andy, Brett, Chas and Donny)** – and ran a series of head-to-head competitive tests. The competition results are summarised in the directed graph below.

Each arrow shows the winner of a session played in the competition. For example, the arrow from C to A shows that Chas' squad defeated Andy's squad.



In the competition, each squad was given a ranking that was determined by calculating the sum of their one-step and two-step dominance.

a) Construct a **one-step dominance matrix** for this competition, with rows representing winners and columns representing losers.

		LOSER			
		A	**B**	**C**	**D**
WINNER	**A**	[			]
	B				
	C				
	D				

(1 mark)

b) Construct a **two-step** **inance** matrix for this competition.

(1 mark)

c) Construct a matrix showing the sum of the one-step and two-step matrices in order to determine the ranking of the squads.

(1 mark)

Finishing Place	Name
1st	
2nd	
3rd	
4th	

(1 mark)

Question 3 (4 marks)

Historical records of the first season for the Bears and Hoppers in 1975 showed they played each other three times and the number of tickets sold for those matches was summarised as follows.

Match	Adults	Children	Pensioners	Total Match Takings $
Round 2	245	310	76	720.40
Round 9	120	44	0	311.00
Round 16	321	410	102	945.80

However no records of ticket prices were found.

(a) Three simultaneous equations were set up to find the ticket prices charged back in 1975.

Complete the final equation, given

***x* = adult ticket prices**

***y* = children ticket prices**

***z* = pensioner ticket prices**

First Equation: $245x + 310y + 76z = \$720.40$

Second Equation: $120x + 44y = \$311.00$

Third Equation:

(1 mark)

(b) Write the three simultaneous equations in matrix form.

(1 mark)

(c) Determine the prices of the tickets in 1975 using your matrix equation from **(b)**.

Give the resultant matrix and then list the prices of each ticket.

(2 marks)

Question 4 (8 marks)

The town folks were constantly changing allegiance for their two local teams when clearly one team performed far better than the other team in a season. Over the years the degree of change was determined and can be summarised as follows.

If Bears was the better performing team
90% of Bears' supporters remained Bears supporters the following season.
40% of Hopper's supporters became Bears supporters the following year.

If Hoppers was the better performing team
95% of Hoppers supporters remained Hoppers supporters the following season.
30% of Bears supporters became Hoppers supporters the following year.

In the town of 20 000 supporters, assume the supporters initially are **equally divided** between the two teams.

(a) Write down the initial state matrix concerning supporter numbers for each team.

$$S_0 = [\ \ ] \begin{matrix} B \\ H \end{matrix}$$

(1 mark)

Now consider **that Bears were to perform better than Hoppers** for a three year period.

(b) Write the transition matrix for when **Bears are the better side.**

$$\begin{matrix} & B & H \\ B & & \\ H & & \end{matrix} \begin{bmatrix} & \\ & \end{bmatrix}$$

(1 mark)

(c) Set up an appropriate matrix equation and find the number of **supporters for the two teams** after **three (3) years** of the Bears performing better than the Hoppers.

(1 mark)

Now consider that **Hoppers were to perform better than Bears** for a three year period.

(d) Write the transition matrix for when the Hoppers are the better side.

$$\begin{matrix} & B & H \\ B & & \\ H & & \end{matrix} \begin{bmatrix} & \\ & \end{bmatrix}$$

(1 mark)

(e) Set up an appropriate matrix equation and find the number of supporters for the two teams after three (3) years of the Hoppers performing better than the Bears.

(1 mark)

(f) Which team would in the long term have the largest supporter base if they continued to perform better?

(3 marks)

Question 5 (8 marks)

The gym at the Bears Football and Netball Club has a number of fitness activities.

The football players are advised to vary their training each week to maximise their fitness outcomes, and can choose between the treadmills (T), the weight circuit (W) or football skills training (F).

During the first week of the season, 100 players from various levels attended training. All of the players started on the treadmill. The state matrix for week 1, is:

$$W_1 = \begin{bmatrix} 100 \\ 0 \\ 0 \end{bmatrix} \begin{matrix} T \\ W \\ F \end{matrix}$$

The transition matrix showing the movement between training regimes each week is:

$$T = \begin{bmatrix} 0 & 0.2 & 0.4 \\ 0.5 & 0 & 0.6 \\ 0.5 & 0.8 & 0 \end{bmatrix} \begin{matrix} T \\ W \\ F \end{matrix} \quad To$$

$$From \quad T \quad W \quad F$$

a) Three elements in the transition matrix have been highlighted. Explain the meaning of these three elements.

(1 mark)

b) Determine the number of players who will do football skills training in the **third week** of training.

(1 mark)

c) The pattern continues. Show that in the long run, 41 players will do football skills training each week if values are rounded to the nearest whole number.

(1 mark)

The Tungamah Football and Netball Club has football teams playing at each of three levels; firsts, seconds and thirds.

Some players are promoted or dropped a level during the season and others are unavailable due to injury or return from injury.

The number of players, P_n , at each level for the nth week of one particular season is modeled by the equation :

$$P_{n+1} = \begin{bmatrix} 0.8 & 0.2 & 0 \\ 0.2 & 0.6 & 0.4 \\ 0 & 0.2 & 0.6 \end{bmatrix} \times P_n + \begin{bmatrix} -2 \\ 5 \\ -3 \end{bmatrix}, \quad P_1 = \begin{bmatrix} 40 \\ 40 \\ 40 \end{bmatrix} \begin{matrix} Firsts \\ Seconds \\ Thirds \end{matrix}$$

d) If P_1 is the amount of players available for the first week, calculate P_2, which shows the amount of players available for each level in the second week.

(1 mark)

e) How many players would be available at each level during the **third week?**

(1 mark)

f) The club would like to retain the same number of players that they had at each **level during week 2.**

$$\begin{bmatrix} -2 \\ 5 \\ -3 \end{bmatrix}$$

The matrix above represents the number of players lost or returned due to injury at that level each week.

Using the matrix $[x\ y\ z]$ instead of $[-2\ 5\ -3]$ will ensure that the numbers do not change after week 2.

What are the values of x, y and z?

(3 marks)

END OF TASK